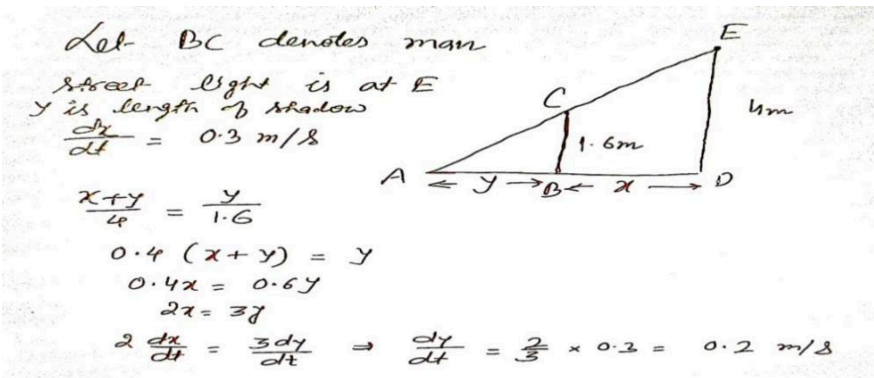
MARKING SCHEME
CLASS XII
MATHS
SECTION-A

1	2	3	4	5	6	7	8	9	10
C	B	C	D	C	B	A	C	C	D
11	12	13	14	15	16	17	18	19	20
A	D	D	C	C	B	B	B	D	A

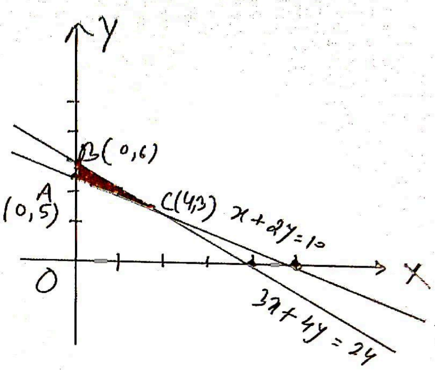
SECTION-B

21	**Applying tanx=tan(**$2\pi - x$)**=tan(**$\pi + x$) **(**$-\pi/6$) **OR** $-1 \le 2x - 1 \le 1$ $0 \le x \le 1$ **, [0,1]** **[0,** π]	**1** **1** **1** **0.5** **0.5**
22	$x^m y^n = (x + y)^{m+n}$ $mlogx + nlogy = (m+n)log(x+y)$ $\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}(1 + \frac{dy}{dx})$ $\frac{dy}{dx} = \frac{y}{x}$ **Or** $y = (sinx)^{log\,x}$ $log\,y = logx.\,log(sinx)$ $\frac{1}{y}\frac{dy}{dx} = logx.cotx + \frac{log(sinx)}{x}$ $\frac{dy}{dx} = y[logx.cotx + \frac{log(sinx)}{x}]$	**0.5** **0.5** **1** **0.5** **1** **0.5**
23		**1** **1**
24	**For correct f'(x)** **For f'(x)>0 in** $(0, \frac{\pi}{4})$ OR **For correct f'(x)** **For critical points x=-1,0,1**	**1** **1** **0.5** **0.5**

	In [-1,1] decreasing **In R-(-1,1) increasing**	**0.5** **0.5**
25	$\vec{a} \times \vec{b} = 32\hat{i} - \hat{j} - 14\hat{k}$ Area of parallelogram=$\sqrt{1221}$ sq unit	**1** **1**

SECTION-C

26	$\frac{dx}{dt} = 2\cos 2t \; and \; \frac{dy}{dt} = 2a\frac{\cos^2 2t}{\sin 2t}$ $\frac{dy}{dx} = \cot 2t$ $\frac{d^2y}{dx^2} =- 2cosec^2 2t\frac{dt}{dx}$ $at\ t = \frac{\pi}{6}, \frac{d^2y}{dx^2} =- \frac{8}{3}$	**1** **0.5** **1** **0.5**
27	y (1 + e^x) dy = (y + 1) e^x dx $\frac{y}{y+1}dy = \frac{e^x}{1+e^x}dx$ **Let y+1=t, dy=dt** **On integrating** $y + 1 - log(y + 1) = log\left(1 + e^x\right) + c$ OR $x\frac{dy}{dx} - y = x^2 sinx$ **Dividing by x,** $\frac{dy}{dx} - \frac{y}{x} = xsinx$ $P =- \frac{1}{x}\ and\ Q = x\, sinx$ Integrating factor=1/x General solution of equation y/x=-cos x+c	**1** **0.5** **1.5** **0.5** **0.5** **1** **1**
28	**Expressing** $\frac{x^2}{\left(x^2+1\right)(x^2+4)} = \frac{1}{3}\left[\frac{4}{x^2+4} - \frac{1}{\left(x^2+1\right)}\right]$ Integrating w.r.to x $\frac{1}{3}\left[2tan^{-1}\frac{x}{2} - tan^{-1}x\right]$ **OR**	**1.5** **1.5** **1** **0.5** **1** **0.5**

	$\int \frac{1}{\sqrt{(a-x)(x-b)}}\,dx$ $\quad a>b$ Let $\quad a-x=t^2$ $x=a-t^2$ $dx=-2t\,dt$ $=\int \frac{-2t\,dt}{\sqrt{t^2(a-b-t^2)}}$ $=\int \frac{-2\,dt}{\sqrt{(a-b)-t^2}} = -2\int \frac{dt}{\sqrt{(\sqrt{a-b})^2-t^2}}$ $= -2\sin^{-1}\frac{t}{\sqrt{a-b}} + c$ $= -2\sin^{-1}\sqrt{\frac{a-x}{a-b}} + c$	
29	$\overrightarrow{PQ} = \hat{i}+9\hat{j}-9\hat{k} = \vec{a}$ let $\overrightarrow{PR} = 3\hat{i}+3\hat{j}+3\hat{k} = \vec{b}$ let $\vec{a}\times\vec{b} = 54\hat{i}-30\hat{j}-24\hat{k} = 6(9\hat{i}-5\hat{j}-4\hat{k})$ Area of triangle $= \frac{1}{2}\lvert\vec{a}\times\vec{b}\rvert = \frac{1}{2}\times 6\sqrt{81+25+16}$ $= 3\sqrt{122}$ sq. units	**0.5** **0.5** **1** **1**
30	$Z = 200x+500y$ $x+2y \geq 10$ $3x+4y \leq 24$ $x \geq 0,\ y \geq 0$ ABC is feasible region  Corner points | Z A (0,5) | 2500 B(0,6) | 3000 → Max. C (4,3) | 2300 → Min. **OR**	**For correct graph** **1.5** **1.5**

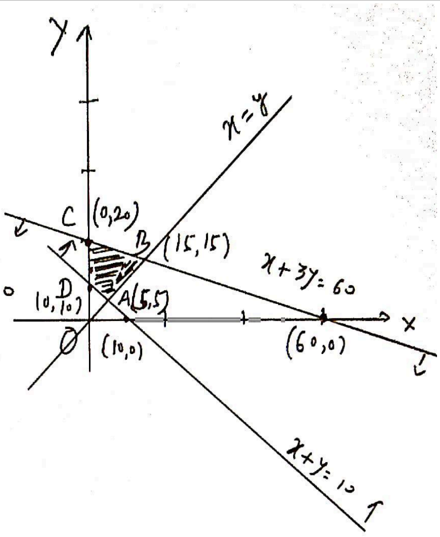
<table>
<tr>
<td></td>
<td>

$Z = 3x + 9y$

$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y, \quad x \geq 0, \quad y \geq 0$



ABCD is feasible region

Corner points	Z
A (5,5)	60
B (15,15)	180
C (0,20)	180
D (0,10)	90

B and C: $\rightarrow$ Maximum

</td>
<td>

For correct graph

1.5

1.5

</td>
</tr>
<tr>
<td>31</td>
<td>

$P(A) = \frac{1}{2}, \quad P(A \cup B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{2}{3} = \frac{1}{2} + P(B) - \frac{1}{6}$

$P(B) = \frac{2}{3} - \frac{1}{2} + \frac{1}{6} = \frac{4-3+1}{6} = \frac{2}{6} = \frac{1}{3}$

(i) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{1/3} = \frac{1}{2}$

(ii) $P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B-A)}{P(B)}$

$= \frac{P(B) - P(A \cap B)}{P(B)} = \frac{\frac{1}{3} - \frac{1}{6}}{\frac{1}{3}}$

$= \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$

</td>
<td>

0.5

1

1.5

</td>
</tr>
</table>

SECTION-D

32

$$A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$$

$|A| = 2(-4+4) - 3(6-5) + 1(12-10)$ exp. along R_1

$= 2\times 0 - 3\times 1 + 1\times 2 = -3+2 = -1 \neq 0$ **1**

so A^{-1} exists.

$$\text{adj } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}$$ **1.5**

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-1}\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ 1 & 9 & 5 \\ -2 & -23 & -13 \end{bmatrix}$$ **0.5**

Now matrix form of given equations

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \quad -(1)$$

Let $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = X$, $\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = B$ **0.5**

Then we have $A'X = B \Rightarrow X = (A')^{-1}\cdot B$

$\Rightarrow X = (A^{-1})'B$ **1**

$$\Rightarrow X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$ **1**

$$\Rightarrow X = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x=1,\ y=2,\ z=3$$ **0.5**

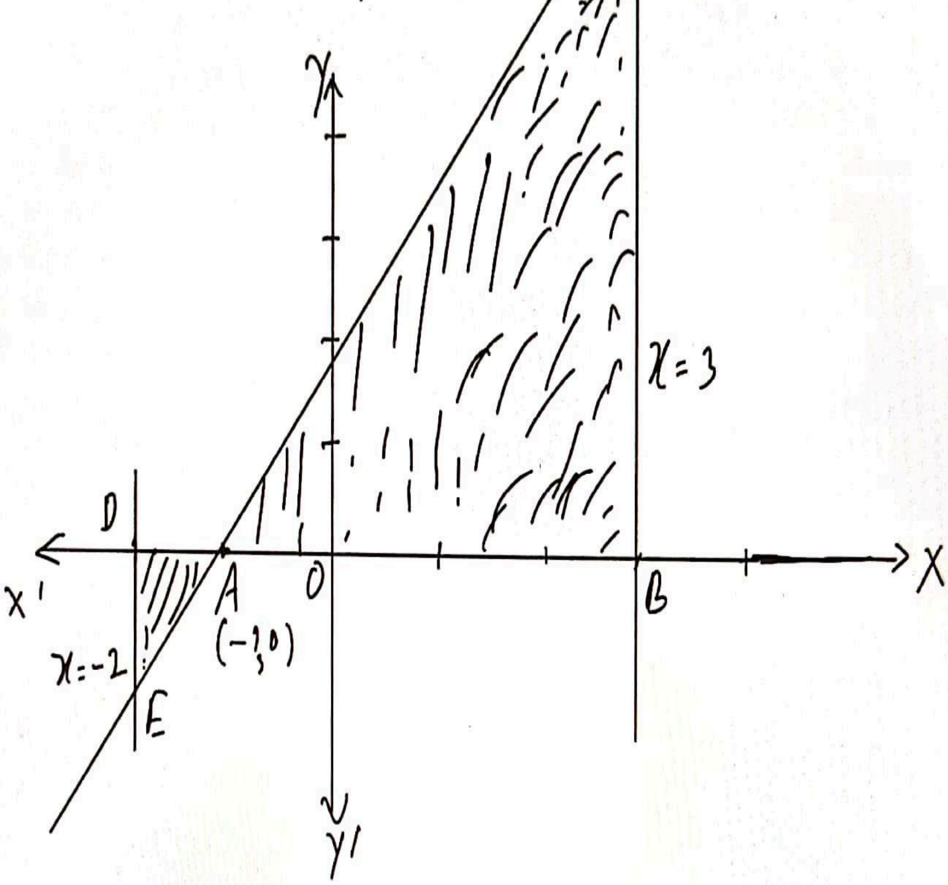
33	$\int_{-1}^{2} \lvert x^3 - x \rvert \, dx$ $\qquad$ $x^3 - x = 0$	
	$x(x^2-1) = 0$	
	$x(x+1)(x-1) = 0$	**0.5**
	$x = 0, -1, 1$	
	$+ \quad - \quad +$	**0.5**
	$-1 \quad 0 \quad 1 \quad 2$	
	$= \int_{-1}^{0} (x^3 - x)\, dx + \int_{0}^{1} (x - x^3)\, dx + \int_{1}^{2} (x^3 - x)\, dx$	**1**
	$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$	**1**
	$= \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\left(\frac{1}{2} - \frac{1}{4}\right) - 0\right] + \left[(4-2) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$	**1**
	$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$	**1**
	OR	
	Let $\quad I = \int_{0}^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x)\, dx$	
	$= \int_{0}^{\pi/2} \log \frac{\sin^2 x}{\sin 2x}\, dx$	**1**
	$= \int_{0}^{\pi/2} \log \frac{\sin^2 x}{2 \sin x \cdot \cos x}\, dx$	**1**
	$I = \int_{0}^{\pi/2} \log \left(\frac{\tan x}{2}\right) dx \quad —(1)$	**1**
	applying $\int_{0}^{a} f(x)\, dx = \int_{0}^{a} f(a-x)\, dx$	
	$I = \int_{0}^{\pi/2} \log \left(\frac{\cot x}{2}\right) dx \quad —(2)$	**1**
	adding (1) & (2)	
	$2I = \int_{0}^{\pi/2} \log \left(\frac{\tan x}{2} \cdot \frac{\cot x}{2}\right) dx$	**0.5**
	$2I = \int_{0}^{\pi/2} \log\left(\frac{1}{4}\right) dx$	
	$2I = \frac{\pi}{2} \log \frac{1}{4} \Rightarrow I = \frac{\pi}{4} \log \frac{1}{4}$	

		0.5

<table>
<tr>
<td>34</td>
<td>

$2x - y + 2 = 0 \Rightarrow y = 2x + 2$

$x = -2$, $x = 3$



Req. Area = ar ABC + ar ADE

$= \int_{-1}^{3} y\,dx + \left|\int_{-2}^{-1} y\,dx\right|$

$= \int_{-1}^{3} (2x+2)\,dx + \left|\int_{-2}^{-1} (2x+2)\,dx\right|$

$= \left[x^2 + 2x\right]_{-1}^{3} + \left|\left[x^2 + 2x\right]_{-2}^{-1}\right|$

$= (9+6) - (1-2) + \left|(1-2) - (4-4)\right|$

$= 15 + 1 + 1 = 17$ sq. units.

</td>
<td>

For each correct line 0.5

Colouring the required region 1

1

1

</td>
</tr>
</table>

		1.5

35

$A(1,2,-3)$, $B(4,0,3)$, $C(6,4,9)$

(i) Equation of BC: $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$$\vec{r} = (4\hat{i} + 3\hat{k}) + \lambda\left[(6\hat{i} + 4\hat{j} + 9\hat{k}) - (4\hat{i} + 3\hat{k})\right]$$

$$\vec{r} = 4\hat{i} + 3\hat{k} + \lambda(2\hat{i} + 4\hat{j} + 6\hat{k}) \quad \text{— (1)}$$ **1.5**

(ii) D is mid point of BC

0.5

$D(5, 2, 6)$

Equation of Median AD

$$\vec{r} = \vec{a} + \mu(\vec{b} - \vec{a})$$

$$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \mu\left[(5\hat{i} + 2\hat{j} + 6\hat{k}) - (\hat{i} + 2\hat{j} - 3\hat{k})\right]$$

$$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \mu(4\hat{i} + 9\hat{k}) \quad \text{— (2)}$$ **1.5**

(iii) Let θ be the angle between BC & AD

from (1) $\vec{b}_1 = 2\hat{i} + 4\hat{j} + 6\hat{k}$

from (2) $\vec{b}_2 = 4\hat{i} + 9\hat{k}$

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \left|\frac{8 + 54}{\sqrt{4+16+36}\,\sqrt{16+81}}\right|$$

$$\cos\theta = \frac{62}{\sqrt{56}\sqrt{97}} = \frac{31}{\sqrt{1358}}$$

$$\theta = \cos^{-1}\frac{31}{\sqrt{1358}}$$

	OR	**1.5** **1** **1** **2**

	Sol. Given point A (1,1,1) and given equations of lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ — (1) & $\frac{x+2}{1}=\frac{3-y}{-2}=\frac{z+1}{4} \Rightarrow \frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ — (2) from (1) $\vec{b_1}=2\hat{i}+3\hat{j}+4\hat{k}$ $\vec{b_2}=\hat{i}+2\hat{j}+4\hat{k}$ vector $\perp^r$ to both lines is given by $\vec{b}=\vec{b_1}\times\vec{b_2}=4\hat{i}-4\hat{j}+\hat{k}$ So the required eqn of line $\perp^r$ to both lines is $\vec{r}=\vec{a}+\lambda\vec{b}$ $\boxed{\vec{r}=\hat{i}+\hat{j}+\hat{k}+\lambda(4\hat{i}-4\hat{j}+\hat{k})}$ or $\boxed{\frac{x-1}{4}=\frac{y-1}{-4}=\frac{z-1}{1}}$	1

SECTION-E

36	MARKING SCHEME (a) Neither one one Nor onto (b)Range = codomain Y=[-3,∞) OR Both one-one Onto	 **1** **1** **1** **1** **1** **1**
37	(a) $\frac{h}{l} = \cos\alpha \Rightarrow h = 110\cos\alpha$ $\frac{r}{l} = \sin\alpha \Rightarrow r = 110\sin\alpha$ (b) $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi \cdot (110\sin\alpha)^2 \cdot 110\cos\alpha$ $V = \frac{(110)^3}{3}\pi \sin^2\alpha \cdot \cos\alpha$ (c) Let $f(\alpha) = \sin^2\alpha \cdot \cos\alpha$ $f'(\alpha) = -\sin^3\alpha + 2\sin\alpha \cdot \cos^2\alpha$ $f'(\alpha) = 0 \Rightarrow -\sin^3\alpha + 2\sin\alpha \cdot \cos^2\alpha = 0$ $\Rightarrow -\sin\alpha(\sin^2\alpha - 2\cos^2\alpha) = 0$ $\Rightarrow \sin\alpha = 0$ or $\sin^2\alpha = 2\cos^2\alpha$ $\alpha = 0$ not possible; $\tan^2\alpha = 2$ $\tan\alpha = \sqrt{2}$ $\alpha = \tan^{-1}\sqrt{2}$ at α, $f''(\alpha) < 0$ so volume is max.	**1** **1** **1** **1**

38	(a) Let A be the event that doctor will be late $P(A) = 0.3 \times 0.25 + 0.2 \times 0.3 + 0.1 \times 0.35 + 0.4 \times 0.1$ $P(A) = 0.075 + 0.060 + 0.035 + 0.040$ $P(A) = 0.21$ (b) Req. Prob. $= \dfrac{0.2 \times 0.3}{0.21} = \dfrac{0.06}{0.21}$ $= \dfrac{6}{21} = \dfrac{2}{7}$ (c) Req. Prob $= \dfrac{0.4 \times 0.1}{0.21} = \dfrac{0.04}{0.21} = \dfrac{4}{21}$	2 2 2

***Alternate solution with correct answers will also be accepted.**