YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020

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CANDIDATE NAME					
CLASS				**INDEX NUMBER**	

MATHEMATICS 4048 / 02
4 Express / 5 Normal Academic

Paper 2

31 August 2020

Candidates answer on the Question Paper. **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions**.**
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
100

This document consists of **18** printed pages.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

Answer **all** the questions.

1 **(a)** Write as a single fraction in its simplest form

(i) $\frac{27p}{q^3} \div \left(\frac{-6p}{q}\right)^3$,

Answer ..... [2]

(ii) $\frac{1}{1-3y} - \frac{2}{y+2}$.

Answer ..... [2]

(b) Simplify $\frac{9-25x^2}{5x^2-12x-9}$.

Answer …… [3]

(c) Solve the equation $\frac{5}{x+1} = 2x - 7$.

Answer x = …… or …… [3]

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2 **(a)** In 2019, Bob earned a total of \$45 600.
The percentage increase in his income from 2018 to 2019 is 3.5%.
Calculate his monthly income in 2018, to the nearest dollar.

Answer \$ …… [2]

(b) A bank offers a savings account with a compound interest rate of 2.2% per annum.
Bob invests \$7000 in his account. Calculate the total amount of interest he earns after 5 years. Give your answer correct to the nearest cent.

Answer $ …… [2]

(c) The exchange rate between Singapore dollars and Macau Pataca (MOP$) is $1 = MOP$5.55.
The exchange rate between Hong Kong dollars (HK$) and Singapore dollars is HK$1 = $0.19.

Bob is planning a trip to Hong Kong and Macau.
He finds these hotel prices on a website.

Hong Kong Hotel	HK$825
Macau Hotel	MOP$825

(i) By comparing the exchange rates, explain which hotel costs lower per night.
Show your workings clearly.

Answer

The ……..…… hotel costs lower per night. [2]

(ii) Bob books 4 nights in the Hong Kong hotel and 2 nights in the Macau hotel.
He pays using his credit card. The credit card company converts the prices to Singapore dollars and charges a fee of k% for the currency conversion.

Given that the total amount Bob pays for the two hotels, including the credit card fee, is $940, find the value of k.

Answer …… [4]

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3 PQR is a triangle. The coordinates of P and Q are (0, 5) and (–2, 1) respectively.
The equation of the line PR is $5y - 2x = 25$.

(a) Given the coordinates of R are $(k, k + 2)$, show that $k = 5$.
Answer

[1]

(b) Find the equation of PQ.

Answer …… [1]

(c) Find the length of diagonal QR.

Answer …… units [2]

(d) Calculate angle PQR.

Answer …… [3]

(e) Calculate the area of triangle PQR.

Answer ..... units^2 [3]

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4 The first four terms in a sequence of numbers are given below.

$$T_1 = 2^2 + 5 = 9$$
$$T_2 = 4^2 + 3 = 19$$
$$T_3 = 6^2 + 1 = 37$$
$$T_4 = 8^2 - 1 = 63$$

(a) Find T_5.

Answer ..... [1]

(b) Explain why the value of T_n must be odd for all values of n.

.....

..... [1]

(c) Show that the nth term of the sequence, T_n, is given by $4n^2 - 2n + 7$.

Answer

[3]

(d) T_k and T_{k+1} are consecutive terms in the sequence.
Find and simplify an expression, in terms of k, for $T_{k+1} - T_k$.

Answer …… [3]

(e) Explain why two consecutive terms of the sequence cannot have a difference of 6.

……

…… [1]

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5 An enrichment centre offers music and dance lessons at basic (B) and advanced (A) levels. Each student has a 8-week block of lessons, with one lesson per week.

The matrix **L** shows the number of students attending the lessons each week.

$$\mathbf{L} = \begin{matrix} \text{B} & \text{A} \\ \end{matrix} \begin{pmatrix} 68 & 76 \\ 43 & 38 \end{pmatrix} \begin{matrix} \text{Music} \\ \text{Dance} \end{matrix}$$

(a) Evaluate the matrix $\mathbf{P} = 8\mathbf{L}$.

Answer …… [1]

(b) The fee for each basic lesson is \$45 and the fee for each advanced lesson is \$65. Represent these fees in a 2×1 column matrix **F**.

Answer …… [1]

(c) Evaluate the matrix $\mathbf{Q} = \mathbf{PF}$.

Answer ……..……..……..…… [2]

(d) State what the elements of **Q** represent.

…….

…… [1]

(e) The enrichment centre increased the fees by 10%. As a result, the number of students attending basic level and advanced level lessons reduced by 12.5% and 6.25% respectively.

Using appropriate matrix multiplication, determine if the enrichment centre has made a profit or a loss in a 8-week block of lessons after the increase in fees.

Answer

The enrichment centre made a …… in a 8-week block of lessons after the increase in fees. [3]

---

6 Yan and Zac run a small business that sells hand-painted ornaments.

(a) Yan takes x minutes to paint one ornament.
Write an expression, in terms of x, for the number of ornaments she paints in one hour.

Answer ..... [1]

(b) Zac takes 6 minutes less than Yan to paint one ornament.
Write an expression, in terms of x, for the number of ornaments he paints in one hour.

Answer ..... [1]

(c) One day, Yan and Zac each work for 6 hours.
Altogether, they paint a total of 27 ornaments.
Write down an equation, in terms of x, to represent this information, and show that it reduces to

$$3x^2 - 98x + 240 = 0 .$$

Answer

[3]

(d) Solve the equation $3x^2 - 98x + 240 = 0$.

Answer x = …… or …… [3]

(e) **(i)** Explain why one of the solutions in part **(d)** must be rejected.

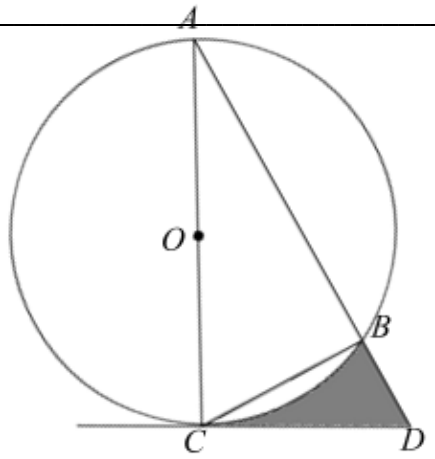
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…… [1]

(ii) Hence, find the number of ornaments Zac paints in one hour.

Answer …… [2]

7



The diagram shows a circle that passes through the points A, B and C, with its centre at O. AC is the diameter of the circle. CD is a tangent that meets AB produced at D.

(a) Show that triangles ACD and CBD are similar.
Give a reason for each statement you make.

……

……

……

…… [3]

(b) The ratio of the area of triangle CBD : area of triangle ACD is 1 : 4.
Show that angle CAD is 30°.

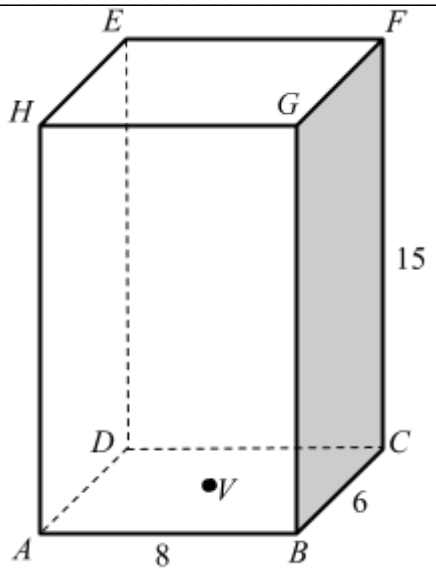
Answer

[2]

(c) Given that the radius of the circle is 5 cm, calculate the shaded area.

Answer ..... cm^2 [4]

8



In the diagram, *ABCDEFGH* is a cuboid with dimensions 8 cm by 6 cm by 15 cm.
V is the centre of the rectangular base.

(a) Show that $EV = 15.8$ cm, correct to 3 significant figures.

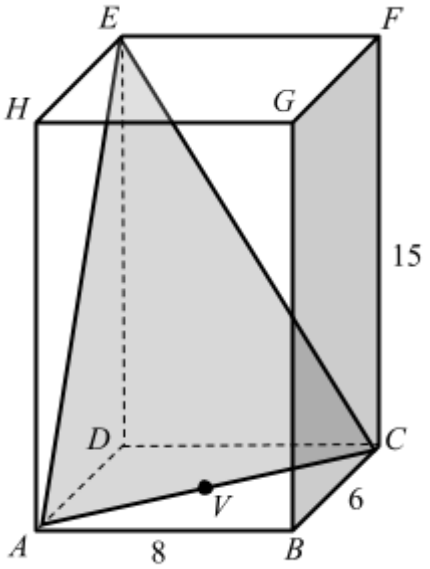
Answer

[2]

(b) Calculate angle *ACE*.

Answer …… [4]

(c) A pyramid *EDAC* is cut out from the cuboid.



(i) Find the total surface area of the pyramid.

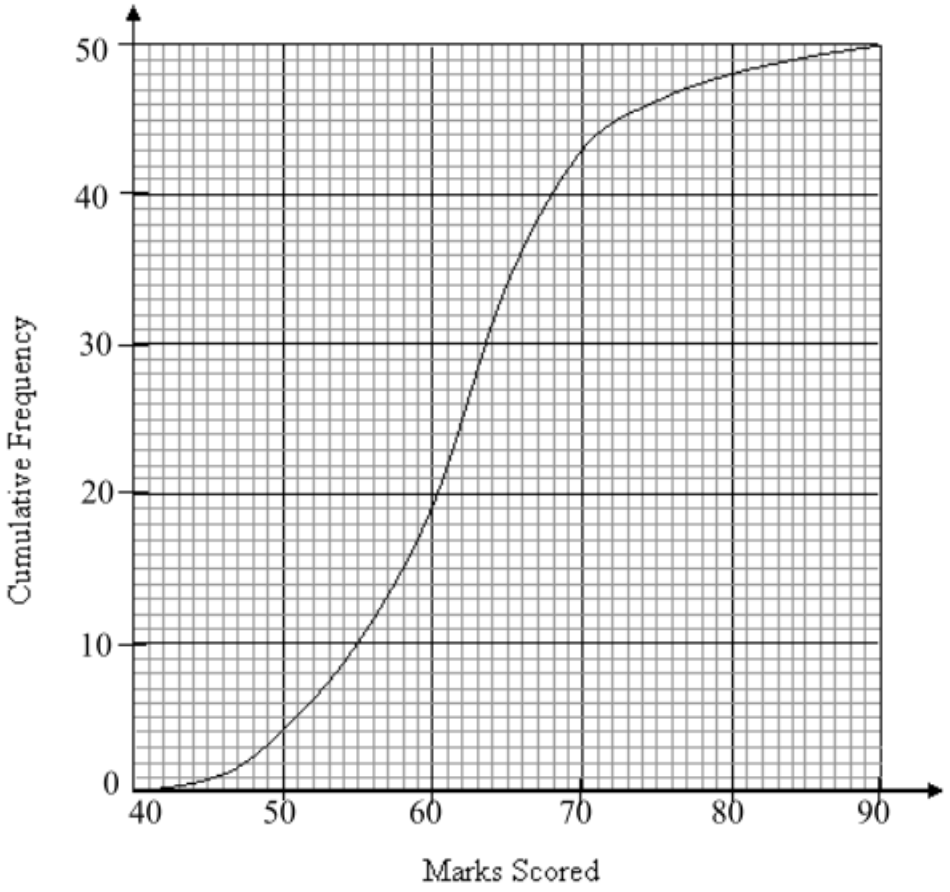
Answer …… cm^2 [3]

(ii) Another pyramid is to be made with volume half of pyramid *EDAC*.
Given that the two pyramids are geometrically similar, find the vertical height of the smaller pyramid.

Answer …….. cm [2]

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9 **(a)** 50 students from School *P* took part in a Mathematics competition.
The cumulative frequency curve below shows the distribution of the marks they scored.



(i) Complete the grouped frequency table of the marks scored by the students. [1]

Marks (x)	$40 \le x < 50$	$50 \le x < 60$	$60 \le x < 70$	$70 \le x < 80$	$80 \le x < 90$
Frequency	4	15			

(ii) Calculate an estimate of the mean mark.

Answer …… [1]

(iii) Calculate an estimate of the standard deviation.

Answer …… [1]

(iv) The minimum mark for a student to be awarded a certificate of merit is 65.
Find the number of students who were awarded a certificate of merit.

Answer …… [1]

(v) Another group of 50 students from school *Q* took part in the same competition and had the same interquartile range of marks as School *P*. However, half of the students from School *Q* scored at least 68 marks.

Describe how the cumulative frequency curve for School *Q* may differ from the curve for School *P*.

……

…… [1]

(b) This table shows information about a group of students.

	Wears spectacles	Does not wear spectacles
Lower Secondary	3	9
Upper Secondary	7	5

(i) One of the students is selected at random.
Find, as a fraction in its lowest terms, the probability that the student
(a) is in upper secondary,

Answer …… [1]

(b) does not wear spectacles.

Answer ..... [1]

(ii) Two of the students are selected at random.
Find, as a fraction in its lowest terms, the probability that
(a) both students are in lower secondary,

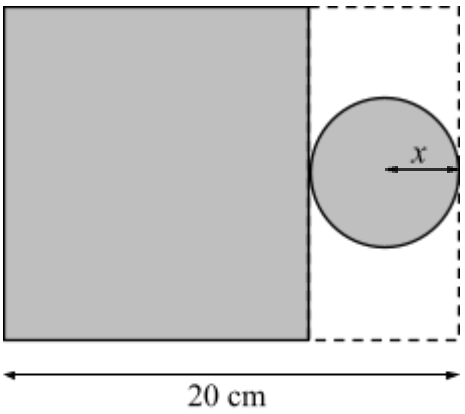
Answer ..... [2]

(b) at least one of them wears spectacles.

Answer ..... [2]

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10 Kate is designing open cylindrical gift boxes with base radius x cm.
She uses rectangular pieces of cardboard of length 20 cm and a variable width.



She cuts the net of the gift boxes out from each piece of cardboard.
By changing the base radius of the cylinders, she can change the volume of the gift boxes.

(a) Work out the volume of a gift box of base radius 2 cm.

Answer ..... cm^3 [2]

(b) Show, in terms of x, the volume of the gift boxes she makes is given by $2\pi x^2(10-x)$.

Answer

[1]

(c) Explain why $0 < x < 10$.

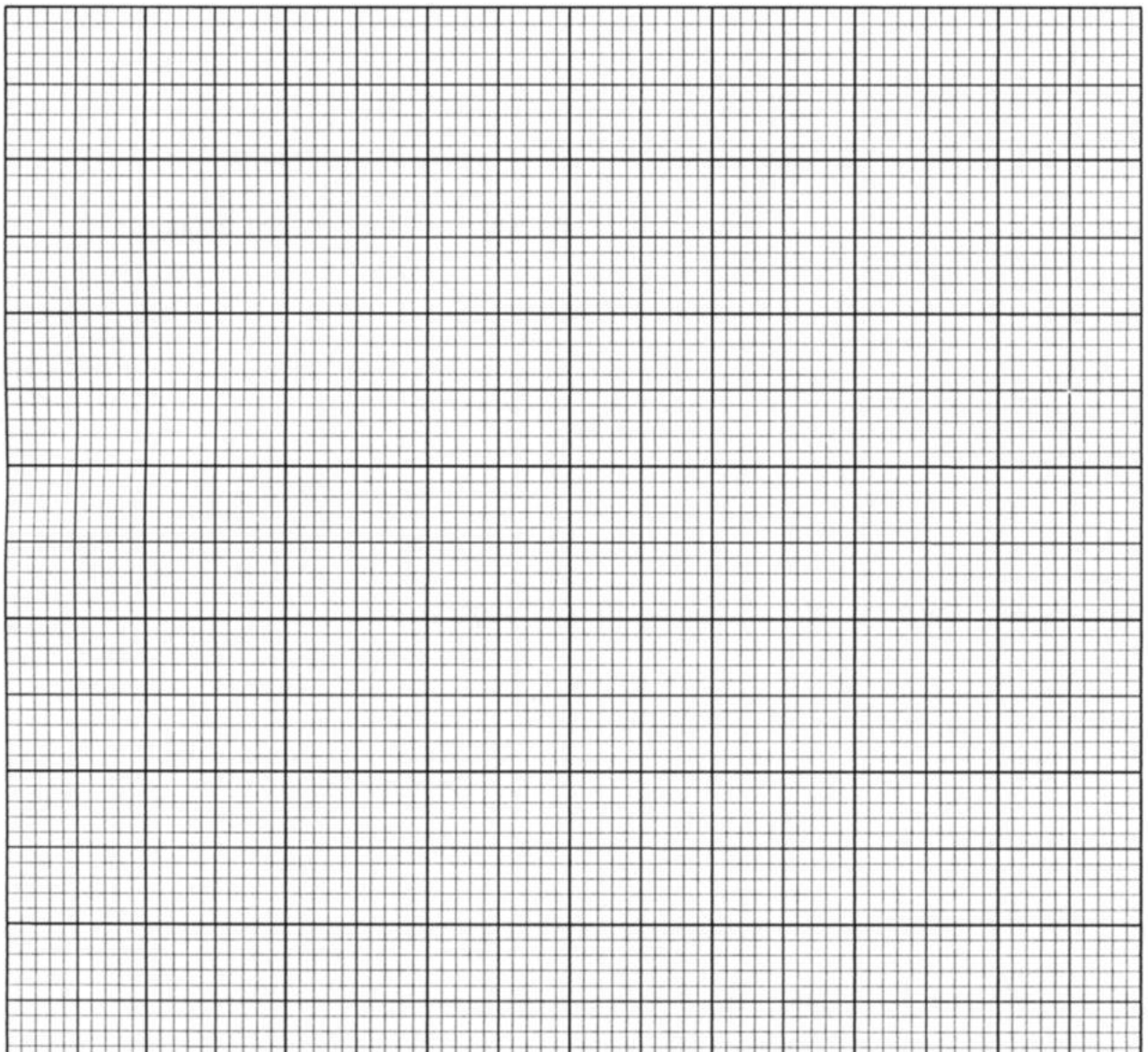
……

…… [1]

(d) Kate paints the external curved surface area of the gift boxes with acrylic paint.
With one bottle of acrylic paint, Kate can paint up to 0.6 m^2 of cardboard.
At the same time, she wants the volume of the each gift box to be at least 600 cm^3.

By drawing a suitable graph, work out the maximum number of gift boxes she can paint with one bottle of acrylic paint.

Use the grid on page 17 and answer space on page 18. [7]



Answer for Question 10(d)

End of Paper