Name		( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2020

Subject : Additional Mathematics
Paper : 4047/01
Level : Secondary Four Express
Date : 25 August 2020
Duration : 2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correcting fluid / tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use

This paper consists of 28 printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

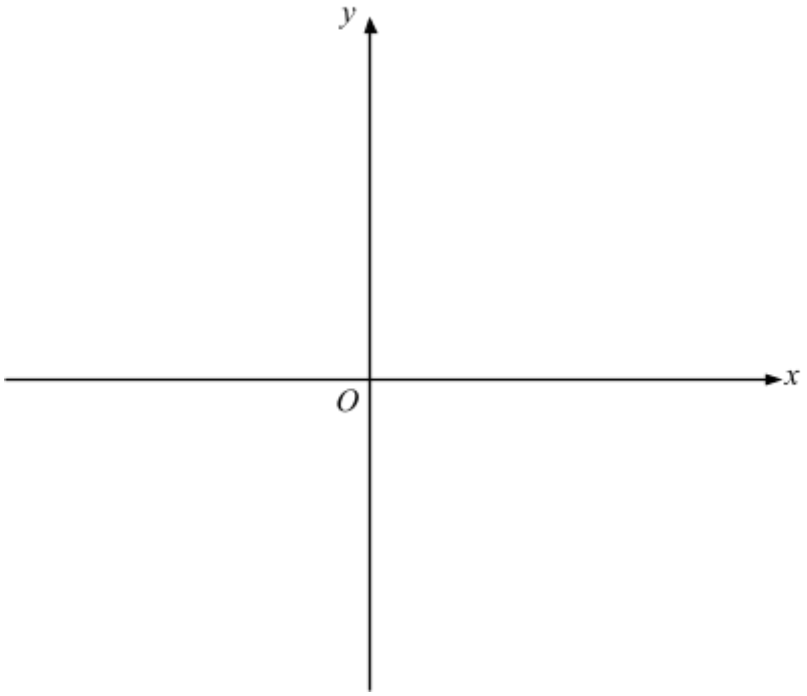
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 **Without using a calculator**, express $\text{cosec}^2 75°$ in the form of $a+b\sqrt{3}$ where a and b are integers. [4]

2 A curve has equation $y^2 = 9x.$

(i) Sketch the graph of $y^2 = 9x.$ [1]



(ii) The curves $y^2 = 9x$ and $y = kx^{-\frac{3}{2}}$ meet at point M where $x = 4.$ Find the value(s) of k. [3]

3 Given $A = \tan^{-1}\left(-\frac{3}{4}\right)$, find, **without using a calculator**, the value of $\sin\left(A - \frac{\pi}{2}\right)$. [3]

4 Given that $p + q = k + 1$ and $pq = 1$, where k is a constant, find the range of values of k for which p and q are distinct and real numbers. [4]

5 A function is defined by the equation $y = \mathrm{f}(x)$ such that $\mathrm{f}'(x) = \dfrac{1}{x-2} + e^{-2x}$, for $x > 2$.

(i) Given that the curve passes through the point $\left(3, -\dfrac{1}{e^6}\right)$, find an expression for $\mathrm{f}(x)$. [4]

(ii) Explain, with reasons, whether the function $f'(x)$ is increasing or decreasing. [3]

6 The equation of a curve is $y = mx^2 + (m+n)x + n$, where m and n are constants, and $m > 0$.
Explain why the curve will not lie completely above the x-axis.
[3]

7 **(a)** Solve the equation $2x^3 - 3x^2 - 3x + 4 = 0$. [4]

(b) It is given that $x-5$ is a factor of $\mathrm{p}(x)+1$, where $\mathrm{p}(x)$ is a polynomial. Find the remainder when $\mathrm{g}(x)=2x^3-\mathrm{p}(x)+5$ is divided by $x-5$. [2]

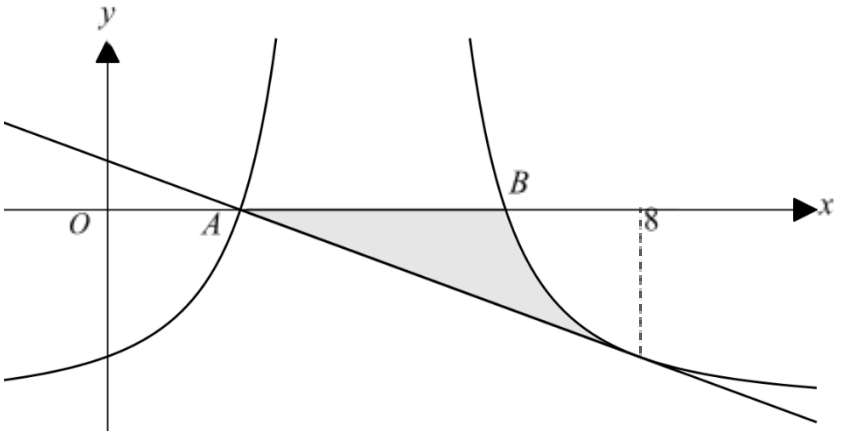
8 The roots of the quadratic equation $4x^2 - 2x + 1 = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. Find a quadratic equation whose roots are $\frac{\beta}{\alpha^2}$ and $\frac{\alpha}{\beta^2}$. [5]

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9 **(i)** Show that $\frac{d}{dx}\left(\frac{2\ln x}{x^3}\right) = \frac{2}{x^4} - \frac{6\ln x}{x^4}$.
[3]

(ii) Integrate $\frac{\ln x}{x^4}$ with respect to x. [3]

10



The diagram shows part of the curve $y = \frac{32}{(4-x)^2} - 8$, intersecting the x-axis at A and B. The tangent to the curve at $x = 8$ meets the x-axis at A.

(i) Find the equation of the tangent. [4]

(ii) Find the area of the shaded region bounded by the tangent, the curve and the x-axis. [5]

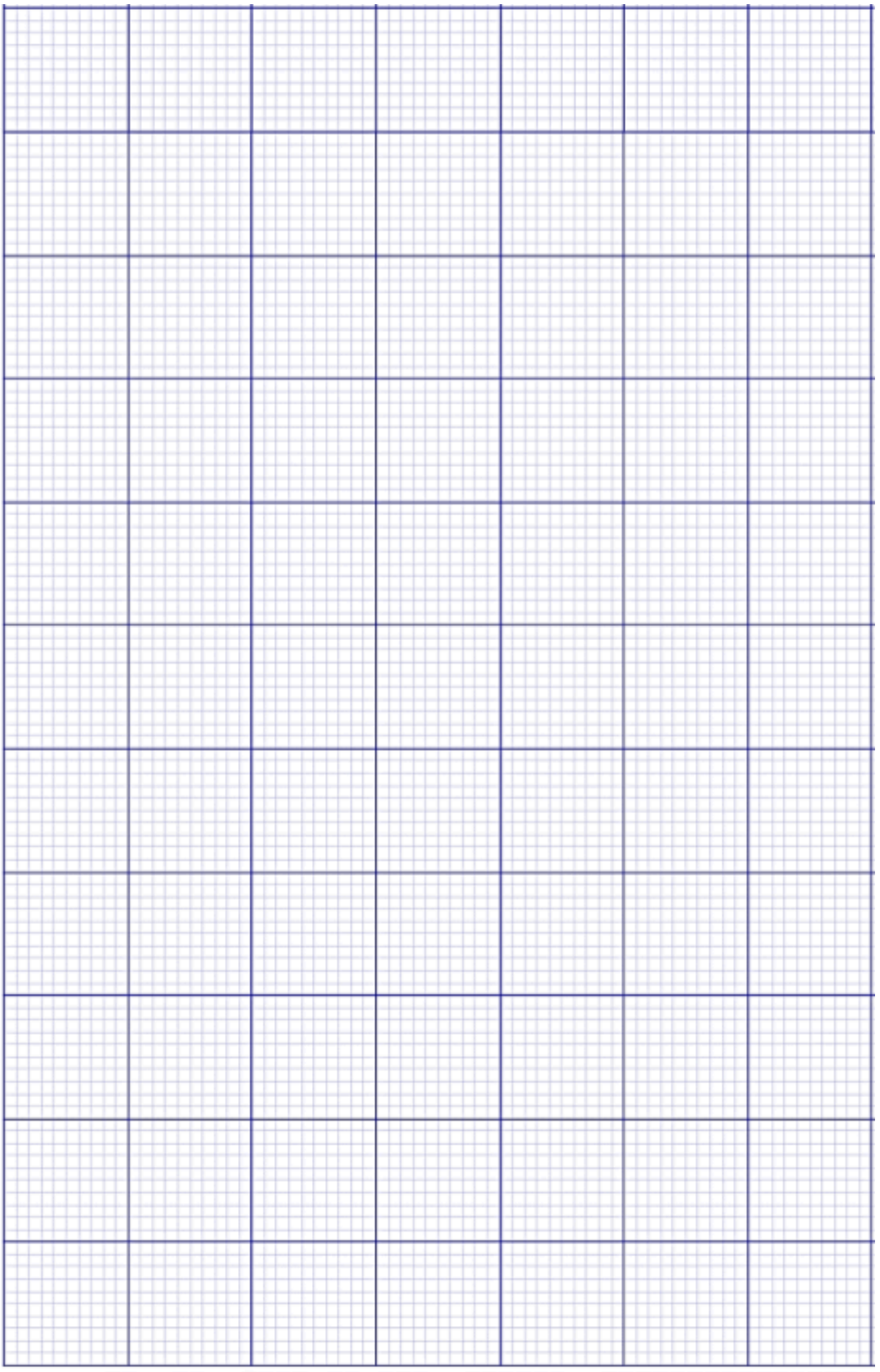
11 The table shows experimental values of two variables x and y.

x	1.00	1.22	1.29	1.32	1.34
y	1	2	3	4	5

It is known that x and y are related by the equation $y = \dfrac{1}{ax^2 + b}$ for $x > 0$, where a and b are constants.

(i) Plot $x^2 y$ against y to obtain a straight line graph. [2]

11(i)



(ii) Use your graph to estimate the values of a and b.
[4]

(iii) By drawing a suitable line on your graph, solve the equation $2.5 = \frac{1}{ax^2 + b}$.
[2]

12 A circle has equation $x^2 + y^2 - 20x + 10y + 100 = 0$. Find

(i) the radius and coordinates of the centre of the circle, [3]

(ii) the equations of the tangents from the origin to the circle, [5]

(iii) **without solving simultaneous equations**, the coordinates of the points at which these tangents meet the curve. [3]

13 **(i)** Solve the equation $2\cos 2A - 3\sin A - 1 = 0$ for $0 \le A \le 2\pi$. [4]

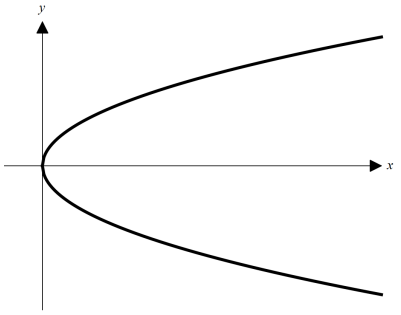
(ii) On the same axes sketch, for $0 \le x \le 4\pi$, the graphs of

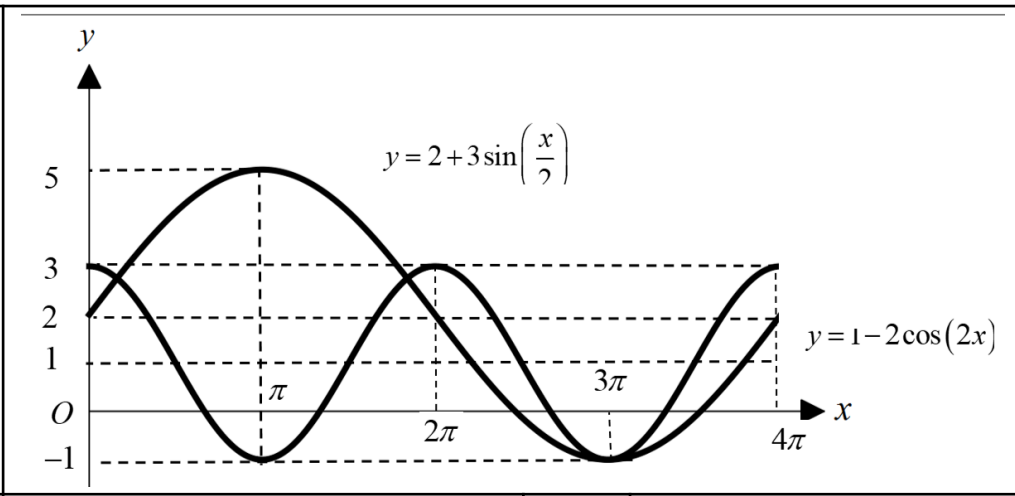
$$y = 2 + 3\sin\frac{x}{2} \text{ and } y = 2\cos x + 1.$$ [4]

(iii) Explain how the solutions of the equation in part **(i)** could be used to find the x-coordinates of the points of intersection of the graphs of part **(ii).** [2]

~ End of Paper ~

Answer Key

1	$8-4\sqrt{3}$ **2**	**2(i)**	
2(ii)	$k=\pm 48$	**3**	$-\frac{4}{5}$
4	$k<-3$ or $k>1$	**5(i)**	$\therefore \mathrm{f}(x)=\ln(x-2)-\frac{1}{2e^{2x}}-\frac{1}{2e^{6}}$
5(ii)	The function $\mathrm{f}'(x)=\frac{1}{x-2}+e^{-2x}$ is decreasing.		
7(a)	$\therefore x=1, x=1.69$ or -1.19	**7(b)**	256
8	$x^2+x+\frac{1}{4}=0$ or $4x^2+4x+1=0$	**9(ii)**	$\int \frac{\ln x}{x^4}dx=-\frac{1}{9x^3}-\frac{\ln x}{3x^3}+c$
10(i)	$y=-x+2$	**10(ii)**	10 unit^2
11(ii)	$\frac{1}{a}=-1\pm 0.1$ $a=-1$ (accept -1.11 to -0.909) $-\frac{b}{a}=$ Gradient $=2\pm 0.1$ $b=a\times gradient$	**11(iii)**	Accept from 1.24 to 1.28
12(i)	$(10,-5)$	**12(ii)**	$y=0$ or $\mathrm{y}=-\frac{4}{3}x$
12(iii)	$(10,0)$ and $(6,-8)$	**13(i)**	$\therefore A=\frac{3\pi}{2}$, 0.253 or 2.89

13(ii)			
13(iii)	Replacing x with 2A.		