**Roll No……………………..** **Total No. of Printed Pages: 2**

**Total No. of Questions: [09]**

**B. Tech EE (Semester – 4th)**

**MATHEMATICS - III (Probability & Statistics)**

**Subject Code: BMATH3-301**

**Paper ID: [18111519]**

**Time: 03 Hours** **Maximum Marks: 60**

**Instruction for candidates:**

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

**Section – A** **(2 marks each)**

Q1. Attempt the following:

a) The probability density function of a random variable $X$ is $(x) = \frac{k}{1+x^2}$ ; $-\infty < x < \infty$ . Determine $k$ and the distribution function.

b) The probabilities of hitting a target by three soldiers are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. Find the probability that one and only one of them will hit the target when they fire simultaneously.

c) If $X$ and $Y$ are two random variables having joint density function:

$$f(x) = \{\frac{1}{8}(6 - x - y),\ 0 \le x < 2,\ 2 \le x < 4\ 0,\ otherwise$$

Find $P(X + Y < 3)$.

d) The sum of mean and variance of a binomial distribution is 4.8 for five trials. Find the distribution.

e) Distinguish between "Skewness" and "Kurtosis" and bring out their importance in describing frequency distribution.

f) For a random variable $X$ with the following probability distribution:

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| P(x) | 0.05 | 0.10 | 0.30 | 0 | 0.30 | 0.15 | 0.10 |

Compute E($X$).

g) A sample of 400 male students is found to have a mean height of 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and S.D. 1.30 inches?

h) Define a continuous random variable and give an example.

i) The number of students in a class is 100. The average marks scored by 64 boys is 66 with a S.D. of 10, while the average marks scored by 36 girls is 70 with a S.D. 8. Test at 5% level of significance whether the girls perform better than the boys.

j) If $A$ and $B$ are two independent events, show that $A$ and $\overline{B}$ are also independent.

**Section – B** **(5 marks each)**

Q2. Find $\overline{X}$, $\overline{Y}$, and $r$ from the following two regression equations:

$$3X + 2Y = 26, \; 6X + Y = 31$$

Q3. Two independent samples of sizes 7 and 6 had the following values:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Sample A | 28 | 30 | 32 | 33 | 31 | 29 | 34 |
| Sample B | 29 | 30 | 30 | 24 | 27 | 28 | |

Examine whether the samples have been drawn from normal population having the same variance.

Q4. The joint distribution of two random variables $X$ and $Y$ is given by:

$f(x, y) = 4xy\, e^{-(x^2+y^2)}$ ; $x \geq 0,\ y \geq 0$.

Test whether $X$ and $Y$ are independent. For the above joint distribution, find the conditional density of $X$ given $Y = y$.

Q5. Derive the Poisson distribution as a limiting case of the Binomial distribution.

Q6. In a normal distribution, 7% of the items are under 35, and 89% are under 63. What are the mean and standard deviation of the distribution?

**Section – C** **(10 marks each)**

Q7. a) Calculate the Rank correlation coefficient for the following distribution:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $X$ | 11 | 6 | 9 | 13 | 6 | 27 | 15 | 16 | 17 | 10 | 1 |
| $Y$ | 21 | 30 | 37 | 40 | 29 | 34 | 39 | 24 | 20 | 40 | 38 |

b) Define the exponential distribution and find its mean.

Q8. Fit a Binomial distribution to the following data and test for goodness of fit at the 5% level of significance.

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| $f$ | 38 | 144 | 342 | 287 | 164 | 25 |

Q9. State and prove Chebyshev's inequality.