Solubility - Model exercise

Exercise

At a given temperature, the solubility of aluminum hydroxide is 1 mg/L.
(a) Determine the solubility product constant (Ks)
(b) 2 liters of NaOH 0.05 M and 6 liters of $AlCl_3$ 0.001 M are added: will a precipitate of aluminum hydroxide appear?

Resolution

SECTION A
First, we will write the dissociation equation for aluminum hydroxide:

a)

$$Al(OH)_3 (s) \rightleftharpoons \underset{x}{Al^{3+}(aq)} + \underset{3x}{3\,OH^-(aq)}$$

$$\downarrow$$

$$K_S = [Al^{3+}][OH^-]^3$$

$$[OH^-] = 3\,[Al^{3+}]$$

This way, we will know: (1) what is the expression of Ks, and (2) that the concentration of hydroxide ions is three times that of aluminum ions.

Next, we will determine the molarity of aluminum hydroxide in a saturated solution. Next, we will determine the concentrations of both ions (aluminum and hydroxide ions) and deterine the value of Ks.

$$Mm\,(Al(OH)_3) = 27 + (3\times16) + (3\times1) = 78\ g/mol$$

$$[Al(OH)_3] = 1\ \frac{mg}{L} \times \frac{1\,g}{1000\,mg} \times \frac{1\,mol}{78\,g} = 1.28\times10^{-5}\ M$$

$$\hookrightarrow [Al^{3+}] = [Al(OH)_3] = 1.28\times10^{-5}\ mol/L$$

$$\hookrightarrow [OH^-] = 3\times[Al(OH)_3] = 3.84\times10^{-5}\ mol/L$$

$$\downarrow$$

$$K_s = [Al^{3+}][OH^-]^3 = (1.28\times10^{-5})\times(3.84\times10^{-5})^3 =$$

$$= \boxed{7.25\times10^{-19}}$$

SECTION B

When two solutions are mixed, we should (1) determine the ionic product ($Q=[Al^{3+}]\ [OH^-]^3$) and (2) compare it against the value of Ks, in order to know if a precipitate will be formed.

b)

$$NaOH \rightarrow Na^+ + OH^-$$

$$n(OH^-) = 2L \times \frac{0.05\ mol\ NaOH}{1L} \times \frac{1\ mol\ OH^-}{1\ mol\ NaOH} = 0.1\ mol\ OH^-$$

$$[OH^-] = \frac{0.1\ mol\ OH^-}{8L} = 0.0125\ M\ OH^-$$

$$AlCl_3 \rightarrow Al^{3+} + 3\,Cl^-$$

$$n(Al^{3+}) = 6L \times \frac{0.001\ mol\ AlCl_3}{1L} \times \frac{1\ mol\ Al^{3+}}{1\ mol\ AlCl_3} = 0.006\ mol\ Al^{3+}$$

$$[Al^{3+}] = \frac{0.006\ mol\ Al^{3+}}{8L} = 7.5\times10^{-4}\ M\ Al^{3+}$$

The comparison of both values gives this result: Q > Ks. Thus, a precipitate will be formed.

$$Q = [Al^{3+}][OH^-]^3 = (7.5 \times 10^{-4}) \times (1.25 \times 10^{-2})^3 = 1.46 \times 10^{-9}$$

$$Q > K_s \quad \ldots \quad Al(OH)_3 \boxed{\downarrow}$$