Polynomial and Rational Expressions

Guided Notes

This resource is designed to help you actively engage with key mathematical concepts. By completing these notes, you'll establish connections between concepts, identify patterns, and develop a deeper understanding of the material.

Fill in the blanks, solve examples, and note questions as you go. Writing in your own words strengthens your memory and helps identify areas needing more attention. Bring your questions to class discussions where your instructor can address specific concerns. Don't focus on perfection—you'll learn math best through active engagement!

Identifying Polynomial Functions

Key Concept: Polynomial Functions

Complete the definitions:

1. A polynomial function:
   - Consists of: ____
   - Can be written as: ____
   - Example: ____
2. Classification by Terms:
   - Monomial: ____
     - Example: ____
   - Binomial: ____

      - Example: ____
   - Trinomial: ____
      - Example: ____

Practice Identifying Polynomials

Determine if each expression is a polynomial function and explain why:

1. $f(x) = 2x^3 \cdot 3x + 4$
   - Is it a polynomial? ____________________________________
   - Explanation: ____
2. $g(x) = -x(x^2 - 4)$
   - Is it a polynomial? ____________________________________
   - Explanation: ____
3. $h(x) = 5\sqrt{x} + 2$
   - Is it a polynomial? ____________________________________
   - Explanation: ____

Identifying Polynomial Functions

Key Concept: Terminology

Fill in the definitions:

1. Degree of a polynomial:
   - Definition: ____
   - How to find it: ____
2. Leading Term:
   - Definition: ____
   - Example from $3 + 2x^2 - 4x^3$: ______________________________
3. Leading Coefficient:
   - Definition: ____
   - Example from $5t^5 - 2t^3 + 7t$: ______________________________

Practice with Terminology

For each polynomial, identify:

1. $f(x) = 3 + 2x^2 - 4x^3$
   - Degree: ____________________
   - Leading term: ____________________
   - Leading coefficient: ____________________
2. $g(t) = 5t^5 - 2t^3 + 7t$
   - Degree: ____________________
   - Leading term: ____________________
   - Leading coefficient: ____________________
3. $h(p) = 6p - p^3 - 2$
   - Degree: ____________________
   - Leading term: ____________________
   - Leading coefficient: ____________________

Operations with Polynomials

Key Concept: Adding and Subtracting

Steps for adding/subtracting polynomials:

1. ____
2. ____
3. ____

Practice Identifying Polynomials

Solve and show your work:

1. $(4x^2 - 5x + 1) + (3x^2 - 8x - 9)$
   - Identify like terms: ____________

- Rearrange terms: ______________
- Final answer: ________________

2. $(9m^2 - 7m + 4) - (m^2 - 3m + 8)$
   - Distribute negative: ____________
   - Combine like terms: ____________
   - Final answer: ________________

Key Concept: Multiplication

Complete the steps for multiplying polynomials:

1. ______________________________
2. ______________________________
3. ______________________________

What does FOIL stand for?

- F: ____
- O: ____
- I: ____
- L: ____

Practice Multiplication

Use FOIL to multiply $(2x - 18)(3x + 3)$

- First: _______________ × _______________ = _______________
- Outer: _______________ × _______________ = _______________
- Inner: _______________ × _______________ = _______________
- Last: _______________ × _______________ = _______________
- Combined result: ____________________

Greatest Common Factor (GCF)

Key Concept: GCF

Greatest Common Factor: ____

Steps to find GCF in polynomials:

1. ____
2. ____
3. ____

Practice Finding GCF

Factor out the GCF in each expression:

1. $25b^3 + 10b^2$
    - GCF of coefficients: ________________
    - GCF of variables: ________________
    - Combined GCF: ________________
    - Final factored form: ________________
2. $6x^3y^3 + 45x^2y^2 + 21xy$
    - GCF of coefficients: ________________
    - GCF of variables: ________________
    - Combined GCF: ________________
    - Final factored form: ________________

Factoring Trinomials

Key Concept: When Leading Coefficient is 1

A trinomial of the form $x^2 + bx + c$ can be written in factored form as $(x + p)(x + q)$ where:

- *p* and *q* are numbers where:
  - Product ($p \times q$) = ____________
  - Sum ($p + q$) = ____________

Practice with Leading Coefficient of 1

Factor $x^2 + 2x - 15$:

- List factors of *c*: ____
- Find pair with sum = *b*: ______________________________________
- Write factored form: ____

Key Concept: When Leading Coefficient ≠ 1

Steps for factoring $ax^2 + bx + c$:

1. ____
2. ____
3. ____
4. ____

Practice with Leading Coefficient ≠ 1

Factor $5x^2 + 7x - 6$:

- Find factors of *ac*: ____
- Find pair that sums to *b*: _______________________________________
- Factor by grouping: ____
- Final factored form: ____

Think About It

Why do we use different methods for trinomials with leading coefficient of 1 versus other numbers?

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Special Factoring Patterns

Key Concept: Perfect Square Trinomials

Complete the patterns:

- $(a+b)^2$ = ____________________
- $(a-b)^2$ = ____________________

Key characteristics of perfect square trinomials:

1. ____
2. ____
3. ____

Practice with Perfect Squares

Identify if $25x^2+20x+4$ is a perfect square trinomial:

- Check first term: ______________________________
- Check last term: ______________________________
- Check middle term: ____________________________
- If yes, write factored form: ______________________

Key Concept: Difference of Squares

Complete the pattern:

- a^2-b^2 = __________________________________

Practice with Difference of Squares

Factor $81y^2 - 100$:

- Identify a^2: ______________________________
- Identify b^2: ______________________________
- Write factored form: __________________________

Key Concept: Sum and Difference of Cubes

Complete these patterns:

- Sum of Cubes: $a^3 + b^3$ = ______________________________
- Difference of Cubes: $a^3 - b^3$ = _________________________

Remember SOAP for the signs:

- S means: ____________________________
- O means: ____________________________
- A means: ____________________________
- P means: ____________________________

Practice with Cubes

Factor $x^3 + 512$:

- Identify the cube terms:
  - First term a^3: ___________
  - Second term b^3: __________
- Apply the formula:
  - First factor: _____________
  - Second factor: ___________
- Final factored form: _________

Factor $8x^3 - 125$:

- Identify the cube terms:
  - First term a^3: ____________
  - Second term b^3: ___________
- Apply the formula:
  - First factor: ______________
  - Second factor: ____________
- Final factored form: __________

Think About It

Why can we factor both sum and difference of cubes, but only difference of squares?

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How can you quickly recognize a perfect square trinomial without having to check all three terms? What patterns do you look for?

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Key Concept: Factoring with Fractional or Negative Exponents

Complete these rules:

- When factoring with fractional exponents:
  ____
- When factoring with negative exponents:
  ____
- The GCF should have: ____

Practice with Special Exponents

Factor: $3x(x+2)^{-\frac{1}{3}} + 4(x+2)^{\frac{2}{3}}$

- Identify term with lowest exponent: ____________________
- Factor out GCF: __________________________________
- Simplify remaining terms: ___________________________
- Final factored form: ______________________________

Factor: $2x^{\frac{1}{4}} + 5x^{\frac{3}{4}}$

- Identify common factor: ____________________________
- Factor out GCF: __________________________________
- Write final expression: ____________________________

Think About It

How does factoring with fractional exponents differ from factoring with integer exponents?

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Why do we need to consider the lowest power when factoring expressions with negative exponents?

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Understanding Rational Expressions

Key Concept: Defining Rational Expressions

Rational Expression: ____

Rational Expression are written as:

$\frac{P(x)}{Q(x)}$ where:

- *P(x)* is ________________________________
- *Q(x)* is ________________________________

Key Rule: A rational expression is undefined when ________________________________

Practice Finding Undefined Values

For each expression, find the undefined values:

1. $\frac{x^2 - 4}{x - 2}$
   - Set denominator to zero: ______________________________
   - Solve: ____
   - Undefined at *x* = ____________________________________
2. $\frac{x^2 + 3x}{x^2 - 4}$
   - Factor denominator: ___________________________________
   - Find zeros: ____
   - Undefined at *x* = _______________________________________

Think About It

Why is finding undefined values important for rational expressions?

Operations with Rational Expressions

Key Concept: Multiplication

Steps for multiplying rational expressions:

1. ______________________________
2. ______________________________
3. ______________________________

Practice with Multiplication

Multiply and simplify: $\dfrac{x^2+4x-5}{3x+18} \cdot \dfrac{2x-1}{x+5}$

- Factor numerators: ______________________________
- Factor denominators: ______________________________
- Cancel common factors: ______________________________
- Final result: ______________________________

Key Concept: Division

Steps for dividing rational expressions:

1. ______________________________
2. ______________________________
3. ______________________________

Practice with Division

Divide and simplify: $\dfrac{2x^2+x-6}{x^2-1} \div \dfrac{x^2-4}{x^2+2x+1}$

- Rewrite as multiplication: ______________________________
- Factor all terms: ______________________________
- Final result: ______________________________

Key Concept: Addition/Subtraction

Steps for adding/subtracting rational expressions:

1. ______________________________
2. ______________________________
3. ______________________________

Practice with Addition/Subtraction

Add: $\frac{5}{x} + \frac{6}{y}$

- Find LCD: ______________________________
- Rewrite with LCD: ______________________________
- Add numerators: ______________________________
- Simplify: ______________________________

Subtract: $\frac{6}{x^2 + 4x + 4} - \frac{2}{x^2 - 4}$

- Factor denominators: ______________________________
- Find LCD: ______________________________
- Rewrite with LCD: ______________________________
- Subtract numerators: ______________________________
- Simplify: ______________________________

Complex Rational Expressions

Key Concept: Complex Rational Expressions

Definition: ______________________________

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Steps for simplifying:

1. ______________________________

2. ____________________
3. ____________________
4. ____________________

Practice with Complex Rationals

Simplify: $\dfrac{y + \frac{1}{x}}{\frac{x}{y}}$

- Combine numerator: ____________________
- Rewrite as division: ____________________
- Convert to multiplication: ____________________
- Final result: ____________________

Think About It

How are complex rational expressions different from simple rational expressions?

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Why do we need to find a common denominator before adding/subtracting?

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Key Takeaways

Summarize the most important concepts from each section:

Polynomial Basics:

- ____________________

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Factoring Polynomials:

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Rational Expressions:

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Questions to Ask in Class:

1.
2.
3.

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