Whiteboard: 1/60, f4.5, iso 640, EV+1.3, Auto Focus, H4n **INPUT** @ **95%** gain.

BBB 1/60, f6.7, iso 640, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT** @ **95%** gain.

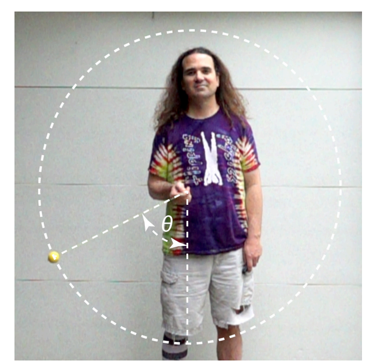
BBB Check Horizontal Level & Billy's Lights!! Middle line at bottom 1/3 of desk & remember Billy and Bo chair locations!

Nonuniform Circular Motion - Force of Tension in a Rope **Switch audio to 96kHz??**

Mr.p: (60 seconds of listening for this group of videos)

Mr.p: [No 4k] Good morning. We already discussed the general magnitudes and directions of forces acting on a ball moving in a vertical circle in non-uniform circular motion like this.[1] Today we are going to solve for an equation for the force of tension in the string. Bobby, please read the problem and Bo please translate.

- Bobby: [♪ Flipping Physics ♪] "A ball of



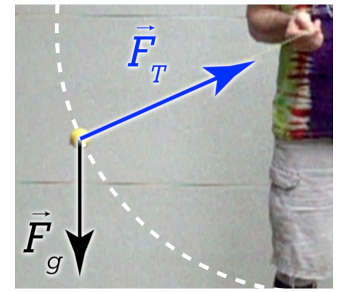
[1] http://www.flippingphysics.com/non-uniform-circular-motion-ball.html

mass m on the end of a string of length L moves in a vertical circle with a non-constant angular speed ω. The string forms an angle θ with the vertical as shown. Determine the tension in the rope in terms of m, L, ω, θ, and known constants.

- Bo: Our knowns are mass m, rope length L, angular speed omega, and angle theta. … And force of tension equals question mark. … That's it.
- **Mr.p: (Right. Billy, what should we do next?)**
- Billy: We need to draw the free body diagram, but you kinda already had it there for us just a few moments ago.
- **Mr.p: (Yeah, I know. That's from last time.**

Please just tell me the forces in the free body diagram.)

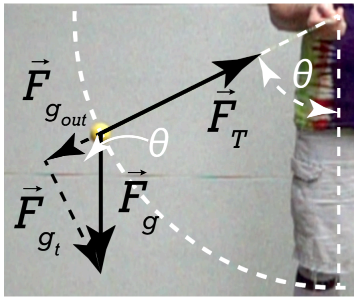
- Billy: Absolutely! The force of gravity is straight down and the force of tension is in toward the center of the circle.



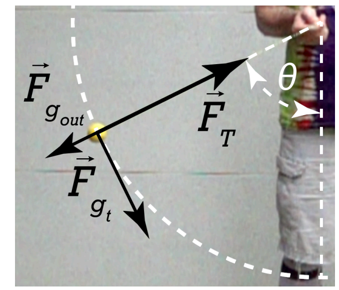
- **Mr.p: (And what should we do next?)**
- Billy: Eventually we are going to sum the forces in the in direction, or the direction along the radius of the circle which is where the string is. And we are going to sum the forces in a direction perpendicular to the in direction which is the tangential direction because it is tangent to the

circle and in the direction of the motion of the ball. So … we need to break forces into components in the in and tangential directions.

- Bobby: Actually, the force of tension is always pointed in the in direction, so we do not need to break that force into components.
- Billy: Right. So, we only need to break the force of gravity into components in the in and tangential directions. … Actually, the force of gravity component in the "in-direction" is actually directed outward. So I guess that is the outward component of the force of gravity. … Oh, and you



can see, because the outward component of the force of gravity is along the radius and the force of gravity is straight down, that the angle theta in the vector addition triangle which has the components of the force of gravity in it … is the same angle theta as shown in the problem. So, sine theta equals opposite over hypotenuse or the tangential component of force of gravity over the force of gravity. Therefore, the tangential component of the force of gravity equals mass times acceleration due to gravity times sine theta. And cosine theta equals adjacent over hypotenuse or the outward component of the force of gravity over force of gravity. Therefore, the outward component of the force of gravity equals mass times acceleration due to gravity times cosine theta. … Oh, and we should redraw the free body diagram with the components of the force of gravity rather than the force of gravity.



- $$\sin\theta = \frac{O}{H} = \frac{F_{g_t}}{F_g} \Rightarrow F_{g_t} = mg\sin\theta \;\&\; \cos\theta = \frac{A}{H} = \frac{F_{g_{out}}}{F_g} \Rightarrow F_{g_{out}} = mg\cos\theta$$
- Bo: Now we can sum the forces in the tangential direction. The only force acting in the tangential direction is the force of gravity tangential which then equals mass times tangential acceleration. …

That means mass times acceleration due to gravity times sine theta equals mass times tangential acceleration. And …

- BBB: Everybody brought mass to the party! {dancing}

Mr.p: {dancing} Bo, keep going. {… Ah, déjà vu!}

- Bo: Now we know the tangential acceleration of the ball equals acceleration due to gravity times sine theta. … I guess we put that in our equation holster.
- $\sum F_t = F_{g_t} = ma_t \Rightarrow mg\sin\theta = ma_t \Rightarrow a_t = g\sin\theta$
- Bobby and Billy: Equation holster!
- **Mr.p: (Bobby, what now?)**

- Bobby: Now we sum the forces in the in direction. So, net force in the in direction equals … force of tension, which is positive because forces acting inward are positive, … minus outward force of gravity, minus because forces which act outward are negative. … All that equals mass times the acceleration in the in direction … which is centripetal acceleration.
- Billy: … Uh, the force of gravity component which acts outward in the bottom semicircle of the path, acts inward instead when the ball is in the top semicircle of the path. How do we account for that?

- Bo: … When the ball is in the top semicircle of the path, theta is between 90 and 270 degrees and the cosine of an angle between 90 and 270 degrees is negative, which changes the direction of the force from outward to inward, and the equations still work.
- Billy: Cool. That makes sense. Thanks!
- Bo: You are welcome.
- Bobby: … So, force of tension equals the outward force of gravity plus mass times centripetal acceleration. … We have stuff we can substitute in for those. Uh … mass times acceleration due to gravity times cosine theta plus

mass times radius times angular speed squared. … And the radius equals L ,the length of the string. So, the force of tension equals mass times acceleration due to gravity times cosine theta plus mass times length times angular speed squared.

$$\sum F_{in} = F_T - F_{g_{out}} = ma_c \Rightarrow F_T = F_{g_{out}} + ma_c = mg\cos\theta + mr\omega^2 \Rightarrow F_T = mg\cos\theta + mL\omega^2$$

Mr.p: Very nice y'all. Now, I do want to point out that the equation we just derived matches what we previously showed visually. … The term mass times length times angular velocity squared is always positive. … And the term mass times acceleration due to gravity times cosine theta oscillates back and forth between positive and negative values as theta changes. … When the ball is at the bottom of the circle, theta equals zero, the cosine of zero equals 1, and that term has its maximum value, therefore the force of tension is at its maximum value when the ball is at the bottom of the circle. … When the ball is at the top of the circle, theta equals 180 degrees, the cosine of 180 degrees equals negative 1, and that term has its minimum value, therefore the force of tension is at its minimum value when the ball is at the top of the circle.

Bo: Cool. That is actually what we see in the animation.

Mr.p: Yep. But notice we can also solve for the minimum angular speed to keep the ball moving in a circle. Bo, how do we do that?

Bo: We are solving for the minimum angular speed to keep the ball moving in a circle. If the ball moves any slower than that minimum angular speed, it won't make it all the way to the top to form a circular path. … Well then, when the ball is at the top of the circle, at that minimum speed, the force of tension is reduced down to zero and … the angle is 180°. So, we just substitute those values into the equation for the force of tension. We get … zero equals mass times acceleration due to gravity times the cosine of 180 degrees plus mass times length times angular speed squared. … We can rearrange that equation to get negative mass times acceleration due to gravity times negative one equals mass times length times angular speed squared. And …

- BBB: Everybody brought mass to the party! {dancing}

Mr.p: {dancing} Bo, keep going. … Ah, déjà vu!

- Bo: We get acceleration due to gravity equals length times angular speed squared … and we can solve for angular speed. The minimum angular speed to keep the ball moving in a vertical circle equals the square root of the quantity acceleration due to gravity over the length of the rope.

- Billy: Hey, that's exactly what we got for the minimum speed to keep water in the vertically revolving bucket.[2] That's Cool!

Mr.p: Yes, Billy that is correct. {hear Bo} Yes, Bo?

Bo: (hold up mr.p!) We never used the equation I put in the equation holster; the tangential acceleration of the ball equals acceleration due to gravity times the sine of theta. What's up with that?

Mr.p: Bo, you are correct, we never did end up using that equation. It turns out that summing the forces

[2] https://www.flippingphysics.com/water-bucket-minimum-speed.html

in the tangential direction was an unnecessary step when solving this problem. {Bo grunt} Sometimes, we end up doing steps which are correct, however, not necessary when solving for what is asked for in the problem. That's just how solving problems goes sometimes. {Bo grunt} Yeah. Thank you very much for learning with me today, I enjoy learning with you.

Bo: {grunt}

Bo: {larger grunt}