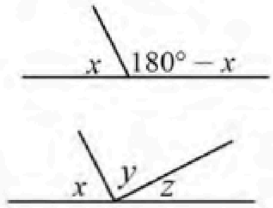
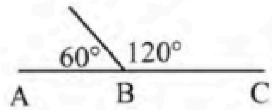
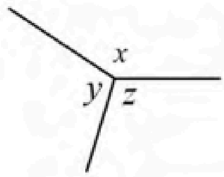
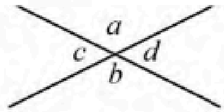
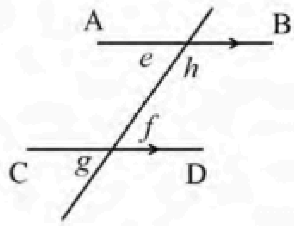
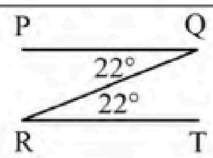
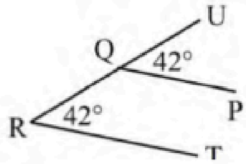
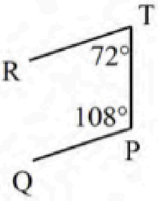
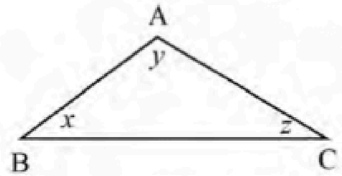
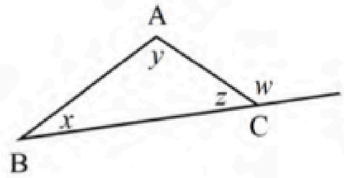
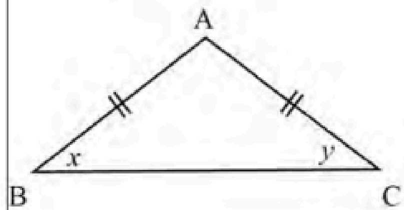
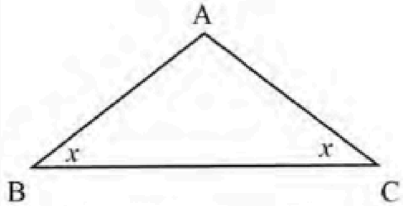
Mathematics, Grade 9, Euclidean Geometry

LINE AND ANGLES

THEOREM STATEMENT	**ACCEPTABLE REASON(S)**	**DIAGRAM**
Two adjacent angles on a straight line are supplementary The sum of two or more angles on a straight line is 180° $(x + y + z = 180°)$	$\angle$s on a str line $\angle$s on a str line	
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj $\angle$s supp	
The angles in a revolution add up to 360°. $(x + y + z = 360°)$	$\angle$s round a pt **OR** $\angle$s in a rev	
Vertically opposite angles are equal.	vert opp $\angle$s = (a and b are vertically opp $\angle$s) (c and d are vertically opp $\angle$s)	
If AB || CD, then the alternate angles are equal.	alt $\angle$s; AB || CD (e and f are alternate $\angle$s)	
If AB || CD, then the corresponding angles are equal.	corresp $\angle$s; AB || CD (e and g are corresponding $\angle$s)	
If AB || CD, then the co-interior angles are supplementary. $(f + h = 180°)$	co-int $\angle$s; AB || CD (f and h are co-interior $\angle$s)	
If the alternate angles between two lines are equal, then the lines are parallel.	alt $\angle$s = (You can prove that lines PQ and RT are parallel if you can show that the alternate angles are equal)	

If the corresponding angles between two lines are equal, then the lines are parallel.	corresp $\angle$s = (You can prove that lines RT and QP are parallel if you can show that the corresponding angles are equal)	
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint $\angle$s sup (You can prove that lines RT and QP are parallel if you can show that the co-interior angles are supplementary)	

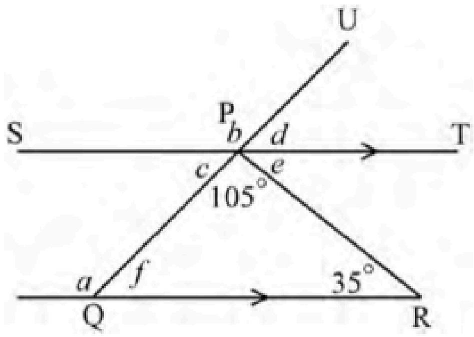
TRIANGLES

THEOREM STATEMENT	ACCEPTABLE REASON(S)	DIAGRAMS
The interior angles of a triangle are supplementary.	$\angle$ sum in Δ **OR** sum of $\angle$s in Δ **OR** Int $\angle$s Δ	
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext $\angle$ of Δ ($w = x + y$)	
The angles opposite the equal sides in an isosceles triangle are equal.	$\angle$s opp equal sides (You can prove that $x = y$) (You may **NOT** say ISOSCOLES Δ as the reason)	
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal $\angle$s (You can prove that AB = AC) (You may NOT say ISOSCOLES Δ as the reason)	

Study the examples below and answer the questions that follow:

Worked example:

In the figure below $Q\hat{P}R = 105^\circ$ and $\hat{R} = 35^\circ$. Find, in order, the sizes of angles *a* to *f*.



$a = 140^\circ$	exterior $\angle$ of Δ PQR
$b = 140^\circ$	corresponding $\angle$s, ST || MR
$c = 40^\circ$	<'s on straight line or co-interior $\angle$s, ST || MR
$d = 40^\circ$	vertically opposite $\angle$s
$e = 35^\circ$	alternate $\angle$s, ST || MR
$f = 40^\circ$	<'s on straight line or alternate $\angle$s, ST || MR or sum <'s in Δ PQR

Solve for *x* by setting up an equation and then solving it. Give appropriate reasons.

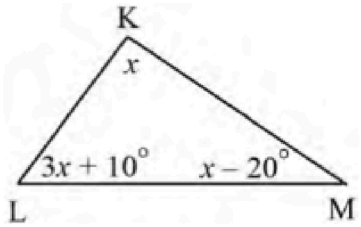
Worked example

$x + 3x + 10^\circ + x - 20^\circ = 180^\circ$ sum<'s in ΔKLM

$5x = 180^\circ - 10^\circ + 20^\circ$

$5x = 190^\circ$

$x = 38^\circ$

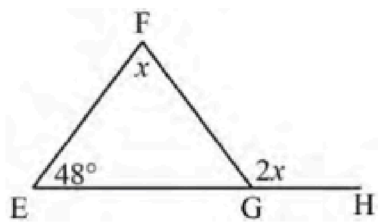


Worked example

$2x = x + 48^\circ$ ext $\angle$ of ΔEFG

$2x - x = 48^\circ$

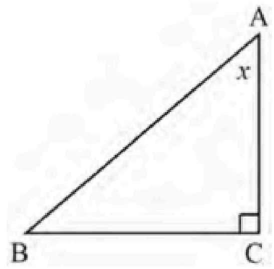
$x = 48^\circ$



Writing angles in term of x and y

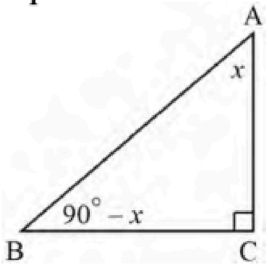
Fill in the size of each of the missing angles in terms of x or y.
(Although no working needs to be shown, you may still show your calculations if you find it helps you to find the values of the missing angles.)

Worked example

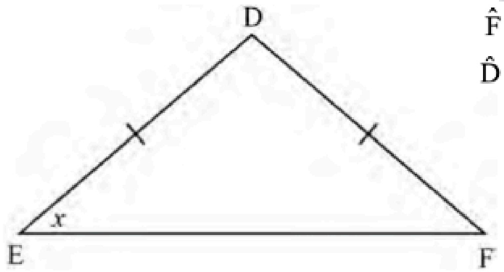


$$\hat{B} = 180^\circ - (90^\circ + x)$$
$$= 90^\circ - x$$

Optional Calculation:

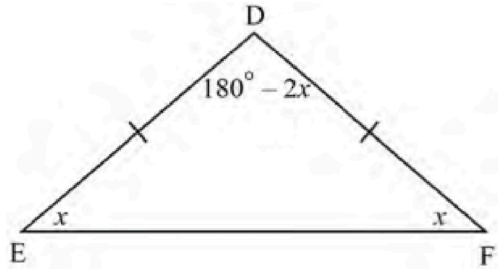


Worked example

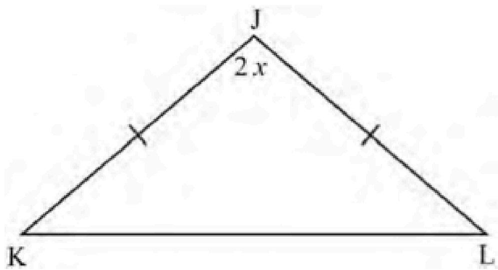


Optional calculations:

$$\hat{F} = x$$
$$\hat{D} = 180^\circ - (x + x)$$
$$= 180^\circ - 2x$$

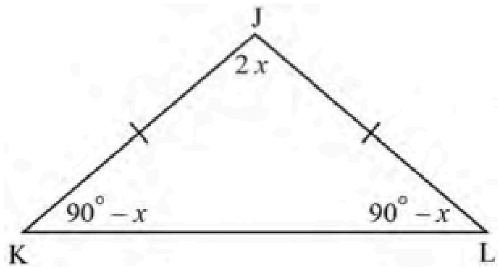


Worked example

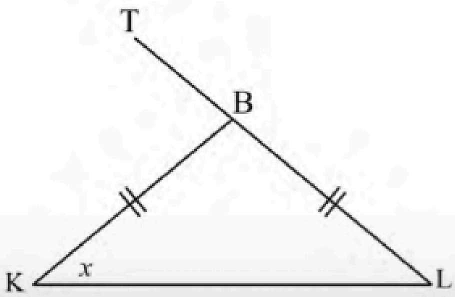


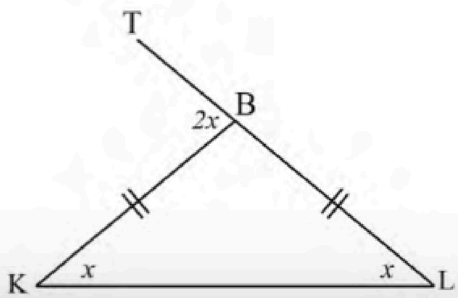
Optional calculations:

$$\hat{K} = \hat{L} = \frac{180^\circ - 2x}{2}$$
$$= 90^\circ - x$$



Worked example



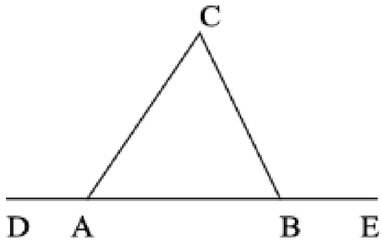


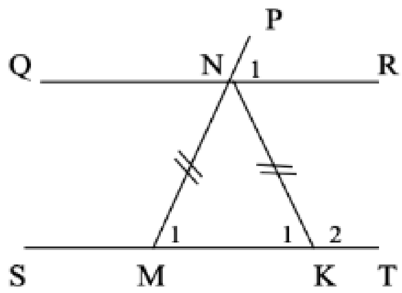
Geometric Proofs

If A = B and A = C This logic is used often in geometric proofs

then B = C and is called deductive reasoning.

Worked Example:





If $\hat{CAB} = \hat{CBA}$,

prove that $\hat{CAD} = \hat{CBE}$

PROOF: Let $\hat{CAB} = x$

$\therefore \hat{CBA} = x$ (given)

$\hat{CAD} = 180^\circ - x$ ($\angle$'s on str lineDE)

$\hat{CBE} = 180^\circ - x$ ($\angle$'s on str line DE)

$\therefore \hat{CAD} = \hat{CBE}$

If $N_1 = K_1$

Prove QR//ST

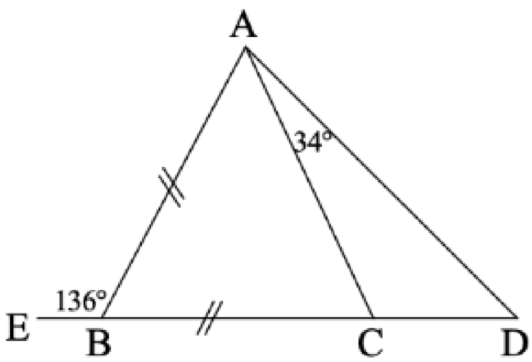
PROOF: Let $N_1 = K_1 = x$

$M_1 = K_1 = x$ $<$'s opp equal sides

$\therefore N_1 = M_1 = x$

$\therefore$ QR//ST corres$\angle$'s =

Worked Example:



Prove : AC = CD

Proof:

$\hat{BAC} = \hat{ACB} = 68^\circ$ exterior angle of isosΔ

$\hat{ADC} = 34^\circ$ exterior angle of isosΔ

$\therefore \hat{CAD} = \hat{ADC} = 34^\circ$

$\therefore AC = CD$ sides opp = angles $\therefore \Delta$ is isos

(mathguide)