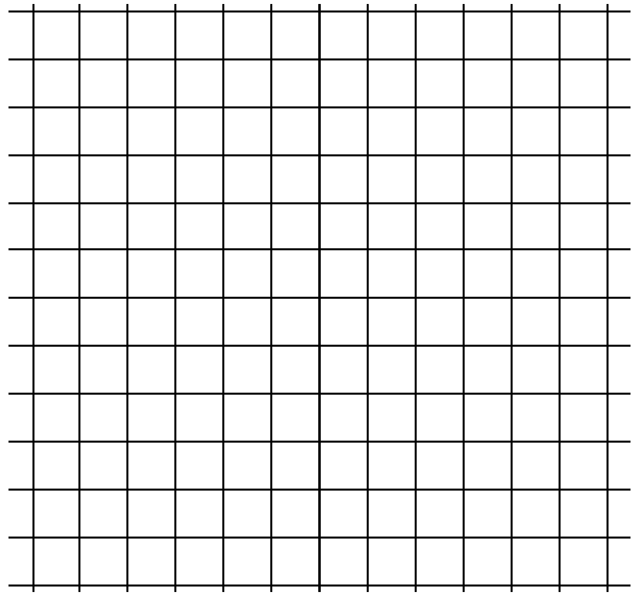
6.6 OPERATIONS WITH ALGEBRAIC VECTORS IN R^2



Defining a vector in R^2.

A second way of writing $\vec{OP} = (a, b)$ is with the use of the unit vector $\vec{i}$ and $\vec{j}$.

The vectors $\vec{i} = (1, 0)$ and $\vec{j} = (0, 1)$ have magnitude 1 and lie along the positive x- and y-axes, respectively, as shown on the graph.

Ex. 1 Write each in term of its components.

a) $\vec{OP} = (4, 5)$

b) Vector $\vec{OQ}$ with Q(-3,0)

c) with points A(-3,2) and B(2,-4)

Representations of Vectors in R^2

The position vector $\vec{OP}$ can be represented as either $\vec{OP} = (a,\ b)$ or $\vec{OP} = a\vec{i} + b\vec{j}$, where $O(0, 0)$ is the origin, $P(a,\ b)$ is any point on the plane, and $\vec{i}$ and $\vec{j}$ are the standard unit vectors for R^2. Standard unit vectors $\vec{i}$ and $\vec{j}$, are unit vectors that lie along the x- and y-axes respectively, so $\vec{i} = (1,\ 0)$ and $\vec{j} = (0,\ 1)$. Every vector in R^2, given in terms of its components, can also be written uniquely in terms of $\vec{i}$ and $\vec{j}$. The unit vectors $\vec{i}$ and $\vec{j}$ are also called the standard basis vectors in R^2.

Ex. 2 Given $\vec{x} = 2\vec{i} - 3\vec{j}$ and $\vec{y} = \quad - 3\vec{j}$ determine:

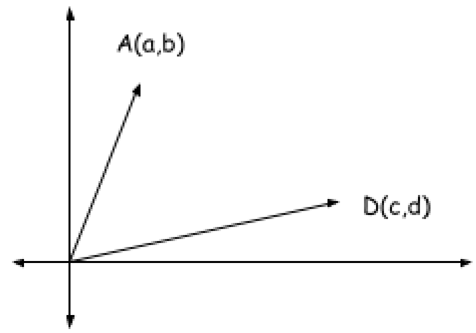
a) $|\vec{x}|$

b) $|\vec{y}|$

c) $\vec{x} + \vec{y}$

Addition of Two Vectors Using Component Form

Determine the sum $\vec{OA} = (a,\ b)$ and $\vec{OD} = (c,\ d)$, where A and D are any two points in R^2.



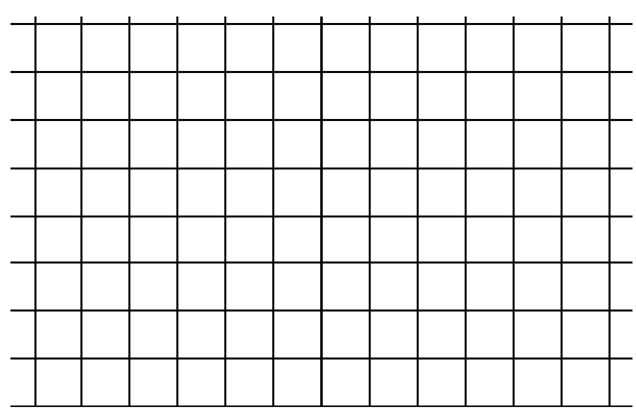
Therefore $\vec{OA} + \vec{OD} =$

And $\vec{OA} - \vec{OD} =$

Scalar Multiplication of Vectors Using Components

Given $\vec{OP} = (a, b)$ determine the coordinates of $m\vec{OP}$ where m is a real number.

Ex.3 Given $\vec{a} = \vec{OA} = (1, 3)$ and $\vec{b} = \vec{OB} = (4, -2)$, determine the components of $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, and illustrate each of these vectors on the graph.



Vectors in R^2 Defined by Two Points

Ex. 4 Given points $A(1, 4)$ and $B(4, -3)$ determine $\vec{AB}$ and $|\vec{AB}|$.

In general, given points $A(x_1, y_1)$ and $B(x_2, y_2)$ determine $\vec{AB}$ and $|\vec{AB}|$.

Position Vectors and Magnitudes in R^2 If $A(x_1, y_1)$ and $B(x_2, y_2)$are two points, then the vector $\vec{AB} = (x_2 - x_1,\ y_2 - y_1)$ is its related position vector $\vec{OP}$ and $|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ex. 5 For the vectors $\vec{x} = 2\vec{i} - 3\vec{j}$ and $\vec{y} = -4\vec{i} - 3\vec{j}$, determine $|\vec{x} + \vec{y}|$ and $|\vec{x} - \vec{y}|$.

Ex. 6 A(-3,7), B(5,22), and C(8,18) are three points in R^2.

a) Calculate the value of $\left|\vec{AB}\right| + \left|\vec{BC}\right| + \left|\vec{CA}\right|$, the perimeter of triangle ABC.

b) Calculate the value of $\left|\vec{AB} + \vec{BC}\right|$,

Ex. 7 If $\vec{a} = (5, -6)$, $\vec{b} = (-7, 3)$, and $\vec{c} = (2, 8)$, calculate $\left|\vec{a} - 3\vec{b} - \frac{1}{2}\vec{c}\right|$