A IAL Pure Maths 4
Differentiation
QP

1.Jan 2025 question 2

2. The curve C has equation

$$3x + 5y^2 + 4x^2y = 10(2^x) + 35 \qquad y > 0$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y

(6)

Curve C cuts the y-axis at the point P

(b) Find the exact value of the gradient of the tangent to C at P

(2)

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2.Jan 2025 question 4

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The volume, V, of a spherical balloon is increasing at a constant rate of $70\pi\,\text{cm}^3\,\text{s}^{-1}$

Find the rate of increase of the radius of the balloon, in cm s^{-1}, at the instant when the radius of the balloon is 5 cm.

[The volume V of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$]

(4)

(ii) The depth of water in a cave is being monitored.

The rate of increase in the depth of water, h cm, at a particular point in the cave is modelled by the differential equation

$$\frac{dh}{dt} = \frac{k}{h^3}$$

where k is a constant and t hours is the time after monitoring began.

Given that

- initially the depth of water was 4 cm
- 5 hours after monitoring began, the depth of water was 6 cm
- T hours after monitoring began, the depth of water was 10 cm

solve the differential equation to find the value of T.

Give your answer to one decimal place.

(6)

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3.Jan 2025

9.

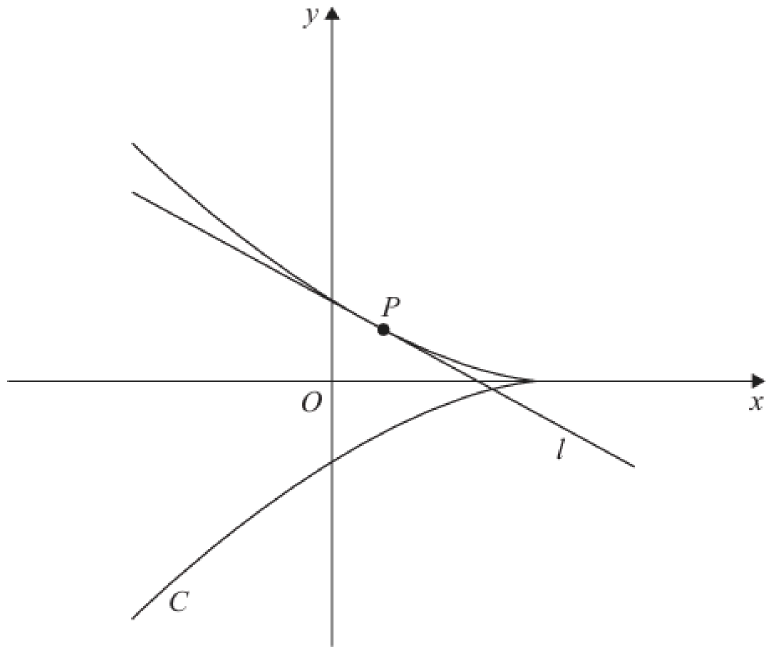

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = \sin^3 t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \frac{\pi}{6}$

The line l, shown in Figure 2, is the tangent to C at P.

(a) Use parametric differentiation to show that

(i) $\frac{dy}{dx} = k\sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q.

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q.

(6)

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4.Jan 2024

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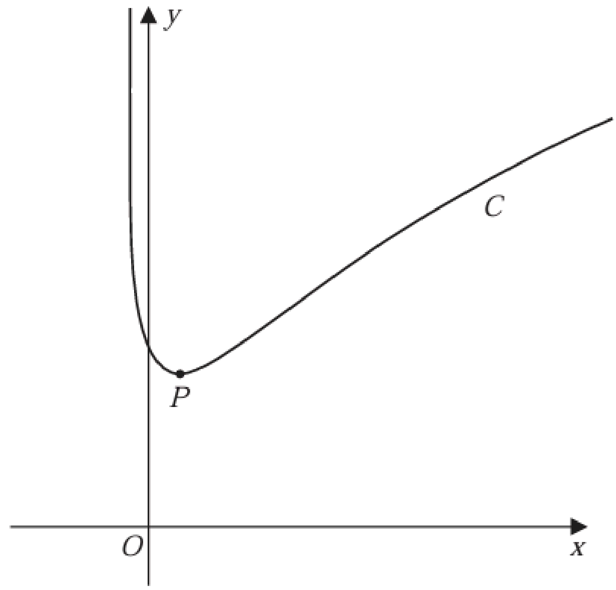

Figure 1

The curve C, shown in Figure 1, has equation

$$y^2x + 3y = 4x^2 + k \qquad y > 0$$

where k is a constant.

(a) Find $\frac{dy}{dx}$ in terms of x and y

(5)

The point $P(p, 2)$, where p is a constant, lies on C.

Given that P is the minimum turning point on C,

(b) find

(i) the value of p

(ii) the value of k

(4)

5.Jan 202

4.

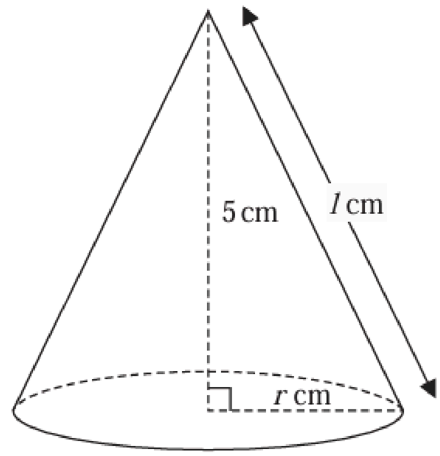

Figure 2

A cone, shown in Figure 2, has

- fixed height 5 cm
- base radius r cm
- slant height l cm

(a) Find an expression for l in terms of r

(1)

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in cm^2 per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi r l$]

(4)

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6.Jan 2024

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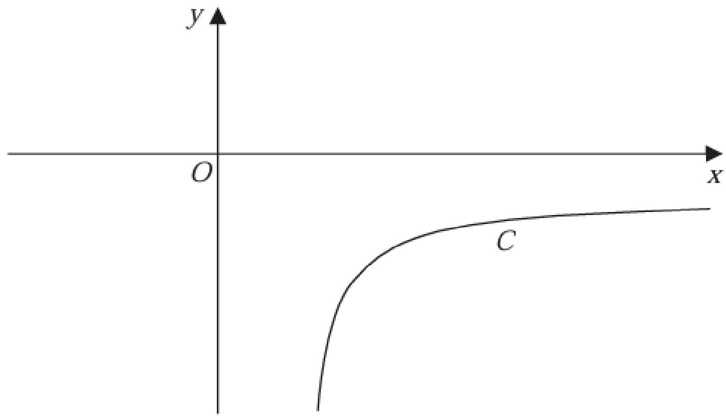

Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \sec t \qquad y = \sqrt{3}\tan\left(t + \frac{\pi}{3}\right) \qquad \frac{\pi}{6} < t < \frac{\pi}{2}$$

(b) Find an equation for the tangent to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are constants.

(4)

(c) Show that all points on C satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where A and B are constants to be found.

(5)

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7.June 2024

3. The curve C is defined by the equation

$$8x^3 - 3y^2 + 2xy = 9$$

Find an equation of the normal to C at the point (2, 5), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

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8.June 2024

7. The current, x amps, at time t seconds after a switch is closed in a particular electric circuit is modelled by the equation

$$\frac{dx}{dt} = k - 3x$$

where k is a constant.

Initially there is zero current in the circuit.

(a) Solve the differential equation to find an equation, in terms of k, for the current in the circuit at time t seconds.
Give your answer in the form $x = f(t)$.

(6)

Given that in the long term the current in the circuit approaches 7 amps,

(b) find the value of k.

(2)

(c) Hence find the time in seconds it takes for the current to reach 5 amps, giving your answer to 2 significant figures.

(3)

9.Oct 2024

3.

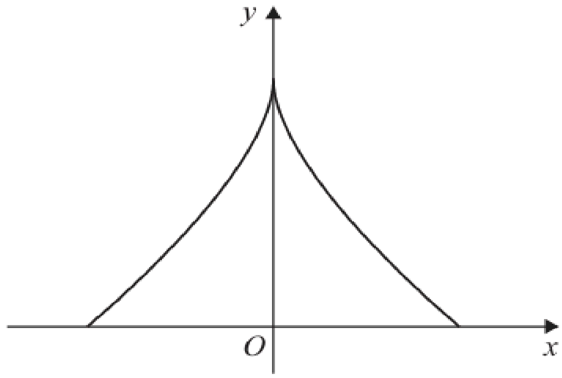

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 3\sin^3\theta \qquad y = 1 + \cos 2\theta \qquad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = k\operatorname{cosec}\theta \qquad \theta \neq 0$$

where k is a constant to be found.

(3)

The point P lies on C where $\theta = \frac{\pi}{6}$

The point P lies on C where $\theta = \frac{\pi}{6}$

(b) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(3)

(c) Show that C has Cartesian equation

$$8x^2 = 9(2 - y)^3 \qquad -q \leqslant x \leqslant q$$

where q is a constant to be found.

(3)

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10.Oct 2024

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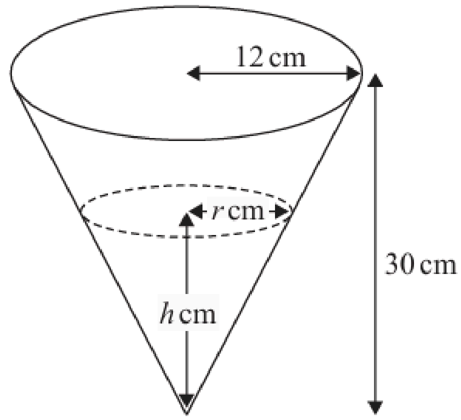

Figure 3

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is $V\text{cm}^3$

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \frac{1}{3}\pi r^2 h$]

(2)

Given that water flows into the container at a constant rate of $2\pi\,\text{cm}^3\,\text{s}^{-1}$

(b) find, in cm s^{-1}, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

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11.Oct 2024

9. (a) Express $\dfrac{1}{x(2x-1)}$ in partial fractions.

(2)

The height above ground, h metres, of a carriage on a fairground ride is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right)$$

where t seconds is the time after the start of the ride.

Given that, at the start of the ride, the carriage is 2.5 m above ground,

(b) solve the differential equation to show that, according to the model,

$$h = \frac{5}{10 - 8\mathrm{e}^{k\sin\left(\frac{t}{10}\right)}}$$

where k is a constant to be found.

(6)

(c) Hence find, according to the model, the time taken for the carriage to reach its maximum height above ground for the 3rd time.
Give your answer to the nearest second.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

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12.Jan 2023

2. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

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13.Jan 2023

5.

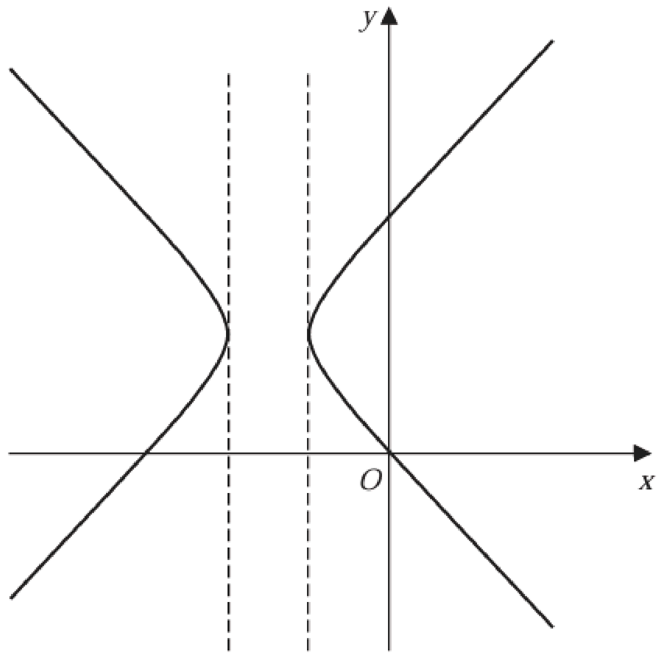

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

The curve is not defined for values of x in the interval (p, q), as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of p and the value of q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

14.Jan 2023

7. The volume $V\text{cm}^3$ of a spherical balloon with radius $r$cm is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\frac{dV}{dr}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

(5)

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

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15.June 2023

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ°C, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine

- is 85 °C at the instant the engine is turned off
- is 40 °C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a, b, c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20 °C. Give your answer to the nearest minute.

(2)

16.June 2023

8.

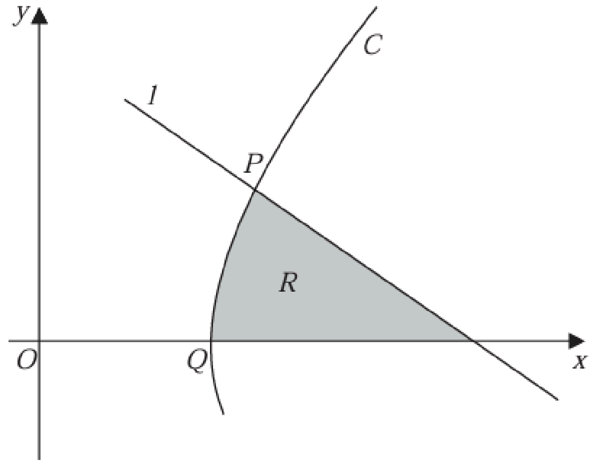

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve C intersects the x-axis at the point Q.

(a) Find the x coordinate of Q.

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

(b) write down the coordinates of P

(1)

(c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

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17.Oct 2023

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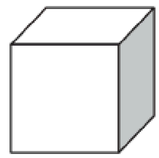

Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the surface area of the cube is S cm^2
- the volume of the cube is V cm^3

Given that the surface area of the cube is increasing at a constant rate of 4 cm^2 s^{-1}

(a) show that $\frac{dx}{dt} = \frac{k}{x}$ where k is a constant to be found,

(4)

(b) show that $\frac{dV}{dt} = V^p$ where p is a constant to be found.

(3)

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18.Oct 2023

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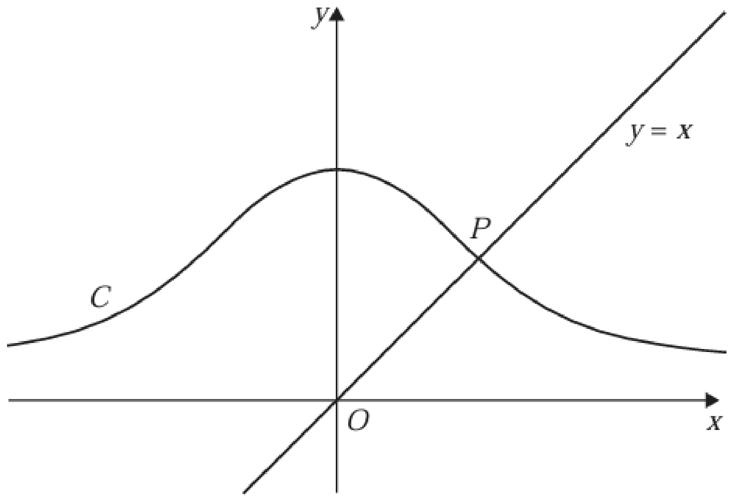

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y^3 - x^2 + 4x^2y = k$$

where k is a positive constant greater than 1

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P lies on C.

Given that the normal to C at P has equation $y = x$, as shown in Figure 2,

(b) find the value of k.

(5)

19.Oct 2023

7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x, in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

(a) Show that $k = 1.8$

(2)

(b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x, in thousands, is modelled by the differential equation

$$3\frac{dx}{dt} = x(9 - 2x)$$

(c) Write $\frac{3}{x(9 - 2x)}$ in partial fraction form.

(3)

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

(5)

(e) Find the long-term population of goats predicted by this **second** model.

(1)

20.Oct 2023

8.

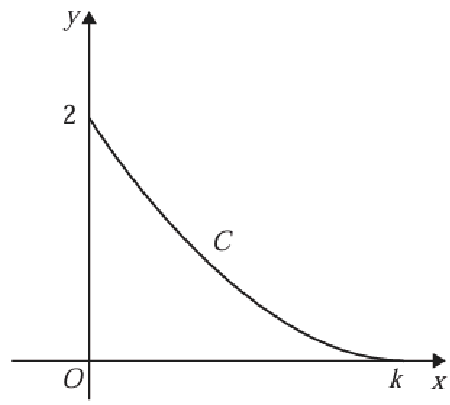

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the y-axis at 2 and the x-axis at k, where k is a constant.

(a) State the value of k.

(1)

(b) Use parametric differentiation to show that

$$\frac{dy}{dx} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The tangent to C at the point P cuts the y-axis at the point N.

(c) Find the exact y coordinate of N, giving your answer in simplest form.

(3)

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21.Jan 2022

3. The curve C has parametric equations

$$x = 3 + 2\sin t \qquad y = \frac{6}{7 + \cos 2t} \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Show that C has Cartesian equation

$$y = \frac{12}{(7 - x)(1 + x)} \qquad p \leqslant x \leqslant q$$

where p and q are constants to be found.

(6)

(b) Hence, find a Cartesian equation for C in the form

$$y = \frac{a}{x + b} + \frac{c}{x + d} \qquad p \leqslant x \leqslant q$$

where a, b, c and d are constants.

(3)

22.Jan 2022

4.

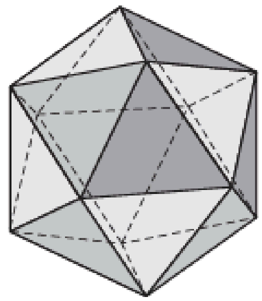

Figure 1

A regular icosahedron of side length x cm, shown in Figure 1, is expanding uniformly.

The icosahedron consists of 20 congruent equilateral triangular faces of side length x cm.

(a) Show that the surface area, A cm^2, of the icosahedron is given by

$$A = 5\sqrt{3}x^2$$

(2)

Given that the volume, V cm^3, of the icosahedron is given by

$$V = \frac{5}{12}\left(3+\sqrt{5}\right)x^3$$

(b) show that $\dfrac{dV}{dA} = \dfrac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$

(3)

The surface area of the icosahedron is increasing at a constant rate of $0.025\,\text{cm}^2\,\text{s}^{-1}$

(c) Find the rate of change of the volume of the icosahedron when $x = 2$, giving your answer to 2 significant figures.

(3)

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23.June 2022

2. (a) Express $\dfrac{1}{(1+3x)(1-x)}$ in partial fractions.

(3)

(b) Hence find the solution of the differential equation

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leqslant \frac{1}{2}$$

for which $x = \dfrac{1}{2}$ when $y = \dfrac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where n is an integer to be found.

(6)

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24.June 2022

3.

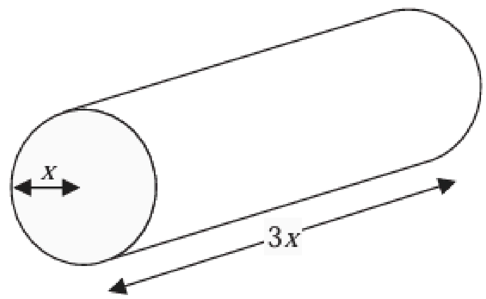

Figure 1

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At t seconds after the tablet is dropped into the water, the radius of the tablet is x mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of $0.5\,\text{mm}^2\,\text{s}^{-1}$

(a) Find $\frac{dx}{dt}$ when $x = 7$

(4)

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$

(4)

25.June 2022

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where k is a constant.

(a) Show that

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

(4)

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of k

(4)

26.June 2022

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \sin t - 3\cos^2 t \qquad y = 3\sin t + 2\cos t \qquad 0 \leqslant t \leqslant 5$$

(a) Show that $\frac{dy}{dx} = 3$ where $t = \pi$

(4)

The point P lies on C where $t = \pi$

(b) Find the equation of the tangent to the curve at P in the form $y = mx + c$ where m and c are constants to be found.

(3)

Given that the tangent to the curve at P cuts C at the point Q

(c) show that the value of t at point Q satisfies the equation

$$9\cos^2 t + 2\cos t - 7 = 0$$

(2)

(d) Hence find the exact value of the y coordinate of Q

(3)

27.Oct 2022

1. A curve C has parametric equations

$$x = \frac{t}{t-3} \qquad y = \frac{1}{t} + 2 \qquad t \in \mathbb{R} \qquad t > 3$$

Show that all points on C lie on the curve with Cartesian equation

$$y = \frac{ax - 1}{bx}$$

where a and b are constants to be found.

(3)

28.Oct 2022

6.

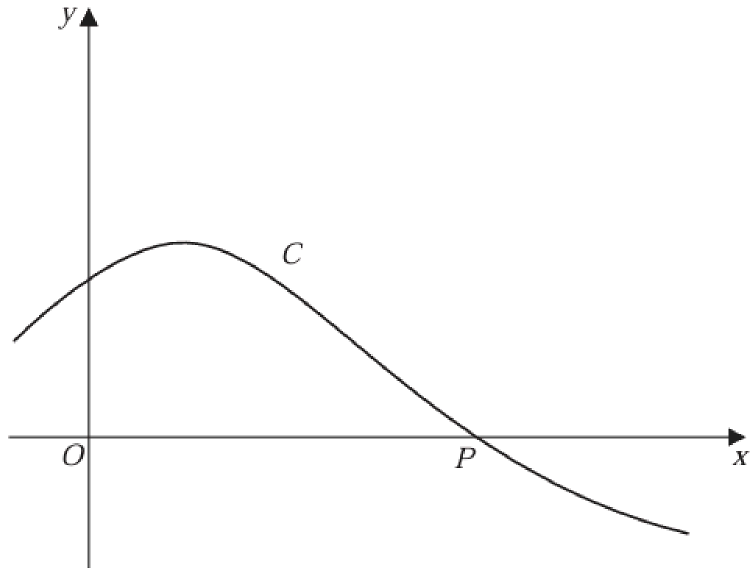

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 + 3\tan t \qquad y = 2\cos 2t \qquad -\frac{\pi}{6} \leqslant t \leqslant \frac{\pi}{3}$$

The curve crosses the x-axis at point P, as shown in Figure 3.

(a) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

The curve C has equation $y = f(x)$, where f is a function with domain $\left[k,\ 1 + 3\sqrt{3}\right]$

(b) Find the exact value of the constant k.

(1)

(c) Find the range of f.

(2)

29.Oct 2022

10. A spherical ball of ice of radius 12 cm is placed in a bucket of water.

In a model of the situation,

- the ball remains spherical as it melts
- t minutes after the ball of ice is placed in the bucket, its radius is r cm
- the rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius
- the radius of the ball of ice is 6 cm after 15 minutes

Using the model and the information given,

(a) find an equation linking r and t,

(5)

(b) find the time taken for the ball of ice to melt completely.

(2)

(c) On Diagram 1 on page 27, sketch a graph of r against t.

(1)

30.Jan 2021

4. The curve C is defined by the parametric equations

$$x = \frac{1}{t} + 2 \qquad y = \frac{1-2t}{3+t} \qquad t > 0$$

(a) Show that the equation of C can be written in the form $y = \mathrm{g}(x)$ where g is the function

$$\mathrm{g}(x) = \frac{ax+b}{cx+d} \qquad x > k$$

where a, b, c, d and k are integers to be found.

(5)

(b) Hence, or otherwise, state the range of g.

(2)

31.Jan 2021

6. A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The curve crosses the y-axis at the origin and at the point P.

(b) Find the equation of the normal to the curve at P, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

32.Jan 2021

10. (a) Write $\dfrac{1}{(H-5)(H+3)}$ in partial fraction form.

(3)

The depth of water in a storage tank is being monitored.

The depth of water in the tank, H metres, is modelled by the differential equation

$$\frac{dH}{dt} = -\frac{(H-5)(H+3)}{40}$$

where t is the time, in days, from when monitoring began.

Given that the initial depth of water in the tank was 13 m,

(b) solve the differential equation to show that

$$H = \frac{10 + 3e^{-0.2t}}{2 - e^{-0.2t}}$$

(7)

(c) Hence find the time taken for the depth of water in the tank to fall to 8 m.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

According to the model, the depth of water in the tank will eventually fall to k metres.

(d) State the value of the constant k.

(1)

33.June 2021

3.

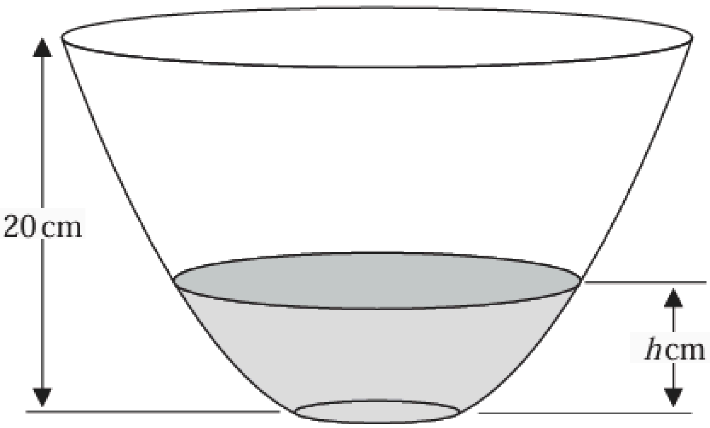

Figure 2

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is h cm, the volume of water in the bowl, V cm³, is modelled by the equation

$$V = \frac{1}{3}h^2(h + 4) \qquad 0 \leqslant h \leqslant 20$$

Given that the water flows into the bowl at a constant rate of $160\,\text{cm}^3\,\text{s}^{-1}$, find, according to the model,

(a) the time taken to fill the bowl,

(2)

(b) the rate of change of the depth of the water, in $\text{cm}\,\text{s}^{-1}$, when $h = 5$

(5)

34.June 2021

5. A curve has equation

$$y^2 = ye^{-2x} - 3x$$

(a) Show that

$$\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y}$$

(4)

The curve crosses the y-axis at the origin and at the point P.

The tangent to the curve at the origin and the tangent to the curve at P meet at the point R.

(b) Find the coordinates of R.

(5)

35.June 2021

8. (a) Given that $y = 1$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{6xy^{\frac{1}{3}}}{e^{2x}} \qquad y \geqslant 0$$

giving your answer in the form $y^2 = g(x)$.

(7)

(b) Hence find the equation of the horizontal asymptote to the curve with equation $y^2 = g(x)$.

(2)

36.Oct 2021

2. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \qquad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

(6)

37.Oct 2021

9.

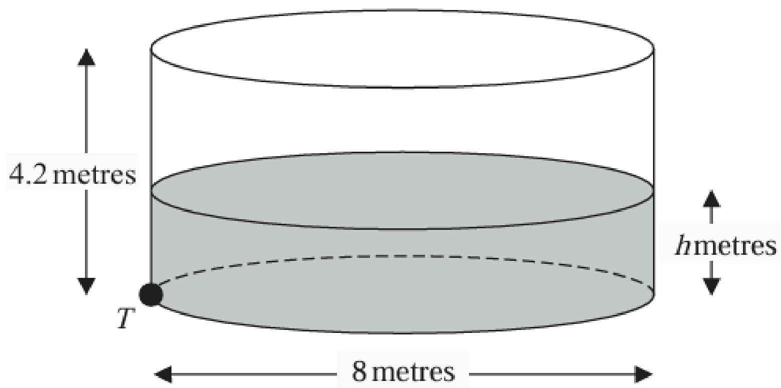

Figure 4

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of (0.6π) m^3 per minute.

There is a tap at point T at the bottom of the tank.

At time t minutes after the tap has been opened,

- the depth of the water is h metres
- the water is leaving the tank at a rate of $(0.15\pi h)$ m^3 per minute

(a) Show that

$$\frac{dh}{dt} = \frac{12 - 3h}{320}$$

(4)

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

(6)

38.Oct 2020

4.

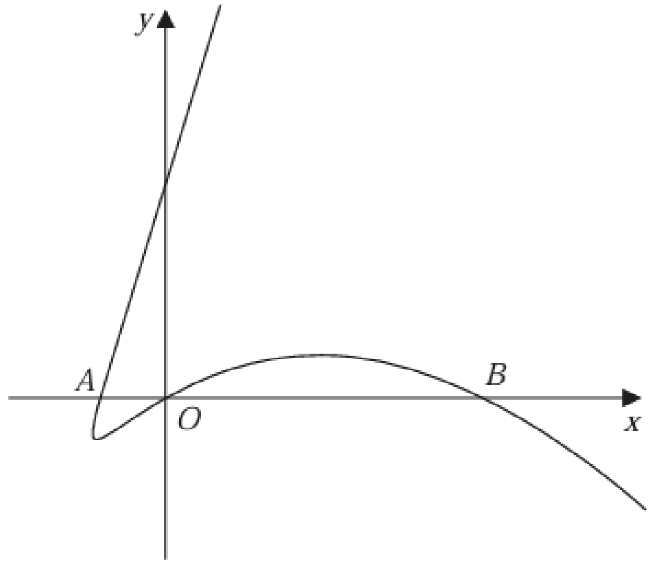

Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B, as shown in Figure 2.

(a) Find the coordinates of A and show that B has coordinates $(20, 0)$.

(3)

(b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x coordinate of P.

(4)

39.Oct 2020

6. A curve C has equation

$$y = x^{\sin x} \qquad x > 0 \qquad y > 0$$

(a) Find, by firstly taking natural logarithms, an expression for $\frac{dy}{dx}$ in terms of x and y.
(5)

(b) Hence show that the x coordinates of the stationary points of C are solutions of the equation

$$\tan x + x\ln x = 0$$

(2)

40.Oct 2020

9. Bacteria are growing on the surface of a dish in a laboratory.

The area of the dish, A cm^2, covered by the bacteria, t days after the bacteria start to grow, is modelled by the differential equation

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{A^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $A = 2.25$ when $t = 3$

(a) show that

$$A = \left(\frac{pt}{qt + r}\right)^2$$

where p, q and r are integers to be found.

(7)

According to the model, there is a limit to the area that will be covered by the bacteria.

(b) Find the value of this limit.

(2)

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