

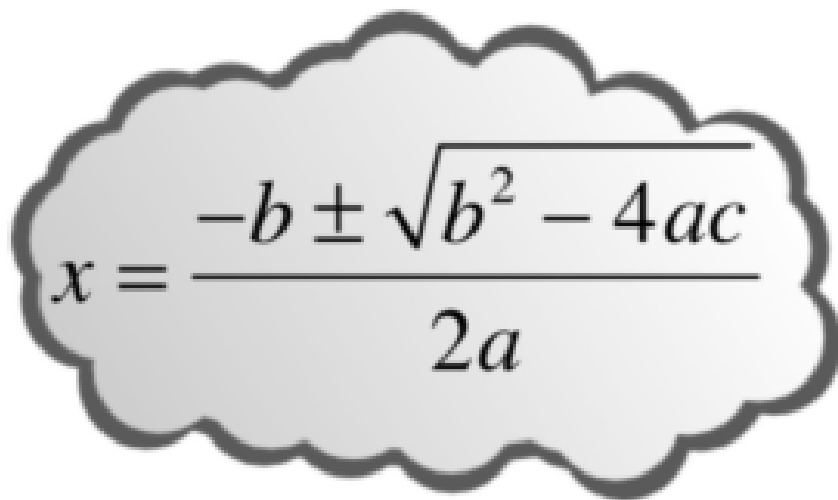
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Learning Outcomes

- Write a quadratic equation in standard form and identify the values of a , b , and c in a standard form quadratic equation.
- Use the Quadratic Formula to find all real solutions of a quadratic equation

You can solve any quadratic equation by **completing the square**—rewriting part of the equation as a perfect square trinomial. If you complete the square on the generic equation $ax^2+bx+c=0$ and then solve for x , you find that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This equation is known as the Quadratic Formula.

The Quadratic Formula


$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic formula

We can derive the quadratic formula by completing the square. First, assume that the leading coefficient is positive; if it is negative, we can multiply the equation by -1 and obtain a positive a . Given $ax^2+bx+c=0$, $a \neq 0$, we will complete the

square as follows:

1. First, move the constant term to the right side of the equal sign:
$$ax^2 + bx = -c$$
2. As we want the leading coefficient to equal 1, divide through by a :
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
3. Then, find $\frac{1}{4}$ of the middle term, and add
$$\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 = \frac{b^2}{4a^2}$$
 to both sides of the equal sign:
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$
4. Next, write the left side as a perfect square. Find the common denominator of the right side and write it as a single fraction:
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
5. Now, use the square root property, which gives
$$\begin{array}{l} x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ x + \frac{b}{2a} = \frac{\pm \sqrt{b^2 - 4ac}}{2a} \end{array}$$
6. Finally, add $-\frac{b}{2a}$ to both sides of the equation and combine the terms on the right side. Thus,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula is very helpful for solving quadratic equations that are difficult or impossible to factor, and using it can be faster than completing the square. The Quadratic Formula can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$.

Writing a Quadratic Equation in Standard Form

Remember that the form $ax^2 + bx + c = 0$ is called standard form of a quadratic equation. Before solving a quadratic equation using the Quadratic Formula, it is *vital* that you be sure the equation is in this form. If you do not, you might use the wrong values for a , b , or c , and then the formula will give incorrect solutions.

The following examples show how to ensure that your quadratic equation is in standard form and then correctly identify the values you will be using for a , b , and c in the Quadratic Formula.

Example

Rewrite the equation $3x + 2x^2 + 4 = 5$ in standard form and identify a , b , and c .

Show Solution

First be sure that the right side of the equation is 0. In this case, all you need to do is subtract 5 from both sides.

$$3x+2x^2+4=5 \parallel 3x+2x^2+4-5=5-5 \end{array} [/latex]$$

Simplify, and write the terms with the exponent on the variable in descending order.

$$\begin{array}{r} 3x+2x^2-1=0 \\ 2x^2+3x-1=0 \end{array}$$

Now that the equation is in standard form, you can read the values of a , b , and c from the coefficients and constant. Note that since the constant 1 is subtracted, c must be negative.

$$\begin{array}{l} 2x^2 + 3x - 1 = 0 \\ \downarrow \\ x^2 + 3x - 1 = 0 \\ \downarrow \\ ax^2 + bx + c = 0 \end{array}$$

Answer

$$2x^2+3x-1=0;a=2,b=3,c=-1$$

Example

Rewrite the equation $2(x+3)^2-5x=6$ in standard form and identify a , b , and c .

Show Solution

First be sure that the right side of the equation is $[0]$.

$$\begin{array}{c} 2 \\ \left(x+3\right)^2-5x=6 \\ 2\left(x+3\right)^2-5x-6=6-6 \end{array}$$

Expand the squared binomial, then simplify by combining like terms.

Be sure to write the terms with the exponent on the variable in descending order.

$$\begin{array}{r} 2(x^2+6x+9)-5x-6=0 \\ 2x^2+12x+18-5x-6=0 \\ 2x^2+7x+12=0 \end{array}$$

Now that the equation is in standard form, you can read the values of a , b , and c from the coefficients and constant.

$$\begin{array}{l} 2x^2 \\ + \\ 7x \\ + \\ 12 \\ = \\ 0 \\ \downarrow \\ a \\ \downarrow \\ b \\ \downarrow \\ c \\ a=2, b=7, c=12 \end{array}$$

Answer

$$2x^2+7x+12=0; a=2, b=7, c=12$$

Try It



[See this interactive in the course material.](#)

Solving a Quadratic Equation using the Quadratic Formula

The Quadratic Formula will work with *any* quadratic equation, but *only* if the equation is in standard form, $ax^2+bx+c=0$. To use it, follow these steps.

1. Put the equation in standard form first.
2. Identify the coefficients, a , b , and c . Be sure to include negative signs if the bx or c terms are subtracted.
3. Carefully substitute the values noted in step 2 into the equation. To avoid needless errors, use parentheses around each number input into the formula.
4. Simplify as much as possible.
5. Use the $\pm$ in front of the radical to separate the solution into two values: one in which the square root is added and one in which it is subtracted.
6. Simplify both values to get the possible solutions.

That is a lot of steps. Let us try using the Quadratic Formula to solve a relatively simple equation first; then you will go back and solve it again using another factoring method.

Example

Use the Quadratic Formula to solve the equation $x^2+4x=5$.

Show Solution

First write the equation in standard form.

$$x^2+4x=5, \quad x^2+4x-5=0, \quad a=1, \quad b=4, \quad c=-5$$

Note that the subtraction sign means the constant c is negative.

$$\begin{array}{l} x^2 \\ +4x \\ -5 \\ =0 \end{array} \quad \begin{array}{l} \downarrow \\ a \\ \downarrow \\ b \\ \downarrow \\ c \end{array} \quad \begin{array}{l} x^2 \\ +bx \\ +c \\ =0 \end{array}$$

Substitute the values into the Quadratic Formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{array}{l} x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-5)}}{2(1)} \end{array}$$

Simplify, being careful to get the signs correct.

$$x = \frac{-4 \pm \sqrt{16+20}}{2}$$

Simplify some more.

$$x = \frac{-4 \pm \sqrt{36}}{2}$$

Simplify the radical: $\sqrt{36}=6$.

$$x = \frac{-4 \pm 6}{2}$$

Separate and simplify to find the solutions to the quadratic equation. Note that in one, 6 is added and in the other, 6 is subtracted.

$$\begin{array}{c} x = \frac{-4+6}{2} = \frac{2}{2} = 1 \text{ or } x = \frac{-4-6}{2} = \frac{-10}{2} = -5 \\ \text{end{array}} \end{array}$$

The solutions are $x=1$ or $x=-5$.

You can check these solutions by substituting 1 and -5 into the original equation.

$\begin{array}{l} x=1 \\ x^2+4x=5 \\ \left(1\right)^2+4\left(1\right)=5 \\ 1+4=5 \\ 5=5 \end{array}$	$\begin{array}{l} x=-5 \\ x^2+4x=5 \\ \left(-5\right)^2+4\left(-5\right)=5 \\ 25-20=5 \\ 5=5 \end{array}$
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You get two true statements, so you know that both solutions work: $x=1$ or $x=-5$. You have solved the equation successfully using the Quadratic Formula!

Try it



[See this interactive in the course material.](#)

Sometimes, it may be easier to solve an equation using conventional factoring methods like finding number pairs that sum to one number (in this example, 4) and that produce a specific product (in this example -5) when multiplied. The power of the Quadratic Formula is that it can be used to solve *any* quadratic equation, even those where finding number combinations will not work.

In our next two video examples, we will see, first, a quadratic equation with two real, rational solutions and, second, a quadratic equation that has irrational solutions and that could not have been solved using factoring.



[Video Link](#)

In the next video example, we show that the quadratic formula is useful when a quadratic equation has two irrational solutions that could not have been obtained by factoring.



[Video Link](#)

Most of the quadratic equations you have looked at have two solutions, like the one above. The following example is a little different.

Example

Use the Quadratic Formula to solve the equation $x^2 - 2x = 6x - 16$.

Show Solution

Subtract $6x$ from each side and add 16 to both sides to put the equation in standard form.

$$x^2 - 2x = 6x - 16 \quad x^2 - 2x - 6x + 16 = 0 \quad x^2 - 8x + 16 = 0$$

Identify the coefficients a , b , and c . $x^2 = 1x^2$, so $a = 1$. Since $8x$ is subtracted, b is negative. $a = 1, b = -8, c = 16$

$$\begin{array}{r} x^2 \\ - 8x \\ + 16 \\ \hline = 0 \end{array} \quad \begin{array}{l} \downarrow \\ a = 1 \\ \downarrow \\ b = -8 \\ \downarrow \\ c = 16 \end{array}$$

Substitute the values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)}$$

Simplify.

$$x = \frac{8 \pm \sqrt{64 - 64}}{2}$$

Since the square root of 0 is 0 , and both adding and subtracting 0 give the same result, there is only one possible value.

$$x = \frac{8 \pm \sqrt{0}}{2} = \frac{8}{2} = 4$$

The answer is $x = 4$.

Again, check using the original equation.

$$x^2 - 2x = 6x - 16 \quad \left(4\right)^2 - 2\left(4\right) = 6\left(4\right) - 16 \\ 16 - 8 = 24 - 16 \quad 8 = 8$$

In the following video we show an example of using the quadratic formula to solve a quadratic equation that has one repeated solution.



[Video Link](#)

In the next example, we will show that some quadratic equations do not have real solutions. As we simplify with the quadratic formula, we may end up with a negative number under a square root, which, as we know, is not defined for real numbers.

Example

Use the Quadratic Formula to solve the equation $x^2 + x = -x - 3$

Show Solution

Add x to both sides and add 3 to both sides to get the quadratic equation in standard form.

$$x^2 + x = -x - 3 \implies x^2 + 2x + 3 = 0$$

Identify a, b, c.

$$a=1, b=2, c=3$$

Substitute values for a, b, c into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \implies x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)}$$

Simplify

$$x = \frac{-2 \pm \sqrt{-8}}{2}$$

Since the square root of a negative number is not defined for real numbers, there are no real number solutions to this equation.

Summary

The Quadratic Formula is a useful way to solve any quadratic equation. The Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, is found by completing the square of the quadratic equation. When you simplify using the quadratic formula and your result is a negative number under a square root, there are no real number solutions to the equation.

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