

هذه قائمة بتكاملات الدوال النسبية

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} \quad (\text{for } n \neq -1)$$

$$\int \frac{c}{ax + b} dx = \frac{c}{a} \ln |ax + b|$$

$$\int x(ax + b)^n dx = \frac{a(n+1)x - b}{a^2(n+1)(n+2)} (ax + b)^{n+1} \quad (\text{for } n \notin \{-1, -2\})$$

$$\int \frac{x}{ax + b} dx = \frac{x}{a} - \frac{b}{a^2} \ln |ax + b|$$

$$\int \frac{x}{(ax + b)^2} dx = \frac{b}{a^2(ax + b)} + \frac{1}{a^2} \ln |ax + b|$$

$$\int \frac{x}{(ax + b)^n} dx = \frac{a(1-n)x - b}{a^2(n-1)(n-2)(ax + b)^{n-1}} \quad (\text{for } n \notin \{1, 2\})$$

$$\int \frac{x^2}{ax + b} dx = \frac{1}{a^3} \left(\frac{(ax + b)^2}{2} - 2b(ax + b) + b^2 \ln |ax + b| \right)$$

$$\int \frac{x^2}{(ax + b)} dx = \frac{1}{a^3} \left(ax + b - 2b \ln |ax + b| - \frac{b^2}{ax + b} \right)$$

$$\int \frac{x^2}{(ax + b)^2} dx = \frac{1}{a^3} \left(\ln |ax + b| + \frac{2b}{ax + b} - \frac{b^2}{2(ax + b)^2} \right)$$

$$\int \frac{x^2}{(ax + b)^n} dx = \frac{1}{a^3} \left(-\frac{(ax + b)^{3-n}}{(n-3)} + \frac{2b(ax + b)^{2-n}}{(n-2)} - \frac{b^2(ax + b)^{1-n}}{(n-1)} \right)$$

$$\int \frac{1}{x(ax + b)} dx = -\frac{1}{b} \ln \left| \frac{ax + b}{x} \right|$$

$$\int \frac{1}{x^2(ax + b)} dx = -\frac{1}{bx} + \frac{a}{b^2} \ln \left| \frac{ax + b}{x} \right|$$

$$\int \frac{1}{x^2(ax + b)^2} dx = -a \left(\frac{1}{b^2(ax + b)} + \frac{1}{ab^2x} - \frac{2}{b^3} \ln \left| \frac{ax + b}{x} \right| \right)$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{a}$$

$$\int \frac{1}{x^2 - a^2} dx = \begin{cases} -\frac{1}{a} \operatorname{arctanh} \frac{x}{a} = \frac{1}{2a} \ln \frac{a-x}{a+x} & (\text{for } |x| < a) \\ -\frac{1}{a} \operatorname{arccoth} \frac{x}{a} = \frac{1}{2a} \ln \frac{x-a}{x+a} & (\text{for } |x| > a) \end{cases}$$

for $a \neq 0$:

$$\int \frac{1}{ax^2 + bx + c} = \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}} & (\text{for } 4ac - b^2 > 0) \\ -\frac{2}{\sqrt{b^2 - 4ac}} \operatorname{arctanh} \frac{2ax + b}{\sqrt{b^2 - 4ac}} = \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b + \sqrt{b^2 - 4ac}}{2ax + b - \sqrt{b^2 - 4ac}} \right| & (\text{for } 4ac - b^2 < 0) \\ -\frac{2}{2ax + b} & (\text{for } 4ac - b^2 = 0) \end{cases}$$

$$\int \frac{x}{ax^2 + bx + c} = \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{2a} \int \frac{dx}{ax^2 + bx + c}$$

$$\int \frac{mx + n}{ax^2 + bx + c} = \begin{cases} \frac{m}{2a} \ln |ax^2 + bx + c| + \frac{2an - bm}{a\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}} & (\text{for } 4ac - b^2 > 0) \\ \frac{m}{2a} \ln |ax^2 + bx + c| - \frac{2an - bm}{a\sqrt{b^2 - 4ac}} \operatorname{arctanh} \frac{2ax + b}{\sqrt{b^2 - 4ac}} & (\text{for } 4ac - b^2 < 0) \\ \frac{m}{2a} \ln |ax^2 + bx + c| - \frac{2an - bm}{a(2ax + b)} & (\text{for } 4ac - b^2 = 0) \end{cases}$$

$$\begin{aligned} \int \frac{1}{(ax^2 + bx + c)^n} dx &= \frac{2ax + b}{(n-1)(4ac - b^2)(ax^2 + bx + c)^{n-1}} + \frac{(2n-3)2a}{(n-1)(4ac - b^2)} \int \frac{1}{(ax^2 + bx + c)^{n-1}} dx \\ \int \frac{x}{(ax^2 + bx + c)^n} dx &= \frac{bx + 2c}{(n-1)(4ac - b^2)(ax^2 + bx + c)^{n-1}} - \frac{b(2n-3)}{(n-1)(4ac - b^2)} \int \frac{1}{(ax^2 + bx + c)^{n-1}} dx \\ \int \frac{1}{x(ax^2 + bx + c)} dx &= \frac{1}{2c} \ln \left| \frac{x^2}{ax^2 + bx + c} \right| - \frac{b}{2c} \int \frac{1}{ax^2 + bx + c} dx \\ \int \frac{dx}{x^{2n} + 1} &= \sum_{k=1}^{2^{n-1}} \left\{ \frac{1}{2^{n-1}} \left[\sin\left(\frac{(2k-1)\pi}{2^n}\right) \arctan\left(x - \cos\left(\frac{(2k-1)\pi}{2^n}\right) \csc\left(\frac{(2k-1)\pi}{2^n}\right)\right) \right] - \frac{1}{2^n} \left[\cos\left(\frac{(2k-1)\pi}{2^n}\right) \ln \left| x^2 - 2x \cos\left(\frac{(2k-1)\pi}{2^n}\right) + 1 \right| \right] \right\} \end{aligned}$$