1) If θ is a parameter, then the parametric equations of the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ are given by

(A) $x = 3 + 4\cos cos\,\theta$ and $y = 2 + 4\sin sin\,\theta$

(B) $x = 3 + 4\cos cos\,\theta$ and $y =- 2 + 4\sin sin\,\theta$

(C) $x = 3 + 4\sin sin\,\theta$ and $y = 2 + 4\cos cos\,\theta$

(D) $x =- 3 + 4\sin sin\,\theta$ and $y =- 2 + 4\cos cos\,\theta$

Topic :- Conic section - Circle

Sol^n :- We have $x^2 + y^2 - 6x + 4y - 3 = 0$

Here $g =- 3,\quad f = 2,\quad c =- 3$

∴ Centre $\equiv (-g, \; -f) \equiv (3, -2) = (h, \; k)$

Radius = $r = \sqrt{g^2 + f^2 - c} = 4$

∴ Parametric equations of circle is $x = h + r\cos cos\,\theta$ and $y = k + r\sin sin\,\theta$

$\therefore x = 3 + 4\cos cos\,\theta$ and $y =- 2 + 4\sin sin\,\theta$

Correct option :- (B)

2) If $A = [2 \; - 1 \; - 1 \; 2\,]$ such that $A^2 - 4A + 3I = 0$ then $A^{-1} =$ _____

(A) $\frac{1}{3}[2\;1\;1\;2\,]$

(B) $\frac{-1}{3}[2\;1\;1\;2\,]$

(C) $\frac{1}{3}[-\,2 \; - 1\;1 \; - 2\,]$

(D) $\frac{-1}{3}[2 \; - 1 \; - 1\;2\,]$

Topic :- Matrices

Sol^n :- We have $A^2 - 4A + 3I = 0$

On pre-multiplying by A^{-1} on both sides

$A^{-1}A - 4A^{-1}A + 3IA^{-1} = 0$

$\therefore A - 4I + 3A^{-1} = 0$

$\therefore 3A^{-1} = 4I - A$

$\therefore A^{-1} = \frac{4I - A}{3}$

$\therefore A^{-1} = \frac{1}{3}(4I - A)$

$= \frac{1}{3}\{[4\;0\;0\;4\,] - [2 \; - 1 \; - 1\;2\,]\}$

$\therefore A^{-1} = \frac{1}{3}\{[2\;1\;1\;2\,]\}$

Correct option :- (A)

3) $\int \frac{dx}{\cos\cos x \sqrt{\cos\cos 2x}} =$

(A) $\frac{1}{2}\log log \left|\tan tan \left(\frac{\pi}{4} + x\right)\right| + c$ (B) $\frac{1}{2}\log log \left|\tan tan \left(\frac{1-\tan\tan x}{1+\tan\tan x}\right)\right| + c$

(C) $2 \log log \left|\tan tan \left(\frac{1+\tan\tan x}{1-\tan\tan x}\right)\right| + c$ (D) $(\tan tan\, x) + c$

Topic :- Indefinite Integration

Sol^n :- We have $\mathrm{I} = \int \frac{dx}{\cos\cos x \sqrt{\cos\cos 2x}} = \int \frac{dx}{\cos\cos x \sqrt{x - x}}$

$$= \int \frac{dx}{\cos\cos x \sqrt{x(1-x)}}$$

$$= \int \frac{dx}{x\sqrt{1-x}}$$

$\therefore\ \mathrm{I} = \int \frac{x\, dx}{\sqrt{1-x}}$ Put $\tan tan\, x = t$ $\therefore x\, dx = dt$

$\therefore\ \mathrm{I} = \int \frac{dt}{\sqrt{1-t^2}} = (t) + c = (\tan tan\, x) + c$

Correct option :- (D)

4) The displacement of a particle at the time 't' is given by $s = \sqrt{1+t}$, then its acceleration 'a' is proportional to

(A) Cube of velocity (B) Square of velocity

(C) $\sqrt{s}$ (D) $\sqrt[3]{s}$

Topic :- Application of derivatives

Sol^n :- We have $s = \sqrt{1+t}$ $\Rightarrow$ $\therefore \frac{ds}{dt} = v = \frac{1}{2\sqrt{1+t}}$

$\therefore \frac{dv}{dt} = a = \frac{-1}{2} \times \frac{1}{(1+t)} \times \frac{1}{2\sqrt{1+t}} = \frac{-1}{4(1+t)^{\frac{3}{2}}} = \frac{-1}{4}(2v)^3$ $\therefore a \propto v^3$

Correct option :- (A)

5) Which of the following matrix is invertible ?

$A_1 = [4\ 2\ 2\ 1]$ $A_2 = [-1\ \ -2\ 3\ 4\ 5\ 7\ 2\ 4\ \ -6]$ $A_3 = [1\ 0\ 0\ 5\ 2\ 1\ 7\ 2\ 1]$ $A_4 = [1\ 0\ 1$

(A) A_1 (B) A_3

(C) A_4 (D) A_2

Topic :- Matrices

Sol^n :- We have, $A_4 = [1\ 0\ 1\ 0\ 2\ 3\ 1\ 2\ 1\ ] \quad = 1(2 - 6) - 0(0 - 3) + 1(0 - 2)$

$= 1(-4) + 0 + 1(-2) = -4 - 2 = -6$

$\therefore A_4^{-1}$ exists $\Rightarrow A_4$ is an invertible matrix

Correct option :- (C)

6) If $O\equiv(0,\ 0,\ 0),\ \ P(1,\ \sqrt{2},\ 1)$ then the acute angles made by the line OP with XOY, YOZ, ZOX planes are respectively –

(A) $60^0, 45^0, 60^0$ (B) $30^0, 30^0, 45^0$

(C) $45^0, 60^0, 30^0$ (D) $45^0, 45^0, 60^0$

Topic :- Line and Plane

Sol^n :- We have $O\equiv(0,\ 0,\ 0), \quad P(1,\ \sqrt{2},\ 1)$

$\overline{OP} = \overline{r} = \hat{i} + \sqrt{2}\hat{j} + \hat{k} \quad \Rightarrow \quad |\overline{r}| = \sqrt{1^2 + (\sqrt{2})^2 + 1^2} = \sqrt{4} = 2$

$\cos cos\ \theta_1 = \frac{\sqrt{x^2+y^2}}{r} = \frac{\sqrt{1^2+(\sqrt{2})^2}}{2} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta_1 = 30^0$

$\cos cos\ \theta_2 = \frac{\sqrt{y^2+z^2}}{r} = \frac{\sqrt{(\sqrt{2})^2+1^2}}{2} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta_2 = 30^0$

$\cos cos\ \theta_3 = \frac{\sqrt{x^2+z^2}}{r} = \frac{\sqrt{1^2+1^2}}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \theta_3 = 45^0$

Correct option :- (B)

7) If Cartesian equation of the line is $x - 1 = 2y + 3 = 3 - z$, then its vector equation is

(A) $\overline{r} = \left(\hat{i} - 3\hat{j} + 3\hat{k}\right) + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$

(B) $\overline{r} = \left(-\hat{i} - 3\hat{j} + 3\hat{k}\right) + \lambda(\hat{i} + \frac{1}{2}\hat{j} - \hat{k})$

(C) $\overline{r} = \left(\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right) + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$

(D) $\overline{r} = \left(-\hat{i} - \frac{3}{2}\hat{j} - 3\hat{k}\right) + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$

Topic :- Line and Plane

Sol^n :- $x - 1 = 2y + 3 = 3 - z$

$\therefore\ x - 1 = 2\left(y + \frac{3}{2}\right) = -(z - 3)$

$\therefore \ \frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{1}{2}} = \frac{z-3}{-1}$

Here $P(x_1,\ y_1,\ z_1) \equiv (1,\ \frac{-3}{2},\ 3)$ and d. r. s. are $1,\ \frac{1}{2},\ -1$ i. e. $2, 1, -2$

Using $\overline{r} = \overline{a} + \lambda\overline{b} = \left(\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right) + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$

Correct option :- (C)

8) The Cartesian equation of the curve given by $x = 6\cos cos\,\theta,\ y = 6\sin sin\,\theta$ is

(A) $x^2 + y^2 = 25$ (B) $x^2 + y^2 = 6$

(C) $x^2 + y^2 = 5$ (D) $x^2 + y^2 = 36$

Topic :- Conic Section - Circle

Sol^n :- Here $x = 6\cos cos\,\theta,\ y = 6\sin sin\,\theta$ Squaring and adding these equations,

$x^2 = 36\theta$ and $y^2 = 36\theta$

$x^2 + y^2 = 36(\theta + \theta) \Rightarrow x^2 + y^2 = 36$

Correct option :- (D)

9) $\int_0^{\frac{\pi}{2}} \frac{\sin sin\,x \cos cos\,x}{1 + x} dx =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{8}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Topic :- Definite Integration

Sol^n :- Here $\mathrm{I} = \int_0^{\frac{\pi}{2}} \frac{\sin sin\,x \cos cos\,x}{1 + x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin sin\,x \cos cos\,x}{1 + (x)^2} dx$

Put $x = t \Rightarrow 2\sin sin\,x\ \cos cos\,x\,dx = dt$

When $x = 0 \rightarrow t = 0$

$x = \frac{\pi}{2} \rightarrow t = 1$

$\therefore \quad \mathrm{I} = \int_0^1 \frac{\frac{dt}{2}}{1 + t^2} = \frac{1}{2}\int_0^1 \frac{dt}{1 + t^2} = \frac{1}{2}[(t)]_0^1$

$= \frac{1}{2}[(1) - (0)] = \frac{1}{2} \times \frac{\pi}{4} = \frac{\pi}{8}$

Correct option :- (B)

10) If $f(x) = 6\beta - 3\alpha x$, if $-4 \le x < -2$

$= 4x + 1$, if $-2 \le x \le 2$ is continuous on $[-4, 2]$, then $\alpha + \beta =$

(A) $\frac{4}{7}$ (B) $\frac{-4}{7}$

(C) $\frac{7}{6}$ (D) $\frac{-7}{6}$

Topic :- Continuity

Sol[n] :- Since $f(x)$ is continuous on $[-4, 2]$, it is continuous at $x = -2$

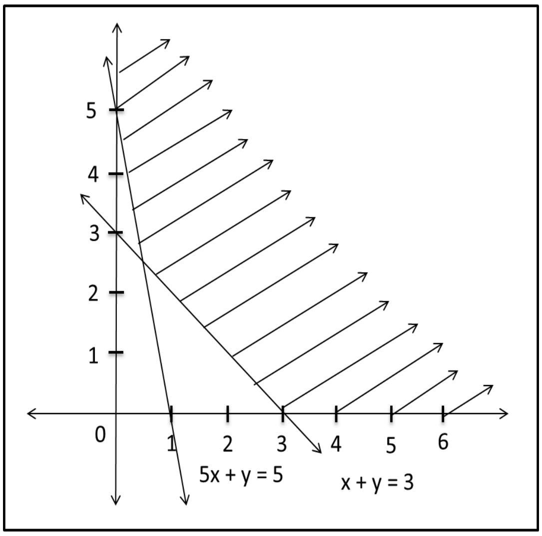
$\therefore f(x) = f(-2)$

$\therefore [6\beta - 3\alpha x] = [4x + 1]_{x=-2}$

$\therefore 6\beta + 6\alpha = -8 + 1 = -7$

$\therefore \alpha + \beta = \frac{-7}{6}$

Correct option :- (D)



11) If $Z = 7x + y$ subject to $5x + y \ge 5$, $x + y \ge 3$, $x \ge 0$, $y \ge 0$ then minimum value of Z is

(A) 5 (B) 3

(C) 6 (D) 2

Topic :- Linear Programing Problem

Sol[n] :- $5x + y \ge 5$ $x + y \ge 3$

x	1	0
y	0	5

x	3	0
y	0	3

Vertices of feasible region are $A(3, 0)$, $B(0, 5)$

and $P\left(\frac{1}{2}, \frac{5}{2}\right)$

Now $Z = 7x + y$

$\therefore Z_A = 7(3) + 0 = 21$

$\therefore Z_B = 7(0) + 5 = 5$

$\therefore Z_P = 7\left(\frac{1}{2}\right) + \frac{5}{2} = 6$ $\therefore$ Minimum value of $Z = 5$

Correct option :- (A)

12) Which of the following statement pattern is tautology ?

$S_1 = \sim p \rightarrow (q \leftrightarrow p)$ $\quad$ $S_2 = \sim p \vee \sim q$

$S_3 = (p \rightarrow q) \wedge (q \rightarrow p)$ $\quad$ $S_4 = (q \rightarrow p) \vee (\sim p \rightarrow q)$

(A) S_4 $\quad$ (B) S_3

(C) S_1 $\quad$ (D) S_2

Topic :- Mathematical Logic

p	q	$\sim p$	$q \rightarrow p$	$\sim p \rightarrow q$	$(q \rightarrow p) \vee (\sim p \rightarrow q)$
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	T

Sol^n :-

Correct option :- (A)

13) The maximum value of the function $y = e^{5 + \sqrt{3}\sin \sin x + \cos \cos x}$ is

(A) e^8 $\quad$ (B) e^2

(C) e^5 $\quad$ (D) e^7

Topic :- Application of derivatives

Sol^n :- $\quad y = e^{5 + \sqrt{3}\sin \sin x + \cos \cos x} = e^5 \times e^{\sqrt{3}\sin \sin x + \cos \cos x}$

Since maximum value of $\sqrt{3}\sin \sin x + \cos \cos x = \sqrt{(\sqrt{3})^2 + 1^2}$

$= \sqrt{3 + 1} = \sqrt{4} = 2$

$\therefore y = e^5 \times e^2 = e^7$

Correct option :- (D)

14) The area of the region bounded by the parabola $y^2 = 8x$ and its Latus rectum is

(A) $\frac{4}{3}$ sq. unit $\quad$ (B) $\frac{8}{3}$ sq. unit

(C) $\frac{32}{3}$ sq. unit $\quad$ (D) $\frac{16}{3}$ sq. unit

Topic :- Application of definite integration

Sol^n :- We have $y^2 = 8x$

$\Rightarrow \quad 4a = 8 \quad \Rightarrow \quad a = 2$

Since area of region bounded by $y^2 = 4ax$ and its latus rectum is $\frac{8}{3}a^2$

$\therefore$ Required area $= \frac{8}{3}(2)^2$

$= \frac{32}{3}$ sq. unit

Correct option :- (C)

15) The equation of plane containing the point $(1, -1, 1)$ and parallel to the plane $2x + 3y - 4z = 17$ is

(A) $\bar{r} \bullet (2\hat{i} + 3\hat{j} - 4\hat{k}) = -15$ (B) $\bar{r} \bullet (3\hat{i} + 4\hat{j} - 2\hat{k}) = -3$

(C) $\bar{r} \bullet (2\hat{i} + 3\hat{j} - 4\hat{k}) = -5$ (D) $\bar{r} \bullet (4\hat{i} + 3\hat{j} - 4\hat{k}) = -3$

Topic :- Line and Plane

Sol^n :- We have $2x + 3y - 4z = 17$

i. e. $2x + 3y - 4z - 17 = 0$

Since required plane is parallel to given plane

The equation of required plane is $2x + 3y - 4z + k = 0$

Since it passes through $(1, -1, 1)$,

$2(1) + 3(-1) - 4(1) + k = 0$

$2 - 3 - 4 + k = 0 \quad \Rightarrow \quad k = 5$

$\therefore$ The equation of required plane is $2x + 3y - 4z + 5 = 0$

i. e. $\bar{r} \bullet (2\hat{i} + 3\hat{j} - 4\hat{k}) + 5 = 0$

Correct option :- (C)

16) If $A = \{2, 4\}$, $B = \{3, 4, 5\}$ then $(A \cap B) \times (A \cup B) =$

(A) $\{(4, 2), (4, 3), (4, 4), (4, 5)\}$ (B) $\{(3, 2), (3, 4), (4, 4), (5, 4)\}$

(C) $\{(4, 3), (4, 4), (4, 5)\}$ (D) $\{(2, 3), (2, 4), (2, 5)\}$

Topic :- Set – relations

Sol^n :- $A \cap B = \{4\}$ and $A \cup B = \{2, 3, 4, 5\}$

$\therefore (A \cap B) \times (A \cup B) = \{(4, 2), (4, 3), (4, 4), (4, 5)\}$

Correct option :- (A)

17) The line through the points $(1, 4)$ and $(-5, 1)$ intersect the line $4x + 3y - 5 = 0$ in the point

(A) $(2, 1)$ (B) $(-1, 3)$

(C) $(-1, -3)$ (D) $\left(\frac{-5}{3}, \frac{5}{3}\right)$

Topic :- Straight line

Sol^n :- The equation of line passing through the points $(1, 4)$ and $(-5, 1)$ is

$\frac{x-1}{1+5} = \frac{y-4}{4-1}$ i.e. $\frac{x-1}{6} = \frac{y-4}{3}$

i.e. $x - 1 = 2y - 8$

i.e. $x - 2y + 7 = 0$ ___________(1)

This line intersect the line $4x + 3y - 5 = 0$ ___________(2)

Solving these equations, we get $x = -1, \ y = 3$

Correct option :- (B)

18) The rate of growth of bacteria is proportional to number present. If initially there were 1000 bacteria and the number double in 1 hour then the number of bacteria after $2\frac{1}{2}$ hours are____.

(Given $\sqrt{2} = 1.414$)

(A) 5056 approximately (B) 5656 approximately

(C) $400\sqrt{2}$ approximately (D) 4646 approximately

Topic :- Differential Equation

Sol^n :- Let x be the number of bacteria at any time t

Given that $\frac{dx}{dt} \propto x$ $\Rightarrow$ $\frac{dx}{dt} = kx$ where k is constant

$\therefore \frac{dx}{x} = kdt$

On integrating, $x = C \cdot e^{kt}$ ___________(1)

Initially $t = 0, \ x = 1000 \Rightarrow C = 1000$

Eq. (1) becomes, $x = 1000 \cdot e^{kt}$ ___________(2)

When $t = 1, \ x = 2000$ $\Rightarrow$ $2000 = 1000 \cdot e^{kt}$

$\Rightarrow e^{k} = 2$

When $t = 2\frac{1}{2} = \frac{5}{2}, \ x = ?$

Eq. (2) becomes, $x = 1000 \cdot e^{\frac{5}{2}k} = 1000 \cdot (e^{k})^{\frac{5}{2}}$

$= 1000\cdot(2)^{\frac{5}{2}}$

$= 1000\cdot(2^2 \times 2^{\frac{1}{2}})$

$= 4000(1.414)$

= 5656 approximately

Correct option :- (B)

19) The value of m if the vectors $\hat{i} - \hat{j} - 6\hat{k}$, $\hat{i} - 3\hat{j} + 4\hat{k}$ and $2\hat{i} - 5\hat{j} + m\hat{k}$ are coplanar is

(A) -3 (B) 3

(C) 1 (D) -1

Topic :- Vectors

Sol^n :- Since given vectors are coplanar, $|1 \ -1 \ -6 \ 1 \ -3 \ 4 \ 2 \ -5 \ m| = 0$

$\Rightarrow \quad 1(-3m + 20) + 1(m - 8) - 6(-5 + 6) = 0$

$\Rightarrow \quad -3m + 20 + m - 8 - 6 = 0$

$\Rightarrow \quad -2m =- 6$

$\Rightarrow \quad m = 3$

Correct option :- (B)

20) The probability that a person win a prize on a lottery ticket is $\frac{1}{4}$. If he purchases 5 lottery tickets at random, then the probability that he wins at least one prize is

(A) $\frac{121}{1024}$ (B) $\frac{223}{1024}$

(C) $\frac{781}{1024}$ (D) $\frac{774}{1024}$

Topic :- Binomial Distribution

Sol^n :- Here $n = 5$ and $p = \frac{1}{4}$ $\therefore q = \frac{3}{4}$

$P(X) = {}^5C_x p^x q^{n-x}$

$P(X\geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$= 1 - {}^5C_0 p^0 q^{n-0} = 1 - 1\left(\frac{3}{4}\right)^5$

$= 1 - \frac{243}{1024} = \frac{781}{1024}$

Correct option :- (C)

21) If $|3x - 2| \leq \frac{1}{2}$ then $x \in$

(A) $\left(\frac{1}{2}, \frac{5}{6}\right]$ (B) $\left[\frac{1}{2}, \frac{5}{6}\right)$

(C) $\left(\frac{1}{2}, \frac{5}{6}\right)$ (D) $\left[\frac{1}{2}, \frac{5}{6}\right]$

Topic :- Set - relation

Sol^n :- $|3x - 2| \le \frac{1}{2}$ $\Rightarrow$ $\therefore \frac{-1}{2} \le (3x - 2) \le \frac{1}{2}$

$\therefore \frac{-1}{2} + 2 \le 3x \le \frac{1}{2} + 2$ $\Rightarrow$ $\therefore \frac{3}{2} \le 3x \le \frac{5}{2}$

$\therefore \frac{1}{2} \le x \le \frac{5}{6}$ $\Rightarrow$ $x \in \left[\frac{1}{2}, \frac{5}{6}\right]$

Correct option :- (D)

22) If $\bar{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$, $\bar{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$ then value of $(2\bar{a} - \bar{b}) \bullet [(\bar{a} \times \bar{b}) \times (\bar{a} + 2\bar{b})] =$

(A) 7 (B) -7

(C) 5 (D) -5

Topic :- Vector

Sol^n :- $(2\bar{a} - \bar{b}) \bullet [(\bar{a} \times \bar{b}) \times (\bar{a} + 2\bar{b})] = (2\bar{a} - \bar{b}) \bullet [(\bar{a} \times \bar{b}) \times \bar{a} + 2(\bar{a} \times \bar{b}) \times \bar{b}]$

$= (2\bar{a} - \bar{b}) \bullet [(\bar{a} \times \bar{b}) \times \bar{a} + 2(\bar{a} \times \bar{b}) \times \bar{b}]$

$= (2\bar{a} - \bar{b}) \bullet [(\bar{a} \bullet \bar{a})\bar{b} - (\bar{a} \bullet \bar{b})\bar{a} + 2(\bar{a} \bullet \bar{b})\bar{b} - 2\bar{a}(\bar{b} \bullet \bar{b})]$

Now $\bar{a} \bullet \bar{a} = |\bar{a}|^2 = \left[\frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})\right]^2 = \frac{10}{10} = 1$

$\bar{a} \bullet \bar{b} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k}) \bullet \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k}) = 0$

$\bar{b} \bullet \bar{b} = |\bar{b}|^2 = \left[\frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})\right]^2 = 1$

$(2\bar{a} - \bar{b}) \bullet [(\bar{a} \times \bar{b}) \times (\bar{a} + 2\bar{b})] = (2\bar{a} - \bar{b}) \cdot [1\bar{b} - 0 + 2\times 0\bar{b} - 2\times 1\bar{a}$

$= (2\bar{a} - \bar{b}) \cdot [\bar{b} - 2\bar{a}]$

$= 2\bar{a} \bullet \bar{b} - 4\bar{a} \bullet \bar{a} - \bar{b} \bullet \bar{b} + 2\bar{b} \bullet \bar{a}$

$= 4\bar{a} \bullet \bar{b} - 4\bar{a} \bullet \bar{a} - \bar{b} \bullet \bar{b}$

$= 4 \cdot 0 - 4 \cdot 1 - 1 = -5$

Correct option :- (D)

23) $\int \frac{dx}{\sqrt{5 + 4x - x^2}} =$

(A) $\log log \left|(x - 2) + \sqrt{5 + 4x - x^2}\right| + c$ (B) $\log log \left|(x + 2) + \sqrt{5 + 4x - x^2}\right| + c$

(C) $\left(\frac{x-2}{3}\right) + c$ (D) $\left(\frac{x+2}{3}\right) + c$

Topic :- Indefinite Integration

Sol^n :- $I = \int \frac{dx}{\sqrt{5+4x-x^2}} = \int \frac{dx}{\sqrt{9-4+4x-x^2}}$

$= \int \frac{dx}{\sqrt{9-(x-2)^2}}$

$= \int \frac{dx}{\sqrt{9-(x-2)^2}} = \int \frac{dx}{\sqrt{3^2-(x-2)^2}}$

$\therefore I = \left(\frac{x-2}{3}\right) + c$

Correct option :- (C)

24) The statement pattern $[(p \vee q) \wedge \sim p] \wedge (\sim q)$ is

(A) equivalent to $p \vee q$ (B) a contradiction

(C) a tautology (D) a contingency

Topic :- Mathematical logic

Sol^n :- $[(p \vee q) \wedge \sim p] \wedge (\sim q) \equiv [(p \wedge \sim p) \vee (q \wedge \sim p)] \wedge (\sim q)$

$\equiv [F \vee (q \wedge \sim p)] \wedge \sim q$

$\equiv [(q \wedge \sim p)] \wedge \sim q$

$\equiv F \wedge \sim q$

$\equiv F$ = contradiction

Correct option :- (B)

25) If the equation $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$ represent a pair of lines, where λ is a real number and θ is angle between them then value of θ is

(A) 9 (B) $\frac{1}{3}$

(C) 3 (D) 10

Topic :-Pair of straight lines

Sol^n :- Given $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$

Here $a = 1$, $h = \frac{-3}{2}$, $b = \lambda$, $g = \frac{3}{2}$, $f = \frac{-5}{2}$, $c = 2$

For a pair of straight lines, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\therefore (1)(\lambda)(2) + 2\left(\frac{-5}{2}\right)\left(\frac{3}{2}\right)\left(\frac{-3}{2}\right) - (1)\left(\frac{-5}{2}\right)^2 - (\lambda)\left(\frac{3}{2}\right)^2 - (2)\left(\frac{-3}{2}\right)^2 = 0$

$\therefore 2\lambda + \frac{45}{4} - \frac{25}{4} - \frac{9\lambda}{4} - \frac{9}{2} = 0 \quad \Rightarrow \quad \lambda = 2$

Angle between pair of lines is given by $\tan tan\,\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

$$\theta = \frac{2\sqrt{\frac{-3}{2}^2 - \lambda}}{1+\lambda} = \frac{2\sqrt{\frac{9}{4} - 2}}{1+2} = \frac{1}{3}$$

$\therefore \cot cot\,\theta = 3 \quad \Rightarrow \quad \theta = 1 + \theta = 1 + 3^2 = 10$

Correct option :- (D)

26) $\int_{-5}^{5} \left[\frac{e^x + e^{-x}}{e^x - e^{-x}}\right] dx =$

(A) $2e^5$ (B) 1

(C) $3e^5$ (D) 0

Topic :-Definite Integration

Sol^n :- $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

$f(-x) = \frac{e^{-x} + e^x}{e^{-x} - e^x}$

$= \frac{e^x + e^{-x}}{-(e^x - e^{-x})}$

$= -\frac{e^x + e^{-x}}{e^x - e^{-x}} = -f(x)$

$\therefore f(x)$ is odd function

$\int_{-5}^{5} \left[\frac{e^x + e^{-x}}{e^x - e^{-x}}\right] dx = 0$ ____________ By property

Correct option :- (D)

27) The solution of differential equation $x^2 \frac{dy}{dx} = y^2 + xy$ is

(A) $\frac{y}{x} - \log log\,|x| + c$ (B) $\frac{x}{y} + \log log\,|x| + c$

(C) $\frac{x}{y} - \log log\,|x| + c$ (D) $\frac{y}{x} + \log log\,|x| + c$

Topic :- Differential equation

Sol^n :- $x^2 \frac{dy}{dx} = y^2 + xy$ __________(1)

$\therefore \frac{dy}{dx} = \frac{y^2+xy}{x^2}$

Put $y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

Equation (1) becomes,

$\therefore v + x\frac{dv}{dx} = \frac{v^2x^2 + xvx}{x^2}$

$\therefore v + x\frac{dv}{dx} = v^2 + v$

$\therefore x\frac{dv}{dx} = v^2$

$\therefore \frac{dv}{v^2} = \frac{dx}{x}$

On integrating , $\frac{-1}{v} = \log log\, x + c$

$\frac{-x}{y} = \log log\, x + c \quad \Rightarrow \quad \log log\, x + \frac{x}{y} = c$

Correct option :- (B)

28) Which of the following functions is not p. d. f. of a continuous random variable X?

F_1 is given by

$f(x) = e^{-x}$ if $0 < x < \infty$

$= 0$ Otherwise

F_2 is given by

$f(x) = \frac{1}{4} \times \frac{1}{\sqrt{x}}$ if $0 < x < 4$

$= 0$ Otherwise

F_3 is given by

$f(x) = 6x(1 - x)$ if $0 < x < 1$

$= 0$ Otherwise

F_4 is given by

$f(x) = \frac{x}{2}$ if $-2 < x < 2$

$= 0$ Otherwise

(A) F_2

(B) F_1

(C) F_4

(D) F_1

Topic :- Probability distribution

Sol^n :- $f(x) = \frac{x}{2}$ if $-2 < x < 2$

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{-2} f(x)dx + \int_{-2}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx$$

$$= \int_{-2}^{2} \frac{x}{2}dx = 0 \neq 1$$

$\Rightarrow \quad f(x)$ is not p.d.f. of continuous r. v. x

Correct option :- (C)

29) A random variable X takes the value 0, 1, 2. Its mean is 1.2. If $P(X = 0) = 0.3$ then $P(X = 1) =$ ______

(A) 0.1 (B) 0.4

(C) 0.2 (D) 0.5

Topic :- Probability distribution

x_i	p_i	$x_i \bullet p_i$
0	0.3	0
1	x	x
2	$0.7 - x$	$2(0.7 - x)$

Mean $= 0 + x + 2(0.7 - x) = 1.2$

$\therefore x + 1.4 - 2x = 1.2$

$\therefore -x = -0.2$

$\therefore x = 0.2$

Sol^n :-

Correct option :- (C)

30) If $\frac{\sin sin (A+B)}{\sin sin (A-B)} = \frac{\cos cos (C+D)}{\cos cos (C-D)}$ then $\tan tan A \cot cot B =$

(A) $\tan tan C \tan tan D$ (B) $- \cot cot C \cot cot D$

(C) $C \tan tan D$ (D) $\cot cot C \cot cot D$

Topic :- Trigonometric functions

Sol^n :- $\frac{\sin sin A \cos cos B + \cos cos A \sin sin B}{\sin sin A \cos cos B - \cos cos A \sin sin B} = \frac{\cos cos C \cos cos D - \sin sin C \sin sin D}{\cos cos C \cos cos D + \sin sin C \sin sin D}$

$\frac{\cot cot B + \cot cot A}{\cot cot B - \cot cot A} = \frac{\cot cot C \cot cot D - 1}{\cot cot C \cot cot D + 1}$

$\cot cot B \cot cot C \cot cot D + \cot cot B + \cot cot A \cot cot C \cot cot D + \cot cot A = \cot cot B$

$\cot cot B + \cot cot A \cot cot C \cot cot D = - \cot cot B - \cot cot A \cot cot C \cot cot D$

$2 \cot cot B + 2 \cot cot A \cot cot C \cot cot D = 0$

$\cot cot B = - \cot cot A \cot cot C \cot cot D \Rightarrow$
$\tan tan A \cot cot B = - \cot cot C \cot cot D$

Correct option :- (B)

31) If $y = (\sec sec x + \tan tan x)$ then $\frac{dy}{dx} =$

(A) $\frac{1}{2}$ (B) -1

(C) 1 (D) $\frac{-1}{2}$

Topic :- Differentiation

Sol^n :- $y = (\sec sec\, x + \tan tan\, x)$

$\therefore y = \left(\frac{1 + \sin sin\, x}{\cos cos\, x}\right)$

$\therefore y = \left[\frac{\left(\cos cos\left(\frac{x}{2}\right) + \sin sin\left(\frac{x}{2}\right)\right)^2}{\left(\frac{x}{2}\right) - \left(\frac{x}{2}\right)}\right] = tan^{-1}\left[\frac{\cos cos\,\frac{x}{2} + \sin sin\,\frac{x}{2}}{\cos cos\,\frac{x}{2} - \sin sin\,\frac{x}{2}}\right]$

$\therefore y = tan^{-1}\left[\frac{1 + \frac{x}{2}}{1 - \tan tan\,\frac{x}{2}}\right]$

$\therefore y = \left[\tan tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] = \frac{\pi}{4} + \frac{x}{2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

Correct option :- (A)

32) Degree of differential equation $e^{\frac{dy}{dx}} + \left(\frac{dy}{dx}\right)^3 = x$ is

(A) 1 (B) Not defined

(C) 3 (D) 2

Topic :- Differential equation

Sol^n :- Degree is not defined

Correct option :- (B)

33) If $\cos cos\, 2\theta = \sin sin\, \alpha$, then $\theta =$

(A) $n\pi \pm \left(\frac{\pi}{4} + \frac{\alpha}{2}\right),\ n \in Z$ (B) $n\pi \pm \left(\frac{\pi}{4} - \frac{\alpha}{2}\right),\ n \in Z$

(C) $\frac{1}{2}\left[n\pi \pm (-1)^n \alpha\right],\ n \in Z$ (D) $2n\pi \pm \left(\frac{\pi}{2} - \alpha\right),\ n \in Z$

Topic :- Trigonometric equation

Sol^n :- $\cos cos\, 2\theta = \sin sin\, \alpha = \cos cos\left(\frac{\pi}{2} - \alpha\right)$

$\therefore 2\theta = 2n\pi \pm \left(\frac{\pi}{2} - \alpha\right)$

Dividing by 2, $\theta = n\pi \pm \left(\frac{\pi}{4} - \frac{\alpha}{2}\right),\ n \in Z$

Correct option :- (B)

34) $\int e^x \left[\frac{x - \sqrt{1 - x^2}}{\sqrt{1 - x^2}}\right] dx =$

(A) $-e^x + c$ (B) $-xe^x + c$

(C) $-e^{x} + c$ (D) $-xe^{x} + c$

Topic :- Indefinite Integration

Sol^n :- $I = \int e^{x}\left[\frac{x-\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]dx$

Put $x = t \quad \Rightarrow \quad \frac{-1}{\sqrt{1-x^2}}dx = dt$

$I = -\int e^{t}\left[\frac{\cos\cos t - \sin\sin t}{1}\right]dt = -\int e^{t}(\cos\cos t - \sin\sin t)dt$

$I = -e^{t}\cos\cos t + c = -xe^{x} + C$

Correct option :- (B)

35) The auxiliary equation of the lines passing through the origin and having slopes $\sqrt{3} + 1$ and $\sqrt{3} - 1$ is

(A) $m^2 + 2\sqrt{3}m - 2 = 0$ (B) $m^2 - 2\sqrt{3}m + 2 = 0$

(C) $m^2 + 2\sqrt{3}m + 2 = 0$ (D) $m^2 - 2\sqrt{3}m - 2 = 0$

Topic :- Pair of straight lines

Sol^n :- $m_1 = \sqrt{3} + 1$ and $m_2 = \sqrt{3} - 1$

Auxiliary equation $bm^2 + 2hm + a = 0$ ____________(1)

$m_1 + m_2 = \sqrt{3} + 1 + \sqrt{3} - 1 = 2\sqrt{3}$

$m_1 \bullet m_2 = (\sqrt{3} + 1)(\sqrt{3} - 1) = (\sqrt{3})^2 - 1 = 2$

But $m_1 + m_2 = \frac{-2h}{b}$ and $m_1 \bullet m_2 = \frac{a}{b}$

$\therefore \frac{-2h}{b} = 2\sqrt{3}$ and $\frac{a}{b} = 2$

$\therefore -2h = 2\sqrt{3}b$ and $a = 2b$

Equation (1) becomes, $bm^2 - 2\sqrt{3}bm + 2b = 0$

i.e. $m^2 - 2\sqrt{3}m + 2 = 0$

Correct option :- (B)

36) If two angles of ΔABC are $\frac{\pi}{4}$ and $\frac{\pi}{3}$ then the ratio of the smallest and greatest side is

(A) $(\sqrt{3} + 1) : 1$ (B) $\sqrt{3} : \sqrt{2}$

(C) $(\sqrt{3} - 1) : 1$ (D) $(\sqrt{3} + 1) : (\sqrt{3} - 1)$

Topic :- Trigonometric functions

Sol^n :- $\angle A = \frac{\pi}{4} = 45^0$, $\angle B = \frac{\pi}{3} = 60^0$ $\Rightarrow$ $\angle C = 75^0$

By Sine rule, $\frac{\sin\sin A}{a} = \frac{\sin\sin C}{c}$ $\Rightarrow$ $\frac{a}{c} = \frac{\sin\sin A}{\sin\sin C}$

$\therefore \frac{a}{c} = \frac{\sin\sin 45^0}{\sin\sin 75^0} = \frac{\frac{1}{\sqrt{2}}}{\frac{(\sqrt{3}+1)}{2\sqrt{2}}} = \frac{2}{\sqrt{3}+1} = \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{2\sqrt{3}-1}{2} = \frac{\sqrt{3}-1}{1}$

Correct option :- (C)

37) If the body cools from 135^0 C to 80^0C at room temperature of 25^0C in 60 minutes then the temperature of body after 2 hours is

(A) $(10.75)^0$C (B) $(52.5)^0$C

(C) $(52.75)^0$C (D) $(10.5)^0$C

Topic :- Differential Equation

Sol^n :- By condition, $\frac{d\theta}{dt} \propto (\theta - \theta_0)$

$\therefore \frac{d\theta}{dt} = - k(\theta - \theta_0)$

$\therefore \frac{d\theta}{(\theta - \theta_0)} = - kdt$

On Integrating, $\log log\,(\theta - \theta_0) = - kt + C_1$

$\theta = \theta_0 + c\cdot e^{-kt}$

Here $\theta_0 = 25^0$ __________ Room temperature

$\theta = 25 + c\cdot e^{-kt}$ ____________(1)

Initially $t = 0,\ \theta = 135^0 C$

$\therefore 135 = 25 + c \Rightarrow c = 110$

Eq. (1) becomes, $\theta = 25 + 110e^{-kt}$ ____________(2)

When $t = 60\ min. = 1\ hour,\ \theta = 80^0 C$ $\therefore 80 = 25 + 110e^{-k}$

$\therefore \frac{55}{110} = e^{-k}$ $\Rightarrow$ $e^{-k} = \frac{1}{2}$

When $t = 2\ hours, \theta = 25 + 110\cdot e^{-2k}$

$\therefore \theta = 25 + 110\cdot\left(e^{-k}\right)^2$

$\therefore \theta = 25 + 110\cdot\frac{1}{4} = 25 + 27.5 = 52.5$

Correct option :- (B)

38) With usual notations, in ΔABC if $a = 2,\ b = 3,\ c = 5$ and $\frac{coscos\ A}{a} + \frac{coscos\ B}{b} + \frac{coscos\ C}{c} = \frac{k+7}{30}$ then k =

(A) 16 (B) 17

(C) 12 (D) 6

Topic :- Trigonometric functions

Sol^n :- $\frac{coscos\ A}{a} + \frac{coscos\ B}{b} + \frac{coscos\ C}{c} = \frac{k+7}{30}$

$$\therefore \frac{b^2+c^2-a^2}{2abc} = \frac{a^2+c^2-b^2}{2abc} = \frac{a^2+b^2-c^2}{2abc} = \frac{k+7}{30}$$

$$\therefore \frac{b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2abc} = \frac{k+7}{30}$$

$$\therefore \frac{a^2+b^2+c^2}{2abc} = \frac{k+7}{30}$$

$$\therefore \frac{4+9+25}{2\times2\times3\times5} = \frac{k+7}{30} \quad \Rightarrow \quad \frac{38}{60} = \frac{k+7}{30}$$

$$\therefore 19 = k + 7 \quad \Rightarrow \quad k = 12$$

Correct option :- (C)

39) The particular solution of differential equation $y\frac{dx}{dy} + x = \cot cot\ y$ when $x = 0,\ y = \frac{3\pi}{4}$ is

(A) $y = 1 + \cot cot\ x$ (B) $xy = \cot cot\ (x + y)$

(C) $x = 1 + \cot cot\ y$ (D) $xy = \cot cot\ (x - y)$

Topic :- Differential equation

Sol^n :- $y\frac{dx}{dy} + x = \cot cot\ y$

$\frac{dx}{dy} + \frac{1}{y}x = \frac{cotcot\ y}{y}$ be linear differential equation

Here $P = \frac{1}{y}$, $Q = \frac{cotcot\ y}{y}$

$$\text{I.F.} = e^{\int P dy} = e^{\int \frac{1}{y} dy}$$

$$= e^{\int y\, dy} = e^{-cotcot\ y}$$

General solution is $x(I.F.) = \int Q(I.F.)dy$

$$\therefore x\cdot e^{-cotcot\ y} = \int \frac{cotcot\ y}{y}\cdot e^{-cotcot\ y} dy$$

On R.H.S. put $-\cot cot\ y = t \quad \Rightarrow \quad y\, dy = dt$

$$\therefore x \cdot e^{-\cot\cot y} = \int - t \cdot e^{t} dt$$

$$\therefore - x \cdot e^{-\cot\cot y} = t \cdot e^{t} - e^{t} + c$$

$$\therefore - x \cdot e^{-\cot\cot y} =- \cot \cot y \cdot e^{-\cot\cot y} - e^{-\cot\cot y} + c$$

$$\therefore - x =- \cot \cot y - 1 + c \cdot e^{\cot\cot y}$$

When $x = 0, \; y = \frac{3\pi}{4}$

$$\therefore 0 =- \cot \cot \frac{3\pi}{4} - 1 + c \cdot e^{\cot\cot \frac{3\pi}{4}}$$

$$\therefore 0 = 1 - 1 + c \cdot e^{-1} \quad \Rightarrow \quad c = 0$$

$\therefore - x =- \cot \cot y - 1$

$\therefore x = 1 + \cot \cot y$ is particular solution

Correct option :- (C)

40) If $\sec \sec x + \tan \tan x = 3, \; x \in \left(0, \frac{\pi}{2}\right)$ then $\sin \sin x =$

(A) $\frac{1}{5}$ (B) $\frac{4}{5}$

(C) -1 (D) 3

Topic :- Trigonometric functions

Sol^{n} :- $\sec \sec x + \tan \tan x = 3$ ____________(1)

$\sec \sec x - \tan \tan x = \frac{1}{3}$ ____________(2)

Adding (1) and (2), $2 \sec \sec x = \frac{10}{3}$

$\therefore \sec \sec x = \frac{5}{3} \quad \Rightarrow \quad \cos \cos x = \frac{3}{5}$

$$x = 1 - x = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$\therefore \sin \sin x = \pm \frac{4}{5}$ But $x \in \left(0, \frac{\pi}{2}\right) \quad \Rightarrow \quad \sin \sin x = \frac{4}{5}$

Correct option :- (B)

41) If $f(x) = |x - 2|, \; x \in [0, 4]$ then the Rolle's theorem can not be applied to the function because

(A) The function is not differentiable at every point in (0, 4)

(B) $f(4) \neq f(0)$

(C) $f(x)$ is not continuous at every point in [0, 4]

(D) function is not well defined in the domain

Topic :- Application of derivative

Sol^n :- $\because$ i) $f(x)$ is continuous at every point in [0, 4]

ii) $f(x)$ is not differentiable at every point in (0, 4)

Correct option :- (A)

42) If $\frac{2+4+6+8+.......upto\ n\ terms}{1+3+5+7+.......upto\ n\ terms} = \frac{37}{36}$ then n =

(A) 23 (B) 29

(C) 37 (D) 36

Topic :- Sequence and series

Sol^n :- $\frac{2+4+6+8+.......upto\ n\ terms}{1+3+5+7+.......upto\ n\ terms} = \frac{37}{36}$

$$\frac{\sum_{r=1}^{n} 2r}{\sum_{r=1}^{n}(2r-1)} = \frac{37}{36} \Rightarrow \frac{n(n+1)}{n(n+1)-n} = \frac{37}{36} \Rightarrow \frac{(n+1)}{n} = \frac{37}{36}$$

$\therefore\ 36 + 36n = 37n \Rightarrow n = 36$

Correct option :- (D)

43) $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} =$

(A) $\frac{-\pi}{60}$ (B) $\frac{\pi}{120}$

(C) $\frac{\pi}{60}$ (D) $\frac{\pi}{80}$

Topic :- Definite Integration

Sol^n :- $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} = \frac{1}{5}\int_0^\infty \left(\frac{1}{x^2+4}\right) - \left(\frac{1}{x^2+9}\right)dx$

$$= \frac{1}{5}\left[\frac{1}{2}\left(\frac{x}{2}\right) - \frac{1}{3}\left(\frac{x}{3}\right)\right]_0^\infty$$

$$= \frac{1}{5}\left[\frac{1}{2}(\infty) - \frac{1}{2}(0) - \frac{1}{3}(\infty) + \frac{1}{3}(0)\right]$$

$$= \frac{1}{5}\left[\frac{1}{2}\frac{\pi}{2} - 0 - \frac{1}{3}\frac{\pi}{2} - 0\right]$$

$$= \frac{1}{5}\left[\frac{\pi}{4} - \frac{\pi}{6}\right] = \frac{1}{5}\left[\frac{2\pi}{24}\right] = \frac{\pi}{60}$$

Correct option :- (C)

44) The angle between the line $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z+7}{2}$ and plane $\bar{r} \bullet (6\hat{i} - 2\hat{j} - 3\hat{k}) = 5$ is

(A) $\left(\frac{4}{21}\right)$ (B) $\left(\frac{5}{7}\right)$

(C) $\left(\frac{5}{7}\right)$ (D) $\left(\frac{4}{21}\right)$

Topic :- Line and Plane

Sol^n :- $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z+7}{2}$

Here $\bar{b} = 2\hat{i} + \hat{j} + 2\hat{k}$ $\therefore |\bar{b}| = 3$

$\bar{r} \bullet \left(6\hat{i} - 2\hat{j} - 3\hat{k}\right) = 5$

Here $\bar{n} = \left(6\hat{i} - 2\hat{j} - 3\hat{k}\right)$ $\therefore |\bar{n}| = 7$

$\bar{b} \bullet \bar{n} = \left(2\hat{i} + \hat{j} + 2\hat{k}\right) \bullet \left(6\hat{i} - 2\hat{j} - 3\hat{k}\right) = 12 - 2 - 6 = 4$

$\therefore \sin sin\, \theta = \frac{\bar{b} \cdot \bar{n}}{|\bar{b}||\bar{n}|} = \frac{4}{3 \times 7} = \frac{4}{21}$ $\therefore \theta = \left(\frac{4}{21}\right)$

Correct option :- (A)

45) The parametric equation of the line passing through $A(3,\ 4,\ -7),\ B(1,\ -1,\ 6)$ are

(A) $x = 3 - 2\lambda,$ $y = 4 - 5\lambda,$ $z =- 7 + 13\lambda$

(B) $x = 3 + \lambda,$ $y =- 1 + 4\lambda,$ $z =- 7 + 6\lambda$

(C) $x =- 2 + 5\lambda,$ $y =- 5 + 4\lambda,$ $z = 13 - 7\lambda$

(D) $x = 1 + 3\lambda,$ $y =- 1 + 4\lambda,$ $z = 6 - 7\lambda$

Topic :- Line and plane

Sol^n :- The equation of line passing through points $A(3,\ 4,\ -7),\ B(1,\ -1,\ 6)$

$\frac{x-3}{1-3} = \frac{y-4}{-1-4} = \frac{z+7}{6+7} = \lambda$ i.e. $\frac{x-3}{-2} = \frac{y-4}{-5} = \frac{z+7}{13} = \lambda$

$x - 3 =- 2\lambda,$ $y - 4 =- 5\lambda,$ $z + 7 = 13\lambda$

$x = 3 - 2\lambda,$ $y = 4 - 5\lambda,$ $z =- 7 + 13\lambda$

Correct option :- (A)

46) If $y = 3e^{5x} + 5e^{3x}$ then $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} =$

(A) $15y$ (B) $10y$

(C) $-15y$ (D) $-10y$

Topic :- Differentiation

Sol^n :- $y = 3e^{5x} + 5e^{3x}$

$\frac{dy}{dx} = 15e^{5x} + 15e^{3x} = 15(e^{5x} + e^{3x})$

$\therefore \frac{d^2y}{dx^2} = 15(5e^{5x} + 3e^{3x})$

$$\therefore \frac{d^2y}{dx^2} - 8\frac{dy}{dx} = 15\left(5e^{5x} + 3e^{3x}\right) - 8\times15(e^{5x} + e^{3x})$$

$$= 15\left[5e^{5x} + 3e^{3x} - 8e^{5x} - 8e^{3x}\right] = -15y$$

Correct option :- (C)

47) If $x = \log log\, t$, $y + 1 = \frac{1}{t}$ then $e^{-x}\frac{d^2x}{dy^2} + \frac{dx}{dy} =$

(A) 1 (B) 0

(C) 2 (D) -1

Topic :-Differentiation

Sol^n :- $x = \log log\, t$, $y + 1 = \frac{1}{t}$

$\therefore t = e^x \Rightarrow y + 1 = \frac{1}{e^x}$ $\therefore y + 1 = e^{-x}$

$\therefore \frac{dy}{dx} = -e^{-x} \Rightarrow \frac{dx}{dy} = -e^x$ ____________(1)

Again differentiating w. r. t. , $\frac{d^2x}{dy^2} = -e^x\frac{dx}{dy}$

$\therefore \frac{1}{e^x}\frac{d^2x}{dy^2} = -\frac{dx}{dy}$ $\therefore e^{-x}\frac{d^2x}{dy^2} + \frac{dx}{dy} = 0$

Correct option :- (B)

48) Two cards are drawn from a pack of well shuffled 52 playing cards one by one without replacement. Then probability that both cards are queen is

(A) $\frac{1}{221}$ (B) $\frac{1}{220}$

(C) $\frac{3}{220}$ (D) $\frac{2}{221}$

Topic :- Probability distribution

Sol^n :- $n(S) = {}^{52}C_2 = \frac{52\times51}{2\times1} = 26\times51$

$n(A) = {}^{4}C_2 = \frac{4!}{2!\times2!} = \frac{4\times3}{2\times1} = 6$

Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{6}{26\times51} = \frac{1}{13\times7} = \frac{1}{221}$

Correct option :- (A)

49) If $[\bar{a}\,\bar{b}\,\bar{c}] = 4$ then volume of parallelepiped with coterminous edges $\bar{a} + 2\bar{b}$, $\bar{b} + 2\bar{c}$, $\bar{c} + 2\bar{a}$ is

(A) $40(units)^3$ (B) $32(units)^3$

(C) $32(units)^3$ (D) $30(units)^3$

Topic :- Vector

Sol^n :- Let $\overline{p} = \overline{a} + 2\overline{b}, \ \overline{q} = \overline{b} + 2\overline{c}, \ \overline{r} = \overline{c} + 2\overline{a}$

Volume of parallelepiped $= \overline{p} \bullet \overline{q} \times \overline{r}$

$= |1\ 2\ 0\ 0\ 1\ 2\ 2\ 0\ 1\ |[\overline{a}\ \overline{b}\ \overline{c}]$

$= [1(1 - 0) - 2(0 - 4) + 0]\times4 = (1 + 8)\times4 = 36$

Correct option :- (C)

50) $\tan tan\ 1^0 \times \tan tan\ 2^0 \times \tan tan\ 3^0 \times . . . . \times \tan tan\ 89^0 =$

(A) $\sqrt{2}$ (B) 2

(C) 1 (D) $\sqrt{3}$

Topic :- Trigonometric functions

Sol^n :- $\tan tan\ 1^0 \times \tan tan\ 2^0 \times \tan tan\ 3^0 \times . . . . \times \tan tan\ 89^0$

$1^0 \times \tan tan\ 2^0 \times \tan tan\ 3^0 \times . . . . \times \tan tan\ 45^0 \times . . . . \times \tan tan\ (90 - 1)^0$

$1^0 \times \tan tan\ 2^0 \times \tan tan\ 3^0 \times . . . . \times \tan tan\ 45^0 \times . . . . \times \cot cot\ 3^0 \times \cot cot\ 2^0 \times$

$= \tan tan\ 45^0 = 1$

Correct option :- (C)