Calculus 2

Q1 Week 6 — September 8–11

Name:

1. FTC Part I — Accumulation

Let $F(x) = \int_a^x f(t)\,dt$.

F(x) represents the

F′(x) represents how fast the

If f is continuous, then

$$F'(x) =$$

Why?

$$F(x+h) - F(x) =$$

$$[F(x+h) - F(x)]/h = (1/h)\int_x^{x+h} f(t)\,dt$$

As h → 0,   av(f) →

Therefore,

$$F'(x) =$$

2. FTC with Changing Bounds

If $F(x) = \int_a^{g(x)} f(t)\, dt$, then

$$F'(x) =$$

For an upper bound:

1. Plug the bound into the integrand.
2. Multiply by

For a lower bound:

1. Plug the bound into the integrand.
2. Multiply by

3. Change the

If both bounds change,

$$d/dx \int g(x) \text{ to } h(x)\ f(t)\ dt =$$

3. Practice

1. Find F′(x).

$$F(x) = \int_2^x (t^3 - 4t + 1)\ dt$$

2. Find F′(x).

$$F(x) = \int_1^{x2+1} \sqrt{(1+t^4)}\ dt$$

3. Find F′(x).

$$F(x) = \int_2^{\sin x} (1+t^2)^3\ dt$$

4. Find G′(x).

$$G(x) = \int \square_{i}\square_{x}^{x2} (1-t^2)\ dt$$

5. Find H′(x).

$$H(x) = \int_x^{2x3+1} \sqrt{(1+t^2)}\ dt$$

4. FTC Part II — Evaluating Definite Integrals

If $F'(x)=f(x)$, then

$$\int_a^b f(x)\,dx =$$

5. Practice

1. Evaluate.

$$\int_0^2 (3x^2-4x+5)\,dx$$

2. Evaluate.

$$\int_1^4 (2\sqrt{x} + 1/x^2)\,dx$$

3. Evaluate.

$$\int_1^4 (\sqrt{x} + 2/x^2)\,dx$$

4. Evaluate.

$$\int_0\text{^}(\pi/2)\ (4\sin x + \cos x)\,dx$$

5. Evaluate.

$$\int_3^{-1} (2x+4)\,dx$$

6. Net Change and Initial Value Problems

$$Q(b) - Q(a) = \int_a^b Q'(t)\,dt$$

Final Amount = Initial Amount +

The integral of a rate gives

7. Net Change vs. Final Value

Suppose $\int_0^5 v(t)dt = 12$. The value 12 represents the

If s(0)=7, then s(5) =

8. Initial Value Problems

If F′(x) is known and F(a) is known, then

$$F(b) =$$

9. Net Change, Total Distance, and Total Area

$$\text{Net Change} = \int_a^b v(t)\,dt$$

The sign of v(t)

$$\text{Total Distance} = \int_a^b |v(t)|\,dt$$

Before finding total distance, first find where v(t) =

10. Practice

1. A particle moves along a line with velocity $v(t)=3t^2-6t+2$. Find the net change in position from t=0 to t=3.

2. Water enters a tank at the rate R(t)=2t+5 gallons per minute. Find the amount of water added during the first 4 minutes.

3. Suppose $F'(x)=3x^2-4x$ and $F(1)=7$. Find $F(3)$.

4. Suppose $P'(t)=4t-3$ and $P(2)=10$. Find $P(5)$.

5. Find the total area between $f(x)=x-2$ and the x-axis on $[0,5]$.

6. Find the total area between $f(x)=x-1$ and the x-axis on $[-1,3]$.

7. A particle has velocity $v(t)=t-2$ on $0 \le t \le 5$. (a) Find the net change in position. (b) Find the total distance traveled. (c) Why are the two answers different.

11. Mixed Practice — 5.2–5.4

1. Evaluate Σ(k=1 to 10)($2k^2-3k+1$).

2. Find A: lim(n→∞) ($1/n^4$) Σ(k=1 to n)(Ak^3+3k^2)=5.

3. Given ∫[-1 to 2]f=6 and ∫[2 to 5]f=−4, evaluate ∫[5 to −1][2f(x)+3]dx.

4. Suppose −2≤f(x)≤3 on [1,5]. Find bounds for ∫[1 to 5][2f(x)−1]dx.

5. Suppose ∫[0 to 6][2f(x)+1]dx=30. Find the average value of f on [0,6].

6. F(x)=∫[1 to x^2+1]$\sqrt{(1+t^4)}$dt. Find F′(x).

7. G(x)=∫[cos x to 3]$(1+t^2)^4$dt. Find G′(x).

8. H(x)=∫[sin x to x^2]($1-t^2$)dt. Find H′(x).

12. Mixed Practice

1. Find A: $\lim_{n\to\infty}(1/n^4)\Sigma(k=1 \text{ to } n)(2Ak^3-5k^2+3k)=7$.

2. Given $\int[-2 \text{ to } 1]f=5$, $\int[1 \text{ to } 4]f=-2$, and $\int[-2 \text{ to } 4]g=3$, evaluate $\int[4 \text{ to } -2][3f(x)-2g(x)+1]dx$.

3. Suppose $-3\le f(x)\le 4$ on $[-1,5]$. Find bounds for $\int[-1 \text{ to } 5][3f(x)+2]dx$.

4. Suppose $\int[1 \text{ to } 7][3f(x)-2]dx=48$. Find the average value of f on [1,7].

5. $F(x)=\int[x^2+1 \text{ to } 5]\ 1/(1+t^4)\ dt$. Find $F'(x)$.

6. $G(x)=\int[\cos x \text{ to } x^3]\sqrt{(1+t^2)}dt$. Find $G'(x)$.

7. A particle moves with velocity $v(t)=2t-4$ on $0\le t\le 5$. (a) Find net change. (b) Find total distance traveled.

8. $F(x)=\int[\sin x \text{ to } x^2+1]f(t)dt$, where $f(x)=x^2+2$. Find $F'(x)$ and simplify.