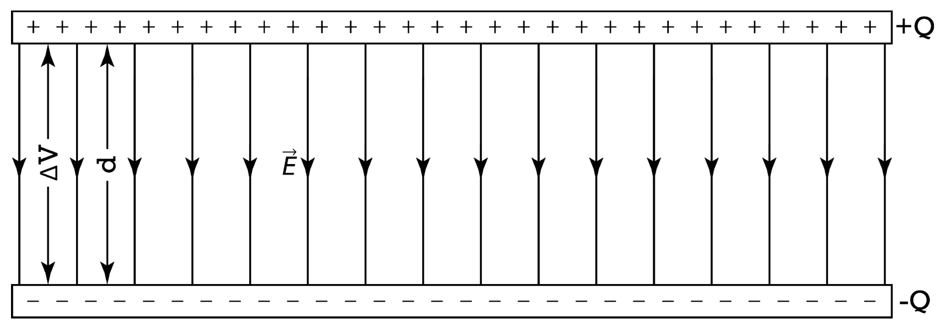
Whiteboard: 1/60, f4.5, iso 640, EV+1.3, Auto Focus, H4n **INPUT** @ **95%** gain.
BBB 1/60, f6.7, iso 640, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT** @ **95%** gain.
BBB Check Horizontal Level & Billy's Lights!! Middle line at bottom 1/3 of desk & remember Billy and Bo chair locations!
Capacitors - Review for AP Physics C: Electricity and Magnetism **Switch audio to 96kHz!!**

Mr.p: [4k, flash] Good morning. Today we are going to review capacitors for AP Physics C: Electricity and Magnetism. [♪ Flipping Physics ♪] A capacitor is a way to store electric potential energy in an electric field. … For example, this camera flash uses chemical energy stored in this rechargeable battery to move electric charges to a capacitor and store energy in the electric field of the capacitor as electric potential energy. … So, right now energy is stored in the capacitor, and I can press this button … to convert that electric potential energy to light and heat energy. {flash}

BBB: Cool. Fun. Yeah.

Mr.p: The capacitance of a capacitor depends only on the capacitor's physical characteristics. For example, the capacitor's shape and material used



$$C \equiv \frac{Q}{\Delta V}$$

to separate the plates of the capacitor. … The simplest form of a capacitor is a parallel plate capacitor. The two plates have equal magnitude charges, but opposite sign, and the electric potential energy is stored in the electric field of the capacitor. … Capacitance, capital C, is defined as the magnitude of the charge stored on one plate divided by the electric potential difference between the two plates. … Capacitance is always positive which means that capital Q, is the charge on the positive plate … or the magnitude of the charge on the negative plate. … And delta V is the positive electric potential difference between the two plates. … Realize this does mean that the net charge on a capacitor is actually zero. When you sum the charge on the positive plate with the charge on the negative plate, you get zero total charge.

$$Q_{\text{total}} = +Q + (-Q) = 0$$

- Bo: Why are there three lines in the equation for the definition of capacitance?
- Billy: Oh, a three horizontal line equal sign

simply means it is a definition. We cannot derive the equation for capacitance because capacitance is just defined as the magnitude of the charge stored on one the plates of the capacitor divided by the electric potential difference across the capacitor. We, pretty much, just made it up.

- Bo: Oh.
- Bobby: … If the total charge on a capacitor is zero, then how can it store electric potential energy?
- Billy: … Uh?

- Bo: … It takes work to separate the charges onto the plates of the capacitor and that work must put electric potential energy into the system.
- Bobby: Oh … right. … The law of charges says the positive and negative charges in the capacitor are attracted to one another, so it takes work to move them apart like they are in the capacitor.
- Billy: Just like it takes work to lift a mass in a gravitational field and that work adds gravitational potential energy to the

mass-Earth system; separating positive and negative charges requires work which adds electric potential energy to the capacitor. Cool.

- Bo: … I thought a capacitor was what made time travel possible.
- Bobby: No, that’s a flux capacitor.
- Bo and Billy: Oh …
- Bo: I bet Gauss is coming back soon then.
- Bobby: If I were Carl, I’d wait for the pandemic to be over.
- Billy: Yeah.
- Bo: Who’s Carl?

- Billy: Johann Carl Friedrich Gauss
- Bo: Okay.

Mr.p: Oh … Kay, … The units on capacitance are coulombs over volts which are called farads, for which the symbol is capital F.

charge, $Q \Rightarrow$ coulombs, C & capacitance, $C \Rightarrow$ farads,

- Bo: Hold up. So, the symbol for capacitance is a capital C. And capacitance is charge over electric potential difference and the units for charge are coulombs, capital C. So, in one equation we have capital C meaning both capacitance and coulombs?

- **Mr.p: (Well, … kind of?)**
- Bo: I don't like that.

Mr.p: [4k, disposable camera, paper clip, iPhone 240 fps] Okay, so yeah, I get it. Unfortunately, I cannot do anything about that. If it really bothers you or you are concerned it might confuse you, you could write Coulomb's name in full for the charge unit, to help distinguish. … Alright, so let's use a disposable camera to demonstrate the basic idea of charging a capacitor. … I have removed the outer shell of this disposable camera. … You can see this black cylinder right here is the capacitor. … Currently there is zero charge on either plate of the capacitor and no electric field between the plates; the capacitor is fully discharged. … Now I add the battery to the camera and press this button which closes the circuit to connect the battery to the terminals of the capacitor and right now the battery is moving charges from one plate of the capacitor to the other plate of the capacitor which is converting chemical energy from the battery to electric potential energy in the electric field of the capacitor. … Now that the capacitor has as much charge separated between the plates of this capacitor as this battery and circuit will allow, I can remove the battery and the capacitor retains the electric potential energy because the electric field is still there between the plates of the capacitor. … That means that right now there is an electric potential difference between the plates of the capacitor. … And in a moment, I am going to take a metal paper clip and place it across the two terminals of the capacitor. … Again, these are the two terminals of the capacitor and I now take a metal paper clip and place it across the two terminals of the capacitor. {spark} I love this demo.

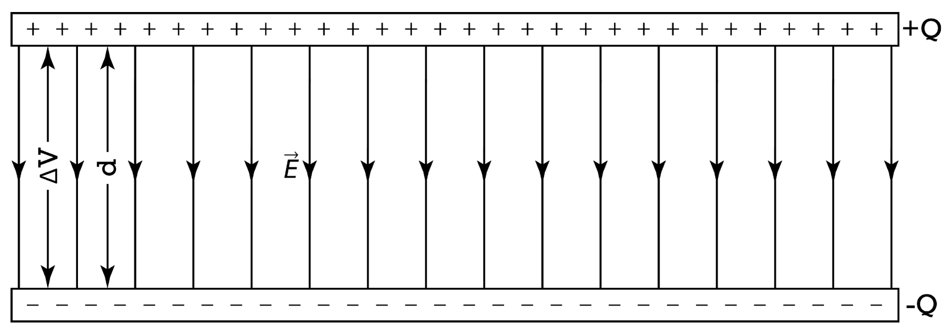
BBB: {reactions} What the … Oh my … Ha ha ha

Mr.p: [4k, disposable camera, paper clip, iPhone 240 fps] One more time, ya know, ‘cause. … Battery. … Charging capacitor. … Removing battery. … Metal paper clip across terminals of the battery and … {spark} Oh yeah!

BBB: Looks dangerous. Oh boy. Nice!

Mr.p: I have converted the electric potential energy stored in the capacitor into light and heat energy. … Just so you know I have burned myself several times doing this demonstration, so I definitely recommend against doing this at home.

- Billy: I saw it. It looked painful.
- Bobby: Yeah, it actually burned a couple of holes in his finger where he accidentally touched the terminals of the capacitor.



- Billy: And he said a bad word.
- Bo: {sardonic smile}
- Bobby: Oh!
- Billy: Ouch!

$$E_{\parallel\ \text{plates}} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \ \ \&\ \ \Delta V_{\text{constant E}} = -Ed \Rightarrow \|\Delta V\| = Ed = \left(\frac{Q}{A\varepsilon_0}\right)d$$

Mr.p: Okay, now let’s derive the equation for the capacitance of a parallel plate capacitor. … We have actually already derived two equations for two parallel, infinitely large, charged plates with equal magnitude, but opposite sign. … The first is that the electric field between the two plates is constant and has a value equal to the magnitude of the surface charge density on one of the plates divided by the electric permittivity of free space. … Now, we know surface charge density equals the magnitude of the charge on one of the plates divided by the area of one of the plates. … The other equation we derived is that the electric potential difference in a constant electric field equals the negative of the magnitude of the electric field times the distance, d, between the two points across which the electric potential difference is measured and that distance is parallel to the direction of the electric field. … In other words, the magnitude of the electric potential difference across the two plates of a parallel plate capacitor equals the electric field between the plates of the capacitor times the distance between the two plates. … Bo, please use those equations to determine the equation for the capacitance of a parallel plate capacitor.

$$\& \quad C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} \Rightarrow C_{\parallel \text{ plate}} = \frac{\varepsilon_0 A}{d}$$

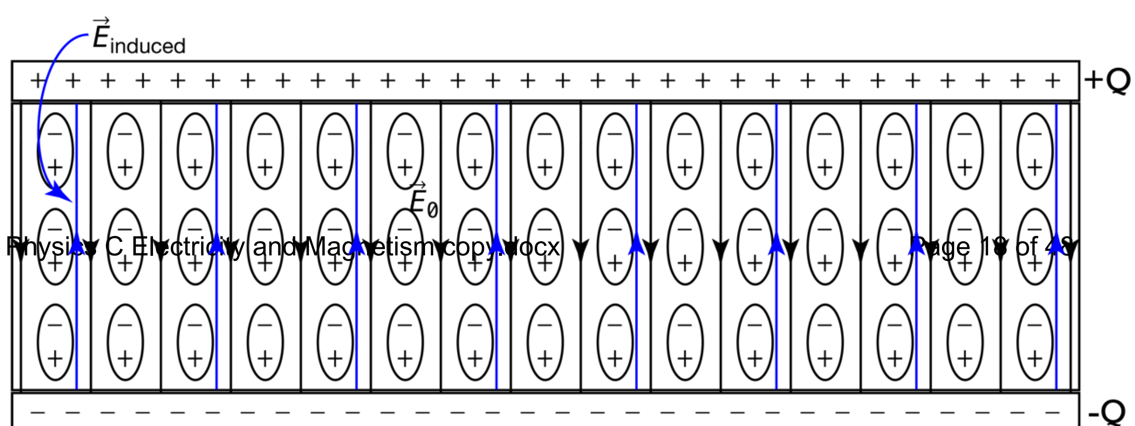
- Bo: Sure. … Don't we just substitute the equation for the constant electric field between the two plates to get an equation for the magnitude of the electric potential difference which exists between the two plates of a parallel plate capacitor. … And then substitute that into the equation for capacitance. … The charge of the capacitor cancels out and we get that the capacitance of a parallel plate capacitor equals the electric permittivity of free space times the area of one plate of the capacitor all over the distance between the plates. … Right?

- **Mr.p: (Bo, that is correct. …)**
- Bo: … Yeah, but we still don't really know what the electric permittivity of free space is.

$$E = E_0 - E_{\text{induced}} \Rightarrow E \downarrow \;\; \& \;\; \|\Delta V\| = Ed$$

$$\Rightarrow \Delta V \downarrow \;\; \& \;\; Q \text{ is constant } \;\& \;\; C = \frac{Q}{\Delta V} \Rightarrow C \uparrow$$

Mr.p: Well, you are about to find out. … An important piece to realize about this



equation for the capacitance of a parallel plate capacitor is that it assumes there is a vacuum between the two plates. … Usually, we place an insulating material between the plates of a capacitor. This is both to help physically separate the two plates and because it increases the capacitance of the capacitor. This insulating material is called a dielectric. … The original electric field in the capacitor is in black in the illustration and I have identified the original electric field as capital E naught. … When a dielectric is

$$\kappa = \frac{\varepsilon}{\varepsilon_0}$$

placed in that electric field, the electric field arranges the molecules in the dielectric according to the law of charges. … The positive sides of the molecules in the dielectric material are attracted to the negative plate and the negative sides of the molecules in the dielectric material are attracted to the positive plate. … This polarizes the dielectric and the polarized molecules in the dielectric induce an electric field opposite the direction of the original electric field E naught. … The induced electric field caused by the dielectric is in blue.

… The magnitude of the net electric field in the capacitor with the dielectric in it is then smaller than the original electric field. … The net electric field typically has no subscript because it is the electric field which exists in the capacitor. … Again, adding a dielectric reduces the electric field in the capacitor. … And because the magnitude of the electric potential difference in the capacitor equals the electric field times distance between plates, the electric potential difference in the capacitor is also decreased when a dielectric is added.

… As long as the capacitor is isolated and not part of a closed electric circuit, adding a dielectric does not change the charge on the capacitor. … Therefore, because capacitance equals charge divided by electric potential difference, adding a dielectric to a capacitor increases the capacitance of the capacitor. … Now, the way we define the effect of a dielectric is with the dielectric constant. The symbol for the dielectric constant is the lowercase Greek letter kappa. … It looks basically like a lowercase k. … The dielectric constant equals the ratio of the electric permittivity of the dielectric to the electric permittivity of free space. … That means the dielectric constant has no units. … And, electric permittivity is the measurement of how much a material is polarized when it is placed in an electric field.

Billy: Oh right, that makes sense. That is exactly what happens to a dielectric when it gets placed in a capacitor. The dielectric gets polarized in the electric field of the capacitor and the electric permittivity of the material is just a measurement of how much the material is able to be polarized.

Mr.p: Exactly Billy. The easier it is for electrons to change configurations in a material, the larger the dielectric constant of that material. … And, sometimes the dielectric constant is called relative permittivity. But, usually on the AP physics exams it is called the dielectric constant. … Bobby, please use the equation we previously derived for the electric field between two parallel plates when there is a vacuum between the plates to determine the dielectric constant in terms of electric fields.

$$E_{\text{vacuum}} = \frac{\sigma}{\varepsilon_0} \;\&\; E_{\text{dielectric}} = \frac{\sigma}{\varepsilon}$$

$$\Rightarrow \frac{E_{\text{vacuum}}}{E_{\text{dielectric}}} = \frac{\frac{\sigma}{\varepsilon_0}}{\frac{\sigma}{\varepsilon}} = \frac{\varepsilon}{\varepsilon_0} = \mathrm{K} \Rightarrow \mathrm{K} = \frac{E_{\text{vacuum}}}{E_{\text{dielectric}}} \Rightarrow \mathrm{K} = \frac{E_0}{E}$$

Bobby: Okay. … Well, we know the electric field between the parallel plates equals the surface charge density of one plate over vacuum permittivity. … Oh, I get it. … That is the electric field between the parallel plates when there is nothing between the two plates. …

The electric field when there is a dielectric between the plates equals the surface charge density divided by the permittivity of the dielectric. … Which, evidently, has no subscript. … The ratio of the vacuum electric field to the dielectric electric field then equals a fraction with those two equations in it. … Surface charge density cancels out and the permittivity of the dielectric over the electric permittivity of free space is what we are left with, … and that equals the dielectric constant. … So, the dielectric constant equals the electric field with a vacuum divided by the electric field with a dielectric.

Mr.p: Correct Bobby. However, the electric field with a vacuum between the plates is electric field naught and the electric field with a dielectric between the plates is the electric field with no subscript. … Billy, please now derive the equation which relates the capacitance with a dielectric to the capacitance with a vacuum between the parallel plates.

$$C_{\parallel \text{ plate}} = C_0 = \frac{\varepsilon_0 A}{d} \;\; \& \;\; \kappa = \frac{\varepsilon}{\varepsilon_0} \Rightarrow \varepsilon = \kappa\varepsilon_0$$

$$\& \;\; C_{\text{dielectric}} = C = \frac{\varepsilon A}{d} = \frac{\kappa\varepsilon_0 A}{d} s \Rightarrow C_{\parallel \text{ plate}} = \frac{\kappa\varepsilon_0 A}{d} \;\; \& \;\; C = \kappa C_0$$

Billy: Absolutely! … Uh, let's start with the capacitance of a parallel plate capacitor which equals the electric permittivity of free space times area of the plate divided by the distance between the plates. … And, I guess we will call that capacitance naught because it is the capacitance with a vacuum between the plates. … I guess we will call the capacitance with a dielectric just capital C with no subscript … and that equals the same equation as the one with a vacuum only with the electric permittivity of the dielectric instead of the electric permittivity of free space. … And because we know the dielectric constant equals the electric permittivity of the dielectric material divided by the electric permittivity of free space, we know the electric permittivity of the dielectric equals the dielectric constant times the electric permittivity of free space … which we can substitute into the equation for the capacitance with a dielectric. … That means the capacitance of a parallel plate capacitor with a dielectric equals the dielectric constant times the permittivity of free space times the area of one plate all divided by the distance between the two plates. … Oh, and that means the capacitance with a dielectric equals the dielectric constant times the capacitance with a vacuum.

Mr.p: Yes Billy. Thanks. … Now, according to the College Board, you are responsible for being able to derive the equations for the capacitance of only the following shapes: parallel-plate capacitors, spherical capacitors, and cylindrical capacitors. … We did the derivation for a parallel-plate capacitor, however, we do not have the time today to derive the equations for the capacitance of spherical and cylindrical capacitors. Sorry.

BBB: {sad faces?}

Mr.p: Next, let’s derive the equation for the energy stored in a capacitor. … Starting with an uncharged capacitor, in other words, charge initial equals zero, we move one, infinitesimally small charge from one plate to the other plate. … Because the electric potential difference between the plates currently is zero, moving this first charge takes zero work. However, … moving the next charge does take work because there now is an electric potential difference between the two plates. … The work it takes to move a charge equals the change in electric potential energy of the capacitor … and it equals the magnitude of the charge which is moved … times the electric potential difference the charge is moved through … which is the electric potential difference across the capacitor because it now has an infinitesimally small electric potential difference across it. … We need to identify the infinitesimally small charge we are moving as dq and the amount of work it takes to move that charge dW. … And take the integral of both sides. … Now we have the work necessary to charge the capacitor equals the integral of the electric potential difference across the capacitor plates with respect to charge. … Bo, take it from here, please.

$$Q_i = 0 \ \& \ W = \Delta U_{\text{elec}} = q\Delta V \Rightarrow \mathrm{d}W = \Delta V \mathrm{d}q$$

$$\& \ C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \frac{Q}{C} \Rightarrow Q = C\Delta V$$

$$W = \int_0^Q \Delta V \,\mathrm{d}q = \int_0^Q \frac{q}{C}\mathrm{d}q = \left[\frac{q^2}{2C}\right]_0^Q = \frac{Q^2}{2C} - \frac{0^2}{2C} \Rightarrow U_C = \frac{Q^2}{2C}$$

$$\Rightarrow U_C = \frac{(C\Delta V)^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\left(\frac{Q}{\Delta V}\right)\Delta V^2 = \frac{1}{2}Q\Delta V$$

$$\Rightarrow U_C = \frac{Q^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}Q\Delta V$$

- Bo: Sure. … But … What am I doing?
- Billy: Determining the work necessary to charge the capacitor to charge capital Q.

- Bo: … How?
- Bobby: By evaluating that integral.
- Bo: Ok, but why can we use that integral? The dielectric is an insulator and should stop charges from crossing the gap between the plates. Can we calculate work with an integral through a path the charges aren't going to actually take?
- **mr.p: (Oh right, this is where the concept of conservative forces helps us. … Remember the electrostatic force is conservative. … It does not matter what**

path we take to calculate the work done by it, or the work done moving charges against it. All that matters is the initial and final states.)

- Bo: Uh, okay, thanks mr.p. … I guess we need to get electric potential difference in terms of charge to be able to take that integral. … Capacitance equals charge over electric potential difference. So, electric potential difference equals charge over capacitance. And we can substitute that into the integral. … But we use a little q for charge to indicate that it is a changing variable. … The limits on the integral are … well the initial charge on the capacitor is zero and the final charge on the capacitor is … capital Q. So, those are the limits. … The capacitance of a capacitor does not depend on charge, so the integral of charge over capacitance with respect to charge equals charge squared over 2 times capacitance. … Substituting in the limits gives us the square of the charge on the capacitor, capital Q, over 2 times capacitance minus zero squared over 2 times capacitance. …

That means the energy stored in the electric field of a capacitor equals the square of the magnitude of the charge on one plate of the capacitor, capital Q, over 2 times the capacitance of the capacitor.

{switch perspectives}

- Mr.p: Thank you Bo. … We also know that charge equals capacitance times electric potential difference. Which we can substitute into that equation to get that the energy stored in the electric field of a capacitor also equals one half times capacitance of the capacitor times the square of the electric potential difference across the capacitor. … And, we can also substitute charge over electric potential difference in for capacitance to show that the electric potential energy stored in the electric field of a capacitor also equals one half times the magnitude of the charge on one plate times the electric potential difference across the two plates. … Yes, the energy stored in the capacitor is stored in the electric field of the capacitor, is equal to the amount of work needed to move the charges from one plate to the other, and we have three different expressions for that electric potential energy.

- Billy: (mr.p?)
- Mr.p: Yes, Billy?

{switch perspective}

- Billy: I have a question.
- **Mr.p: (Sure. Go ahead.)**
- Billy: Going all the way back to the demonstration you did with discharging the charged capacitor of a disposable camera using a metal paper clip.

Mr.p: [4k, disposable camera, paper clip, iPhone

240 fps] You mean this? {spark} He he.

Billy: Yeah. … How come you can hold the metal paperclip in your hand?

Mr.p: Ah, good question. Basically, you are asking "How do I know the charges will not flow through the part of the metal paperclip I am holding?" Right?

Billy: Yeah.

$$C = 120\mu F;\ \Delta V = 330V$$

$$\Rightarrow U_C = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\left(120 \times 10^{-6}\right)(330)^2 = 6.532 \approx 6.5J$$

$$\Rightarrow U_C = 6.532J \times \left(\frac{1\,eV}{1.6x10^{-19}J}\right) = 4.0825 \times 10^{19} \approx 41 \times 10^{18} eV$$

$\Rightarrow U_C \approx 41 \times 10^9 \times 10^9 eV \Rightarrow U_C \approx 41 \text{ billion billion } eV$

- Mr.p: … When I place the metal paper clip across the two terminals of the capacitor, I am providing a path for electrons to flow between the two terminals of the capacitor. … The electrons are attracted to the positive plate of the capacitor and will flow through the short part of the paper clip across the terminals to get there. … Notice that I am wearing shoes which electrically insulate me from the ground. If I were instead standing in a pool of water, I would be providing a path with little resistance for the charges to flow through and likely some of the charges would flow through me to the pool of water and the ground. But, we haven’t defined resistance yet. We will do that next time. … Actually, you know what, let’s look at the numbers for that disposable camera capacitor I discharged. {spark} On the side of the capacitor it states that the capacitance across the capacitor is 120 microfarads and the electric potential difference across the capacitor is 330 volts. … That means there must be a step-up converter in the camera circuitry to step the electric potential difference up from the 1.5 volts across the one AA battery in the camera to the 330 volts across the capacitor. … Now that we have those numbers, we can calculate the energy stored in the capacitor to be roughly 6.5 joules. … That means, when you see this spark, {spark} you are seeing roughly 6.5 joules of electric potential energy being converted to light and heat energy. … But, let’s take a moment to convert that to electron volts. We know 1 electron volt equals 1.6 times 10 to the negative 19 joules. … That means this is roughly 41 times 10 to the 18th electron volts of electric potential energy being released. … But, because 41 times 10 to the 9th times 10 to the 9th is the same as 41 to the 18th …

- **Bo: (Where is this going? This is weird.)**
- **Bobby: (yeah)**
- Mr.p: That means what you are seeing here is roughly 41 billion billion electron volts of energy being converted to light and heat energy.
- **Bo: (So…)**

- Mr.p: So, when, near the end of the movie Ghostbusters: Afterlife, Dan Aykroyd’s character Ray Stantz says “nothing stings like a billion electron volts!”, … the amount of electric potential energy he is referring to is roughly 41 billion times less that the amount of electric potential energy you see here. {spark}

{switch perspective}

- Billy: Oh. That’s kind of funny.
- Bobby: Right.
- Bo: Sure, yeah.
- Billy: Because electron volts are units of

energy generally associated with subatomic particles.

Mr.p: In fact, I would say Ray Stantz mixed up the order of the words in his sentence. He should have said, “a billion electron volts stings like nothing!”

BBB: Yeah. Okay. No, that, yeah.

Mr.p: And that concludes my review of capacitors for AP Physic C: Electricity and Magnetism. Next time we are going to review current, resistance, and simple circuits. … Thank you very much for learning with me today, I enjoy learning with you.