Assignment 3

PM1/04 (Group B)

(Symbols have their usual meanings)

1. Define surface in R^3. Define surface patch in R^3. Give example.

2. Define regular surface patch.

3. Define an allowable surface patch. Define smooth surface.

4. Which of the following are regular surface patches? (in each case $u,v \in R$)

(i) $\sigma(u,v)= (\cos u, \sin u, v)$, (ii) $\sigma(u,v)= (u, v, uv)$, (iii) $\sigma(u,v)= (u, v^2, v^3)$, (iv) $\sigma(u,v)= (u+u^2, v, v^2)$

5. Define derivative of smooth map $f : S \to \tilde{S}$, where S and $\tilde{S}$ being two surfaces in R^3. Write down the expression of derivative of smooth maps at any point p in terms of surface patches.

6. If $f : S \to \tilde{S}$ is a smooth map between surfaces at any point $p \in S$, prove that the derivative $D_p f : T_p S \to T_{f(p)}\tilde{S}$ is linear.

7. If $f : S \to \tilde{S}$ is a smooth map between surfaces. Then prove that f is a local diffeomorphism if and only if for all $p \in S$, the linear map $D_p f : T_p S \to T_{f(p)}\tilde{S}$ is invertible.

8. Define standard unit normal of the surface patch σ at p. When a surface is said to be orientable?