A IAL Pure Maths 4
Binomial Expansion
MS

1.Jan 2025

3(a)	$B = 6 \times 4^{-\frac{1}{2}} = 3$	B1
	$3 \times \left(-\frac{1}{2}\right)\left(\frac{A}{4}\right) = -\frac{1}{4} \Rightarrow A = \frac{2}{3}$	M1 A1
	$C = 3 \times \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2}\left(\frac{A}{4}\right)^2 \Rightarrow C = \frac{1}{32}$	M1 A1
		(5)

Notes:

(a)

B1: For $B = 3$ which may be implied by their expansion e.g. $3 \pm ...$

M1: Attempts to find A condoning the omission of their "3".

Look for $-\frac{1}{2} \times \left(\frac{Ax}{m}\right) = -\frac{1}{4}x$ o.e. where m could be 1 leading to a value for A.

Must be consistent with their $...\left(1 + \frac{A}{m}x\right)^{-\frac{1}{2}}$ but condone the omission of the "3".

Note the omission of the "3" (if correct) leads to $A = 2$.

A1: $A = \frac{2}{3}$ or an exact equivalent or e.g. $0.\dot{6}$

Allow if seen embedded in $6(4 + Ax)^{-\frac{1}{2}}$

M1: Attempt to find C condoning the omission of their "3".

Look for $\frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times \left(\frac{A}{m}\right)^2 = ...$ or $\frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times \left(\frac{Ax}{m}\right)^2 = ...$ oe with their numerical value of A leading to a value for C

Must be consistent with their $...\left(1 + \frac{A}{m}x\right)^{-\frac{1}{2}}$ where m could be 1

Note the omission of the "3" (if correct) with a correct A leads to $C = \frac{1}{96}$.

A1: $C = \frac{1}{32}$ oe e.g. 0.03125

Allow if seen embedded in their expansion.

3(b)	$-6 < x \leqslant 6$	B1ft
		(1)
(c)	coefficient of x^3 is $3\times\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\frac{A}{4}\right)^3 = \ldots$	M1
	$-\frac{5}{1152}$	A1
		(2)

Notes:

(b)

B1ft: $-6 < x \leqslant 6$ but follow through on their A so allow for $-\frac{4}{A} < x \leqslant \frac{4}{A}$

Condone strict or non-strict inequalities for either end e.g. condone $|x| < 6$

or e.g. $-6 < x < 6$ but follow through on their A e.g. condone $|x| \leqslant \left|\frac{4}{"A"}\right|$

Accept alternative notation e.g. $x > -6$ and $x \leqslant 6$, $(-6, 6)$, $(-6, 6]$ etc.

(c)

M1: Attempt to find the coefficient of x^3 or the term in x^3 to obtain a value condoning the omission of their "3".

Look for an attempt at $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\times\left(\frac{A}{m}\right)^3 = \ldots$ with their numerical value of A.

Must be consistent with their $\ldots\left(1+\frac{A}{m}x\right)^{-\frac{1}{2}}$ from part (a) where m could be 1

Note the omission of the "3" with a correct A leads to coefficient of $-\frac{5}{3456}$.

A1: $-\frac{5}{1152}$. Condone $-\frac{5}{1152}x^3$ which may be seen embedded in an expansion.

Correct answer only with correct work in (a) scores both marks.

Expansions for reference

$$6(4+Ax)^{-\frac{1}{2}} = 6\times 4^{-\frac{1}{2}}\left(1+\frac{A}{4}x\right)^{-\frac{1}{2}} = 3\left(1-\frac{A}{8}x+\frac{3A^2}{128}x^2-\frac{5A^3}{1024}x^3+\dots\right)$$

$$=3-\frac{3A}{8}x+\frac{9A^2}{128}x^2-\frac{15A^3}{1024}x^3+\dots$$

or e.g. (direct expansion)

$$6(4+Ax)^{-\frac{1}{2}} = 6\left(4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(Ax)+\frac{(-\frac{1}{2})(-\frac{1}{2}-1)}{2}4^{-\frac{5}{2}}(Ax)^2+\frac{(-\frac{1}{2})(-\frac{1}{2}-1)(-\frac{1}{2}-2)}{3!}4^{-\frac{7}{2}}(Ax)^3+\dots\right)$$

$$=6\left(\frac{1}{2}-\frac{1}{16}(Ax)+\frac{3}{256}(Ax)^2-\frac{5}{2048}(Ax)^3+\dots\right)=3-\frac{3A}{8}x+\frac{9A^2}{128}x^2-\frac{15A^3}{1024}x^3+\dots$$

2.Jan 2024

1	$(1-4x)^{-3} = 1\underline{\pm 3} \times 4x \pm \underline{\frac{-3\times -4}{2}}(...x)^2 \pm \underline{\frac{-3\times -4\times -5}{6}}(...x)^3 + ...$	**M1**
	$(1-4x)^{-3} = 1 + 3\times 4x + \frac{-3\times -4}{2}(-4x)^2 + \frac{-3\times -4\times -5}{6}(-4x)^3 + ...$	**A1**
	$= 1 + 12x + 96x^2 + 640x^3 + ...$	**A1A1**
		(4)
		(4 marks)

Notes:

M1: Attempts the binomial expansion of $(1-4x)^{-3}$ with correct attempts at the (unsimplified, and may be in terms of factorials) binomial coefficients for **at least two** of the x, x^2, x^3 terms. The "−4" may be missing or have incorrect sign and allow for missing brackets. (May be scored if the x^3 term is omitted.)

A1: For a correct unsimplified expansion. (May be in terms of factorials.)

A1: Any two terms correct and simplified (of the four, so includes the 1). The M must have been scored.

A1: Fully correct and all terms simplified. Ignore any higher order terms. ISW after a correct simplified.

Note M1A0A1A0 is possible e.g. if the x and x^3 terms have the wrong signs.

Note: listing terms can score maximum of M1A1A1A0.

3.Oct 2024

1(a)	$(8-3x)^{-\frac{1}{3}}=\frac{1}{2}\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}$	B1
	$\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}=1+\left(-\frac{1}{3}\right)\left(-\frac{3}{8}x\right)+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\left(-\frac{3}{8}x\right)^2+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\left(-\frac{3}{8}x\right)^3+\ldots$	M1
	$(8-3x)^{-\frac{1}{3}}=\frac{1}{2}+\frac{1}{16}x+\frac{1}{64}x^2+\frac{7}{1536}x^3+\ldots$	A1 A1
		(4)
(b)	$\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3+\ldots=\frac{2851}{5184}\Rightarrow\sqrt[3]{6}=\left(\frac{2851}{5184}\right)^{-1}=\ldots$	M1
	$=\frac{5184}{2851}$ or $1\frac{2333}{2851}$	A1
		(2)
		Total 6

Note a misread of $(8-3x)^{\frac{1}{3}}$ can score a maximum of B1M1A0A0 M0A0

(a)

B1: Obtains $\frac{1}{2}(1\pm\ldots x)^{-\frac{1}{3}}$. $8^{-\frac{1}{3}}$ must be evaluated. May be implied by further work e.g. $\frac{1}{2}\pm\ldots$ provided this has not come from an incorrect method.

e.g. $(8-3x)^{-\frac{1}{3}}=\frac{1}{(8-3x)^{\frac{1}{3}}}=\frac{1}{2}+\frac{1}{\frac{1}{2}\sqrt[3]{3x}}+\frac{1}{\frac{1}{2}\left(\sqrt[3]{3x}\right)^2}+\ldots$ is B0.

M1: Attempts the binomial expansion of $(1+kx)^n$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 which may be unsimplified.

Look for $\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\ldots x^2$ or $\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\ldots x^3$ o.e. (you do not need to be concerned with their $-\frac{3}{8}$ which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\begin{pmatrix}-\frac{1}{3}\\1\end{pmatrix}$, $\begin{pmatrix}-\frac{1}{3}\\2\end{pmatrix}$ unless interpreted correctly in further work.

May be implied by correct coefficients with x^2 or x^3

A1: For 2 correct **simplified** terms of $\frac{1}{16}x$, $\frac{1}{64}x^2$, $\frac{7}{1536}x^3$ (allow x^1 for x for this mark) which may be listed.

Condone if coefficients are given as decimals i.e. $0.0625x$, $0.015625x^2$, $0.004557\ldots x^3$

Also allow for $\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}=1+\frac{1}{8}x+\frac{1}{32}x^2+\frac{7}{768}x^3+\ldots$ (Note B0M1A1A0 is possible)

A1: $\frac{1}{2}+\frac{1}{16}x+\frac{1}{64}x^2+\frac{7}{1536}x^3$. Isw following a correct answer.

Condone the position of x^n provided it is not clearly on the denominator. e.g. condone $\frac{1}{16\ x}$ but not $\frac{1}{16x}$. Ignore terms with higher powers of x. Do not accept $\frac{1}{16}x^1$

(b)

M1: Attempts to substitute $x=\frac{2}{3}$ into their expansion from part (a) to achieve a fraction and attempts to find the reciprocal of that fraction. May be implied by their fraction. You may need to check this on your calculator. They may attempt to find an additional term which is fine. Do not be concerned as to whether the fraction is correct for their binomial expansion if they have shown the substitution. Condone slips including losing one of their terms.

$"\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3"=\frac{A}{B}\Rightarrow\frac{B}{A}$ scores M1

$1\div\frac{2851}{5184}$ scores M1

$\frac{1}{"\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3"}$ on its own scores M0

A1: $\frac{5184}{2851}$ or $1\frac{2333}{2851}$. Isw once a correct answer is seen. Correct answer scores M1A1.

4.Jan 2023

1 (a)	$\frac{5x+10}{(1-x)(2+3x)} \equiv \frac{A}{1-x}+\frac{B}{2+3x} \Rightarrow$ Value for A or B	**M1**
	One correct value, either $A=3$ or $B=4$	**A1**
	Correct PF form $\frac{3}{1-x}+\frac{4}{2+3x}$	**A1**
		(3)
(b)(i)	$\frac{A}{1-x}=A(1-x)^{-1}=A\left(1+x+x^2+\ldots\right)$	**B1**
	$\left\{\frac{B}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1}=\left\{\frac{B}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\ldots\right);=\frac{B}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots\right)$	**M1; A1**
	$f(x)= \quad 3\times\left(1+x+x^2+\ldots\right)+ \quad \frac{4}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots\right)$	**M1**
	$=5+\frac{15}{2}x^2+\ldots$	**A1**
		(5)
(b)(ii)	$\lvert x\rvert<\frac{2}{3}$	**B1**
		(1)
		(9 marks)

(b)(i) Alt 1	$(1-x)^{-1}=1+x+x^2+\ldots$	**B1**
	$\left\{\frac{1}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1}=\left\{\frac{1}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\ldots\right);=\frac{1}{2}\left(1-\frac{3x}{2}+\frac{9}{4}x^2+\ldots\right)$	**M1; A1**
	$\frac{5x+10}{(1-x)(2+3x)}=(5x+10)\left(1+x+x^2+\ldots\right)\times\frac{1}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots\right)=5+\ldots x+\ldots x^2$	**M1**
	$=5+\frac{15}{2}x^2+\ldots$	**A1**
		(5)
(b)(i) Alt 2	$\frac{5x+10}{(1-x)(2+3x)}=(5x+10)\left(2+\left(x-3x^2\right)\right)^{-1}=\frac{1}{2}(5x+10)\left(1+\frac{1}{2}\left(x-3x^2\right)\right)^{-1}$	**B1**
	$\left(1+p(x)\right)^{-1}=\left(1\pm p(x)+\frac{(-1)(-2)}{2}\left(p(x)\right)^2+\ldots\right);\ \frac{1}{2}\left(1-\frac{1}{2}\left(x-3x^2\right)+\frac{1}{4}\left(x-3x^2\right)^2+\ldots\right)$	**M1; A1**
	$(10+5x)\left(\frac{1}{2}-\frac{1}{4}x+\frac{3}{4}x^2+\frac{1}{8}x^2+\ldots\right)=5-\frac{5}{2}x+\frac{35}{4}x^2+\frac{5}{2}x-\frac{5}{4}x^2+\ldots$	**M1**

	$=5+\frac{15}{2}x^2+\ldots$	**A1**
		(5)

Notes:

a)

M1: Attempts at correct PF. Correct form identified (may be implicit) and achieves a value for at least one of the constants.

A1: One correct value or term.

A1: Correct PF form $\frac{3}{1-x}+\frac{4}{2+3x}$. This may be awarded if seen in (b) but the correct final form (not just values) must be seen somewhere in the question. Accept at $3(1-x)^{-1}+4(2+3x)^{-1}$

(b)(i)

B1: $\frac{A}{1-x}=A(1-x)^{-1}=A\left(1+x+x^2+\ldots\right)$ which may be unsimplified. Allow with their A or with $A=1$.

M1: Attempts to expand $\frac{1}{2+3x}=(2+3x)^{-1}$ binomially either by taking out the factor 2 first, or directly. Look for $(1+kx)^{-1}=\ldots\left(1\pm kx+\frac{(-1)(-2)}{2}(kx)^2+\ldots\right)$ where $k\neq 1$ following an attempt at taking out a factor 2, or $\frac{1}{2+3x}=(2+3x)^{-1}=\left(2^{-1}\pm 2^{-2}kx+\frac{(-1)(-2)}{2}2^{-3}(kx)^2+\right)$ by direct expansion. Allow missing brackets on kx^2 in either case.

A1: $\frac{B}{2+3x}=\frac{B}{2}\left(1+\frac{3x}{2}\right)^{-1}=\frac{B}{2}\left(1-\frac{3x}{2}+\frac{9}{4}x^2+\right)$ oe with their B from (a) or with $B=1$

M1: Uses their coefficients and attempts to add both series.

A1cao: $5+\frac{15}{2}x^2+\ldots$ Condone additional higher order terms. Terms may be either order.

(b)(ii)

B1: $|x|<\frac{2}{3}$ or exact equivalent. This must be clearly identified as the answer. B0 if both ranges are given with no choice of which is correct. (But B1 if formal set notation with $\cap$ used.)

(b)(i) Alt 1:

B1: $(1-x)^{-1}=1+x+x^2+\ldots$ which may be unsimplified.

M1: Same as main scheme.

A1: Correct expansion (see main scheme, $B=1$ allowed).

M1: Attempts to expand all three brackets, achieving the correct constant term at least.

A1cso: $5+\frac{15}{2}x^2+\ldots$ Condone additional higher order terms. Terms may be either order.

(b)(i) Alt 2

B1: Writes f(x) as $(5x+10)\left(2+\left(x-3x^2\right)\right)^{-1}$ or with the 2 extracted, with the $\left(x-3x^2\right)$ clear.

M1: Attempts the binomial expansion on $\left(1+p(x)\right)^{-1}$ or $\left(2+p(x)\right)^{-1}$ for $p(x)$ of form $ax+bx^2$.
Same conditions as for main scheme.
A1: Correct expansion. For direct expansion $\left(\frac{1}{2}-\frac{1}{4}\left(x-3x^2\right)+\frac{1}{8}\left(x-3x^2\right)^2+...\right)$
M1: Expands the brackets achieving at least the correct constant term.
A1cao: $5+\frac{15}{2}x^2+...$ Condone additional higher order terms. Terms may be either order.

5.June 2023

1(a)	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=8(1-2x)^{-\frac{3}{2}}$	B1
	$(1-2x)^{-\frac{3}{2}}=1+\left(-\frac{3}{2}\right)(-2x)+\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2+\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(-2x)^3+\ldots$	M1 A1
	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=8+24x+60x^2+140x^3+\ldots$	A1, A1
		(5)
(b)	$n=2$	B1
		(1)
(c)	$\left(\frac{1}{4}-\frac{1}{2}x\right)^2=\frac{1}{16}-\frac{1}{4}x+\frac{1}{4}x^2$	B1
	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}=\left(\frac{1}{16}-\frac{1}{4}x+\frac{1}{4}x^2\right)\left(8+24x+60x^2+140x^3+\ldots\right)$ $=8\times\frac{1}{16}+24x\times\frac{1}{16}+60x^2\times\frac{1}{16}-8\times\frac{1}{4}x-24x\times\frac{1}{4}x+8\times\frac{1}{4}x^2$	M1
	$=\frac{1}{2}-\frac{1}{2}x-\frac{1}{4}x^2+\ldots$	A1
		(3)
		Total 9

(a)

B1: Takes out the **correct and simplified** common factor to obtain $8(1\pm\ldots)^{-\frac{3}{2}}$ Implied by an expansion 8 + ...

M1: Attempts the binomial expansion of $(1\pm kx)^{-\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.
The correct binomial coefficient must be combined with the correct power of x.
Look for $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(\pm kx)^2$ or $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(\pm kx)^3$ with or without the brackets on the kx
Even allow with $k=1$

A1: Correct simplified or unsimplified expansion for $(1-2x)^{-\frac{3}{2}}$. (NB simplified is $1+3x+\frac{15}{2}x^2+\frac{35}{2}x^3+\ldots$)

A1: Two correct and simplified terms of $8+24x+60x^2+140x^3$

A1: All correct and simplified $8+24x+60x^2+140x^3$ Ignore extra terms of x^4 and above

(b)

B1: Correct value, $n=2$. Do NOT allow incomplete answers such as $\frac{4}{2}$

(c) **Hence**

B1: Correct expansion of $\left(\frac{1}{4}-\frac{1}{2}x\right)^2$, simplified or unsimplified

M1: Attempts correct strategy to find expansion.
Terms do not need to be collected. There may be other terms as well, for example terms in x^3
Look for an attempt to find 4 or more terms from the following (condoning slips)
$\left(A+Bx+Cx^2\right)\left(E+Fx+Gx^2+Hx^3+\ldots\right)=AE+AFx+AGx^2+BEx+BFx^2+CEx^2+\ldots$

A1: Correct simplified expansion. Ignore extra terms of x^3 and above

www.doteducation.org
+971525465652

(c) **Otherwise:** Applies binomial expansion to $\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}$

B1: Correct simplified expression $\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}=\frac{1}{2}(1-2x)^{\frac{1}{2}}$

M1: Correct structure for 3rd term: $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-2x)^2$ but allow $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(\pm kx)^2$ with or without brackets. Even allow with $k=1$

A1: $\frac{1}{2}-\frac{1}{2}x-\frac{1}{4}x^2+\ldots$ Ignore extra terms of x^3 and above

Direct expansion in (a)

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=\left(\frac{1}{4}\right)^{-\frac{3}{2}}+\left(-\frac{3}{2}\right)\left(\frac{1}{4}\right)^{-\frac{5}{2}}\left(-\frac{1}{2}x\right)+\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{-\frac{7}{2}}\left(-\frac{1}{2}x\right)^2+\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)\times\left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{-\frac{9}{2}}\left(-\frac{1}{2}x\right)^3$$

B1: Obtains an expansion with a constant term of 8

M1: Attempts the binomial expansion of $\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.

Look for $\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{-\frac{7}{2}}\left(\pm\frac{1}{2}x\right)^2$ or $\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)\times\left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{-\frac{9}{2}}\left(\pm\frac{1}{2}x\right)^3$

with or without the brackets on the $\left(\pm\frac{1}{2}x\right)$

A1: Correct simplified or unsimplified expansion .
Expression at the top of the page is acceptable

A1: Two correct and simplified terms of $8+24x+60x^2+140x^3$

A1: All correct and simplified $8+24x+60x^2+140x^3$

Direct expansion in (c)

B1: First two terms which may be unsimplified $\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{1}{2}}+\frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}\left(-\frac{1}{2}x\right)^1+\ldots$

M1: Correct form for the third term $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}\left(-\frac{1}{2}x\right)^2$

A1: Correct simplified expansion $\frac{1}{2}-\frac{1}{2}x-\frac{1}{4}x^2+\ldots$

DOT EDUCATION (KHDA APPROVED)

6.Oct 2023

1(a)	$(2-5x)^{-2} = \frac{1}{4}\left(1-\frac{5x}{2}\right)^{-2}$ or e.g. $\frac{8}{4\left(1-\frac{5x}{2}\right)^{2}}$	B1
	$= 8\times\frac{1}{4}\left(1+(-2)\times\left(-\frac{5x}{2}\right)+\frac{(-2)\times(-3)}{2!}\times\left(-\frac{5x}{2}\right)^{2}+\frac{(-2)\times(-3)\times(-4)}{3!}\times\left(-\frac{5x}{2}\right)^{3}...\right)$	M1A1
	$\frac{8}{(2-5x)^{2}} = 2+10x+\frac{75}{2}x^{2}+125x^{3}...$	A1
		(4)
Alt (a) by direct expansion	$8\times(2-5x)^{-2}$ $= 8\times\left(2^{-2},+(-2)2^{-3}(-5x)^{1}+\frac{-2\times-3}{2!}2^{-4}(-5x)^{2}+\frac{-2\times-3\times-4}{3!}2^{-5}(-5x)^{3}\right)$	B1, M1A1
	$\frac{8}{(2-5x)^{2}} = 2+10x+\frac{75}{2}x^{2}+125x^{3}...$	A1
(b)	$\lvert x\rvert < \frac{2}{5}$ o.e.	B1
		(1)
		(5 marks)

(a)

B1: For taking out a factor of 2^{-2} or $\frac{1}{4}$ **from** $(2-5x)^{-2}$ to obtain e.g. $\frac{1}{4}(1\pm...)^{-2}$, $2^{-2}(1\pm...)^{-2}$.

May be implied by a constant term of 2 or by e.g. $2(1\pm...)^{-2}$

The "8" is likely to be present but it is not required for this mark.

M1: For the form of the binomial expansion $(1+ax)^{-2}$

Requires the correct structure for either term three or term four. Allow a slip on the sign.

So allow for either $\frac{(-2)\times(-3)}{2}(\pm ax)^2$ or $\frac{(-2)\times(-3)\times(-4)}{3!}(\pm ax)^3$ where $a \neq 1$, could be -5 but must be the "a" in their "2"$(1\pm ax)^{-2}$.

Condone missing brackets around the "ax" for this mark.

A1: Any unsimplified or simplified but correct form of the binomial expansion for $\left(1-\frac{5x}{2}\right)^{-2}$

Ignore the factor preceding the bracket for this mark and ignore any extra terms if found.

Score for $1+(-2)\times\left(-\frac{5x}{2}\right)+\frac{(-2)\times(-3)}{2!}\times\left(-\frac{5x}{2}\right)^2+\frac{(-2)\times(-3)\times(-4)}{3!}\times\left(-\frac{5x}{2}\right)^3$ o.e.

Brackets must be present unless they are implied by later work. Allow $\left(\frac{5x}{2}\right)^2$ for $\left(-\frac{5x}{2}\right)^2$.

The simplified form is $1+5x+\frac{75}{4}x^2+\frac{125}{2}x^3+...$ Allow as a list of terms.

A1: cao $2+10x+\frac{75}{2}x^2+125x^3...$ This must be simplified and allow as a list of terms.

PPROVED)

Allow equivalents for $\frac{75}{2}$ e.g. 37.5 and ignore any extra terms.

Do **not** isw and mark the final answer. E.g. Fully correct work leading to $2+10x+\frac{75}{2}x^2+125x^3...$ followed by $=4+20x+75x^2+250x^3...$ loses the final mark.

Alternative to (a) by direct expansion

B1: This is awarded for $(2-5x)^{-2}=2^{-2}+....$ which may be implied by a final answer of 2 + …

M1: For the form of the expansion of $(2-5x)^{-2}$

Requires the correct structure for either term three or term four. Allow a slip on the sign.

So allow for either $\frac{(-2)\times(-3)}{2}(2)^{-4}(\pm 5x)^2$ or $\frac{(-2)\times(-3)\times(-4)}{3!}(2)^{-5}(\pm 5x)^3$

Condone missing brackets around the $5x$.

A1: Any unsimplified or simplified but correct form of the binomial expansion for $(2-5x)^{-2}$

Ignore the factor preceding the bracket for this mark and ignore any extra terms if found.

Score for $2^{-2}+(-2)2^{-3}(-5x)^1+\frac{-2\times -3}{2!}2^{-4}(-5x)^2+\frac{-2\times -3\times -4}{3!}2^{-5}(-5x)^3$ o.e.

Brackets must be present unless they are implied by later work.

The simplified form is $\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+\frac{125}{8}x^3+...$ Allow as a list of terms.

A1: cao $2+10x+\frac{75}{2}x^2+125x^3...$ This must be simplified and allow as a list of terms.

Do **not** isw and mark the final answer. E.g. Fully correct work leading to $2+10x+\frac{75}{2}x^2+125x^3...$ followed by $=4+20x+75x^2+250x^3...$ loses the final mark.

Note: Correct or partially correct answers with <u>no working</u> in (a) should be sent to review.

(b)

B1: $|x|<\frac{2}{5}$ oe e.g. $-\frac{2}{5}<x<\frac{2}{5}$, $x>-\frac{2}{5}$ and $x<\frac{2}{5}$, $\left(-\frac{2}{5},\frac{2}{5}\right)$

But **not** $-\frac{2}{5}<|x|<\frac{2}{5}$, $x<\frac{2}{5}$, $\left|\frac{5x}{2}\right|<1$, $\left|-\frac{5x}{2}\right|<1$, $-1<\frac{5x}{2}<1$

www.doteducation.org
+971525465652

7.Jan 2022

2(a)	$(1+4x^3)^{\frac{1}{3}} = 1+\frac{1}{3}(...x^{...})+...$	**M1**
	$(1+4x^3)^{\frac{1}{3}} = ...+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}(4x^3)^2+...$	**M1**
	$=1+\frac{4}{3}x^3+...$ or $=...-\frac{16}{9}x^6$	**A1**
	$(1+4x^3)^{\frac{1}{3}} = 1+\frac{4}{3}x^3-\frac{16}{9}x^6+...$	**A1**
		(4)
(b)	$1+4x^3 = 1+4\times\left(\frac{1}{3}\right)^3 = \frac{31}{...}$ **or** $1+\frac{4}{3}\left(\frac{1}{3}\right)^3-\frac{16}{9}\left(\frac{1}{3}\right)^6$	**M1**
	$\sqrt[3]{31}\approx 3\times\left(1+\frac{4}{3}\left(\frac{1}{3}\right)^3-\frac{16}{9}\left(\frac{1}{3}\right)^6\right)$	**dM1**
	$=\frac{6869}{2187}$	**A1**
		(3)
		(7 marks)

Notes:

(a)

M1: For $1 \pm \frac{1}{3}\left(4x^{\cdots}\right)$. Allow if the 4 and/or the power of x is incorrect/missing for this mark.

M1: For the correct structure for the third term. Look for the correct binomial coefficient combined with the correct power of $4x^3$ e.g. $\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}\left(4x^3\right)^2$ but condone missing brackets e.g. $\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}4x^{3^2}$

A1: For $1+\frac{4}{3}x^3+\ldots$ **or** for $\ldots-\frac{16}{9}x^6$.

Allow mixed fractions e.g. $1\frac{1}{3}$ for $\frac{4}{3}$ but do not allow '+–' for '–'.

A1: For $1+\frac{4}{3}x^3-\frac{16}{9}x^6+\ldots$. **Ignore any extra terms.**

(b)

M1: Attempts $1+4\times\left(\frac{1}{3}\right)^3$ and reaches $\frac{31}{k}$ (this will usually be seen as e.g. $\sqrt[3]{\frac{31}{27}}$) **or** substitutes $x=\frac{1}{3}$ into their expansion from part (a).

dM1: Attempts $3\times$ (their expansion from part (a) with $x=\frac{1}{3}$ substituted)

Allow if they have included more terms in their expansion for both M's. You need to be convinced that they have attempted this calculation and not just found the cube root of 31 on their calculator e.g. 3.1414…

A1: Cao. Correct fraction.

8.June 2022

1(a)	$A=\frac{1}{9}$	B1
		(1)
(b)	$"3^{-2}"(1+\underline{(-2)(\frac{kx}{3})}+\underline{\frac{(-2)(-3)}{2}(\frac{kx}{3})^2}+\ldots)$ or $x: \quad 3^{-2}(-2)\left(\frac{k}{3}\right)\left(=-\frac{2k}{27}\right)$ and $x^2: \quad 3^{-2}\frac{(-2)(-3)}{2}\left(\frac{k}{3}\right)^2=\frac{k^2}{27}$	B1
	$\frac{(-2)(-3)}{2}\left(\frac{k}{3}\right)^2=3\times(-2)\left(\frac{k}{3}\right)\Rightarrow \ldots k^2=\ldots k$	M1
	$k^2+6k=0$ *	A1*
		(3)
(c)(i)	$k=-6$	B1
(ii)	$3^{-2}\frac{(-2)(-3)(-4)}{3!}\left(\frac{"-6"}{3}\right)^3=\frac{32}{9}$	M1A1
		(3)
		(7 marks)

Notes
Mark parts (a) and (b) as a whole. **(a)** **B1:** $A=\frac{1}{9}$ **(b)** **B1:** Correct unsimplified coefficients for x and x^2 either in an expansion or separate for $(3+kx)^{-2}$ or for $\left(1+\frac{k}{3}\right)^{-2}$ (accept the 3^{-2} missing or incorrect). May be implied. Accept $B=-\frac{2k}{3}$ and $C=\frac{k^2}{3}$ if they forget the multiple outside. B0 if brackets on $\left(\frac{k}{3}\right)^2$ missing unless implied by recovery. **M1:** Sets their coefficient of x^2 equal to 3 times their coefficient of x to produce a two term quadratic equation in terms of k. **A1*:** Achieves given answer from a correct equation, but condone if B and C both missed the 3^{-2}. May be scored if A was incorrect. **(c)(i)** **B1:** $k=-6$ only. The $k=0$ solution must be rejected. **(ii)** **M1:** Substitutes their non-zero value for k into a correct expression for the coefficient of x^3. Must include the 3^{-2} **A1:** $\frac{32}{9}$ oe

9.Oct 2022

4(a)	$\frac{1}{\sqrt{4-x^2}}=\left(4-x^2\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\left(1-...\right)$	B1
	$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{1}{4}x^2\right)^2+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$	M1, A1
	$\frac{1}{\sqrt{4-x^2}}=\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$	A1, A1
		(5)
(b)	$\lvert x\rvert<2$	B1
		(1)
(c)	Substitutes an appropriate value of x in both sides with LHS in terms of $\sqrt{3}$	M1
	E.g. with $x=1$ $\sqrt{3}=\frac{2048}{1181}$ or $\frac{3543}{2048}$	A1
		(2)
		(8 marks)

(a) Note that the first mark in (a) is M1 on epen. We are now marking it as B1
B1: Correct constant term/ factor.

Accept as $4^{-\frac{1}{2}}$ or $\frac{1}{2}$ as the constant term or as the factor with the bracket starting (1....

M1: Correct attempt at the third or the fourth term of binomial expansion of form $\left(1+ax^2\right)^{-\frac{1}{2}}$

Look for a correct binomial coefficient with a correct power of x

E.g. $\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(\pm ax^2\right)^2$ or $\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(\pm ax^2\right)^3$ where a could be 1

A1: Correct unsimplified
For example

$$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(-\frac{1}{4}x^2\right)^2+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$$

A1: Two correct and simplified terms of $\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$

A1: $\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$. Accept as a list, ignore any terms with indices greater than these.

(b)

B1: States a correct range. E.g $|x|<2$ or $-2<x<2$ o.e.

(c) For example: See **

M1: Substitutes an exact value of x into both sides of their expansion that enables $\sqrt{3}$ to be found

E.g. $x=1$ $\dfrac{1}{\sqrt{4-1^2}}=\frac{1}{2}+\frac{1}{16}\times1^2+\frac{3}{256}\times1^4+\frac{5}{2048}\times1^6$

Note that $x=-1$ and $x^2=1$ all work with the same results

Alternatively correctly solves $\dfrac{1}{\sqrt{4-x^2}}=\sqrt{3}$ o.e. and substitutes this into their expansion

A1: With $x=1$ $\sqrt{3}=\frac{2048}{1181}$ or $\frac{3543}{2048}$. This can only be scored from a correct expansion

**There are lots of values of x that work, all more difficult than the one above.

E.g $x^2=\frac{8}{3}\Rightarrow\dfrac{1}{\sqrt{4-x^2}}=\dfrac{\sqrt{3}}{2}$ and $x^2=\frac{11}{3}\Rightarrow\dfrac{1}{\sqrt{4-x^2}}=\sqrt{3}$

In both these cases the fractions become much more difficult to find.

For $x^2=\frac{8}{3}\Rightarrow\sqrt{3}=\frac{43}{27}$ and for $x^2=\frac{11}{3}\Rightarrow\sqrt{3}=\frac{55687}{55296}$

Alt I (a) By direct expansion

$$\frac{1}{\sqrt{4-x^2}}=\left(4-x^2\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)\times4^{-\frac{3}{2}}\left(-x^2\right)^1+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times4^{-\frac{5}{2}}\left(-x^2\right)^2+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\times4^{-\frac{7}{2}}\left(-x^2\right)^3$$

B1: For $4^{-\frac{1}{2}}+$

M1: A correct attempt at the third or fourth terms condoning sign slips on the $-x^2$

A1: Correct and unsimplified expansion. See above

A1: Two correct and simplified terms of $\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$

A1: $\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$. Accept as a list

Alt II (a) By difference of two squares

$$\frac{1}{\sqrt{4-x^2}}=\left(2-x\right)^{-\frac{1}{2}}\times\left(2+x\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\ldots\ldots$$

B1: For $4^{-\frac{1}{2}}+$

M1: For correctly writing $\frac{1}{\sqrt{4-x^2}}=\left(2-x\right)^{-\frac{1}{2}}\times\left(2+x\right)^{-\frac{1}{2}}=\ldots\left(1-\frac{x}{2}\right)^{-\frac{1}{2}}\times\left(1+\frac{x}{2}\right)^{-\frac{1}{2}}=$

with a correct attempt at the third of fourth term in either expansion, followed by an attempt to combine both the expansions.

A1: One correct term in x^2, x^4 or x^6

A1: Two correct and simplified terms of $\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$ with no terms in x, x^3 or x^5

A1: Fully correct

10.Jan 2021

1(a)	$\left(\frac{1}{4}-5x\right)^{\frac{1}{2}}=\frac{1}{2}(...)$	B1
	$=(1-20x)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)\times(-20x)+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(-20x)^2+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(-20x)^3...$	M1A1
	$=\frac{1}{2}-5x-25x^2-250x^3+...$	A1 A1
	Special case: If the final answer is left as $\frac{1}{2}\left(1-10x-50x^2-500x^3+...\right)$ Award SC B1M1A1A1A0	
		(5)
	Alternative by direct expansion	
	$\left(\frac{1}{4}-5x\right)^{\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{1}{2}}+\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-5x)^1+\frac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$	B1M1A1
	$=\frac{1}{2}-5x-25x^2-250x^3+...$	A1A1
(b)	$\left(\frac{1}{4}-\frac{5}{100}\right)^{\frac{1}{2}}=\left(\frac{1}{5}\right)^{\frac{1}{2}}=\frac{1}{2}-5\times\frac{1}{100}-25\left(\frac{1}{100}\right)^2-250\left(\frac{1}{100}\right)^3+...$ $\frac{\sqrt{5}}{5}\approx\frac{1789}{4000}$ or $\frac{1}{\sqrt{5}}\approx\frac{1789}{4000}$ $\Rightarrow\sqrt{5}\approx5\times\frac{1789}{4000}$ or $\sqrt{5}\approx1\div\frac{1789}{4000}$	M1
	$\sqrt{5}\approx\frac{1789}{800}$ or $\frac{4000}{1789}$	A1
		(2)
		(7 marks)

(a)

B1: For taking out a factor of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.

M1: Expands $(1+kx)^{\frac{1}{2}}$, $k\neq\pm1$ with the correct structure for the third or fourth term

e.g. $\pm\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\times(kx)^2$ or $\pm\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\times(kx)^3$ with or without the bracket around the kx

A1: For either term three or term four being correct in any form.

E.g. $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times(20x)^2$ or $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times(-20x)^2$ or $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times(-20x)^3$ or $-\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times(20x)^3$

The brackets must be present unless they are implied by subsequent work. This mark is independent of the B mark.

A1: Two terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow if any of the '–' signs are written as "+–".

A1: All four terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the '–' signs are written as "+–" score A0.

Alternative:

B1: For a first term of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.

M1: For the correct structure for the third or fourth term. E.g. $\frac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(kx)^2$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(kx)^3$

where $k\neq\pm1$

A1: For either term three or term four being correct in any form.

$$\text{e.g. } \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2 \text{ or } \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$$

The brackets must be present unless they are implied by subsequent work.

A1: Two terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow if any of the '–' signs are written as "+–".

A1: All four terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the '–' signs are written as "+–" score A0.

(b)

M1: Attempts to substitute $x=\frac{1}{100}$ into their part (a) and either multiplies by 5 or finds reciprocal.

A1: $\left(\sqrt{5}=\right)\frac{1789}{800}$ or $\frac{4000}{1789}$

11.June 2021

1 (a)	$(1+kx)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)\times(kx)+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(kx)^2+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(kx)^3 \ldots$	
(i)	$\frac{1}{2}k=\frac{1}{8}\Rightarrow k=\frac{1}{4}$	M1A1
(ii)	$A=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times"k"^2=-\frac{1}{128}$ $\quad B=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times"k"^3=\frac{1}{1024}$	M1 A1 A1
		(5)
(b)	Substitutes $x=0.6\Rightarrow\sqrt{1.15}=1+\frac{1}{8}\times0.6-"\frac{1}{128}"\times0.6^2+"\frac{1}{1024}"\times0.6^3=1.072398$	M1 A1
		(2)
		(7 marks)
1(b) alt	$(1+kx)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)\times(kx)-\left(\frac{1}{8}\right)\times(kx)^2+\left(\frac{1}{16}\right)\times(kx)^3$ Substitutes $"kx"=0.15$ $\Rightarrow\sqrt{1.15}=1+\frac{1}{2}\times0.15-\frac{1}{8}\times0.15^2+\frac{1}{16}\times0.15^3=1.072398$	M1A1
		(2)

(a)(i)

M1: Sets $\frac{1}{2}k=\frac{1}{8}$ or $\frac{1}{2}kx=\frac{1}{8}x$ and proceeds to find k. Implied by a correct value for k

A1: $k=\frac{1}{4}$ oe such as $\frac{2}{8}$ or 0.25

(a)(ii)

M1: Correct attempt at 3rd **or** 4th term. Eg. $Ax^2=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(kx)^2$ or $Bx^3=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(kx)^3$ with an attempt at substituting in their value for k to find a value for A or a value for B. Condone a missing bracket around the kx terms so allow with k instead of k^2 or k^3 respectively. These may be simplified so award for $A=-\frac{1}{8}\times("k")^2$ or $B=\frac{1}{16}\times("k")^3$ again, condoning k instead of k^2 or k^3 respectively.

A1: $A=-\frac{1}{128}$ o.e. There is no requirement to simplify the fraction

A1: $B=\frac{1}{1024}$ o.e You may occasionally see $B=\frac{3}{3072}$ which is fine.

(b)

M1: For an attempt to substitute

- either $kx=0.15$ into an expansion of the form $1\pm p\times(kx)\pm q\times(kx)^2\pm r\times(kx)^3$
- or $x=\frac{0.15}{"k"}$ into their $1+\frac{1}{8}x+"A"x^2+"B"x^3$

A1: 1.072398 Must be to 6 decimal places.

12.Oct 2021

4(a)	$\sqrt{1-4x^2} = 1 - \frac{1}{2}\times 4x^2$	B1
	$\frac{\frac{1}{2}\times -\frac{1}{2}\times\left(-4x^2\right)^2}{2}$ or $\frac{\frac{1}{2}\times -\frac{1}{2}\times -\frac{3}{2}\times\left(-4x^2\right)^3}{3!}$	M1
	$= 1 - 2x^2 - 2x^4 - 4x^6 + \ldots$	A1, A1
		(4)
(b)	Substitutes $x = \frac{1}{4}$ into both sides of (a) e.g. $\sqrt{\frac{3}{4}} \approx 1 - 2\times\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right)^4 - 4\left(\frac{1}{4}\right)^6$	M1
	$\sqrt{3} \approx 1.7324$ cao	A1
		(2)
		(6 marks)

B1: Correct first two terms which does not need to be simplified, so $1 + \frac{1}{2}\times -4x^2$ is fine.

M1: Correct attempt at the binomial expansion for term 3 **or** term 4.

Look for $\frac{\frac{1}{2}\times -\frac{1}{2}\times\left(\pm 4x^2\right)^2}{2}$ or $\frac{\frac{1}{2}\times -\frac{1}{2}\times -\frac{3}{2}\times\left(\pm 4x^2\right)^3}{3!}$ but condone a failure to square or cube the 4 (i.e. missing brackets)

Also award this mark for candidates who mistakenly attempt the binomial expansion of

$\sqrt{1-4x}$ to produce either $\frac{\frac{1}{2}\times -\frac{1}{2}}{2}\left(\pm 4x\right)^2$ or $\frac{\frac{1}{2}\times -\frac{1}{2}\times -\frac{3}{2}}{3!}\left(\pm 4x\right)^3$

Or $\sqrt{1-x^2}$ to produce either $\frac{\frac{1}{2}\times -\frac{1}{2}}{2}\left(\pm x^2\right)^2$ or $\frac{\frac{1}{2}\times -\frac{1}{2}\times -\frac{3}{2}}{3!}\left(\pm x^2\right)^3$

A1: Two correct and simplified terms of $-2x^2, -2x^4, -4x^6$

A1: $1 - 2x^2 - 2x^4 - 4x^6$ or exact simplified equivalent such as $1 + (-2)x^2 + (-2)x^4 + (-4)x^6$

Ignore any additional terms. This may be given separately as a list. $1, -2x^2, -2x^4, -4x^6$

(b)

M1: Substitutes $x = \frac{1}{4}$ into both sides of (a) and achieves LHS of $\sqrt{\frac{3}{4}}$ or $\frac{\sqrt{3}}{2}$

It would be implied by $\left(\sqrt{3} =\right)$ $2\times$ their"$\left(1 - 2\times\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right)^4 - 4\left(\frac{1}{4}\right)^6\right)$"

A1: 1.7324 Correct answer only here. This is not awrt

Note that the calculator answer is 1.7321

Alt (a) It is possible to attempt part (a) as follows but it would be very unusual to get the terms up to x^6

$$\sqrt{1-4x^2} = (1-2x)^{\frac{1}{2}}\times(1+2x)^{\frac{1}{2}} = \left(1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3 \ldots\right)\times\left(1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 \ldots\right) = 1\ldots\ldots$$

Score M1 for such an attempt and award the other marks as in the main scheme. As is the main scheme look for a correct attempt at a term 3 or 4 in either bracket followed by an attempt to multiply out

13.Oct 2020

2(a)	$4^{-\frac{1}{2}}$ or $\frac{1}{4^{\frac{1}{2}}}$ or $\frac{1}{2}$	B1
	$(4-5x)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{5x}{4}\right)^{-\frac{1}{2}}$	
	$= \ldots\left(1+\left(-\frac{1}{2}\right)\times\left(-\frac{5x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2!}\times\left(-\frac{5x}{4}\right)^2+\ldots\right)$	M1A1
	$= \frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+\ldots$	A1
		(4)
	$\frac{2+kx}{(2-3x)^3} = (2+kx)\left(\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+\ldots\right)$	
(b)	Compares x terms leading to $k = \ldots.$ E.g. $\frac{10}{16}+\frac{k}{2}=\frac{3}{10} \Rightarrow k=-\frac{13}{20}$	M1 A1
		(2)
(c)	Compares x^2 terms leading to $m = \ldots$ E.g. $m=\frac{75}{128}+\frac{5}{16}\times'-\frac{13}{20}' \Rightarrow m=\frac{49}{128}$	M1 A1
		(2)
		(8 marks)

Alt (a)	$4^{-\frac{1}{2}}$ or $\frac{1}{4^{\frac{1}{2}}}$	B1
	$(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x)+\frac{-\frac{1}{2}\times-\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$	M1A1
	$= \frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+\ldots$	A1
		(4)

(a)

B1: For taking out a factor of $4^{-\frac{1}{2}}$ or $\frac{1}{2}$

For a direct expansion look for $4^{-\frac{1}{2}} + ...$ or equivalent.

M1: For the form of the binomial expansion $(1+ax)^{-\frac{1}{2}}$ where $a \neq 1$ or -5

To score M1 it is sufficient to see either term two or term three. Allow a slip on the sign.

So allow for either $\left(-\frac{1}{2}\right)(\pm ax)$ or $\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}(\pm ax)^2$

In the alternative version look for $\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x)$ or $\dfrac{-\frac{1}{2}\times-\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$ condoning sign slips

A1: Any (unsimplified) but correct form of the binomial expansion for . $\left(1-\frac{5x}{4}\right)^{-\frac{1}{2}}$

Ignore the factor preceding the bracket for this mark

Score for $1+\left(-\frac{1}{2}\right)\times\left(-\frac{5x}{4}\right)+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2!}\times\left(-\frac{5x}{4}\right)^2$ o.e.

In the alternative version look for $(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x) + \dfrac{-\frac{1}{2}\times-\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$

A1: cao $\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+...$ This must be simplified

$$(2+kx)\left(P+Qx+Rx^2+...\right)=1+\frac{3}{10}x+mx^2$$

(b)

M1: For a correct equation in k formed by comparing the x terms. It must lead to a value for k

Follow through on their expansion. So look for $Pk+2Q=\frac{3}{10} \Rightarrow k = ...$ Condone slips, i.e copying errors.

Condone $Pkx+2Qx=\frac{3}{10}x$ as long as it leads to a value for k

A1: $k=-\frac{13}{20}$

(c)

M1: Correctly compares the x^2 terms, following through on their expansion and their value for k leading to a value for m . Condone slips, i.e copying errors.

Look for $Qk+2R=m$ Condone $Qkx^2+2Rx^2=mx^2$ as long as it leads to a value for m

A1: $m = \frac{49}{128}$ oe Condone sight of $\frac{49}{128}x^2$ as evidence for a correct value for m.