

Assignment 3

PM1/04 (Group B)

(Symbols have their usual meanings)

1. Define surface in \mathbb{R}^3 . Define surface patch in \mathbb{R}^3 . Give example.
2. Define regular surface patch.
3. Define an allowable surface patch. Define smooth surface.
4. Which of the following are regular surface patches? (in each case $u, v \in \mathbb{R}$)
(i) $\sigma(u, v) = (\cos u, \sin u, v)$, (ii) $\sigma(u, v) = (u, v, uv)$, (iii) $\sigma(u, v) = (u, v^2, v^3)$, (iv) $\sigma(u, v) = (u+u^2, v, v^2)$
5. Define derivative of smooth map $f : S \rightarrow \tilde{S}$, where S and \tilde{S} being two surfaces in \mathbb{R}^3 . Write down the expression of derivative of smooth maps at any point p in terms of surface patches.
6. If $f : S \rightarrow \tilde{S}$ is a smooth map between surfaces at any point $p \in S$, prove that the derivative $D_p f : T_p S \rightarrow T_{f(p)} \tilde{S}$ is linear.
7. If $f : S \rightarrow \tilde{S}$ is a smooth map between surfaces. Then prove that f is a local diffeomorphism if and only if for all $p \in S$, the linear map $D_p f : T_p S \rightarrow T_{f(p)} \tilde{S}$ is invertible.
8. Define standard unit normal of the surface patch σ at p . When a surface is said to be orientable?