Rich Math Problems...

The overarching goal of this is to allow teachers or students to solve it their own way. And then have them pick another method to solve and compare their results from both strategies.

We are looking for problems that have multiple aspects to them mathematically, could be solved from Algebra 1 - Calculus and incorporate science. Or we want a great science problem that has mathematical aspects including algebraic, geometric, trigonometric, and calculus.

Todd Morstein wrote, “I want to have a couple of these types of problems that have lots of avenues to get to the same place.”
As Jo Boaler said, we want problems that have a low floor, but a high ceiling. Students will learn better if there are various ways to solve the problem. Maths represent nature. See youcubed.stanford.edu for more information.

Mathematical Practices involved: 1. Make sense of problems and persevere in solving them, 2. Reason abstractly and quantitatively, 3. Construct viable arguments and critique reasoning of others, 4. Model with mathematics, 5. Use appropriate tools (technology) strategically, 6. Attend to precision, 7. Look for and make use of structure, 8. Look for and express regularity in repeated reasoning. (see http://www.corestandards.org/Math/Practice/)

NGSS

MS-ETS1 Engineering Design -As part of this work, teachers should give students opportunities to solve quantitative problems and use basic statistics:
HS.PS2 Motion and Instability: Forces and Interactions - As part of this work, teachers should give students opportunities to model with mathematics, use basic algebra, reason quantitatively and use units to solve problems, and apply key takeaways from grades 6–8 mathematics:
HS.PS3 Energy - As part of this work, teachers should give students opportunities to reason quantitatively and use units to solve problems, and apply key takeaways from grades 6–8 mathematics:

Strategies include plotting points on graph paper, using technology to graph data, model the data algebraically. If some students finish before others, have them design another method. Have them present and compare their findings. See inquiry challenges and inquiry lab instructions at the end of this document. We should make these blog entries or video blogs for each of these to help give teachers confidence and ideas.

Keys to all of experiments and models is that students understand variables, dependent, independent, control. These actually are better places to learn domain and range then a math class as well because there is a physical barrier that allows a certain domain and range.

Motion (CBR2)

Terminal Velocity - Coffee filter terminal velocity is a great place to start because of the low entry point. Energy is a thread that is important through all of science. This is a great place to talk about potential and kinetic energy and how something at terminal velocity doesn't meet its potential.

Ball bounce - How much energy is lost with each bounce? Model the data to predict the height of the next bounce.

There are lots of ways to mathematically approach this problem that can help us do this from looking at PE and KE, doing sequences, exponential decay, tangents, maximums at peaks. (In addition to the motion detector CBR2, this can be explored with video analysis.)

Ball rolling up and down an incline: What is the acceleration?

- If using a motion detector you could use the quadratic equation $d(t) = do + vo*t+½ a t^2$ and get the initial velocity, final and initial position for a certain time from the graph or $y= a x^2 + b x + c$
where d is the ...
- Photogates and measure the time between gates at different points. Can we use 3 or 4 in a row? One at the start and end should be good enough, but you can daisy chain photogates together and get more than just 2.
- Slope of velocity-time graph, derivative, slope of the tangent
- Trig: How is the acceleration related to the acceleration of gravity? What fraction of $9.8 m/s^2$ is it? How is this related to the angle?

What is the speed at the peak? What is the acceleration at the peak?

Swinging pendulum: (See my T^3 IC 2014 presentation) Determine the period of a pendulum? What does the period depend upon?

- Stopwatch
- photogates
- CBR
- accelerometer - Note this gives ½ the period. Can you understand why?

What other observations can you make? Justify your conclusions. When is the pendulum moving the fastest? When is it accelerating the most?

Damped harmonic motion - slinky and plate, again I believe this is a great place to talk about energy loss.

Force (Hand Dynamometer)

Grip strength: With a hand dynamometer, squeeze as hard as you can for a certain amount of time. Muscle fatigue. What is your ***average force*** for that time?

- Calculus students could use the integral of the data divided by the time.
- Algebra students could record the force every so many seconds.
- Geometry: (Perhaps the graph can be modeled with a trapezoid.) Estimate the area under the curve. What are the units of the 'area' of this graph. Divide this area by the time.
- Use statistics tools

What rate are you losing strength? Compare your right and left hand? Compare with other students? Write about your maximum and minimum force. How do these compare with the average?

A great key feature is to have people figure out that constants (slopes, rates of change) are conversion factors (acceleration, velocity, coefficient of restitution, coefficient of friction, ideal gas constant, permeability constant as in the slinky electromagnetic lab, etc.)

Friction: What does the friction of a moving object depend upon? Record the weight of a box and the force need to pull it at a constant speed. Put more weight in the box and pull it again. Repeat. Graph the data. Try this again with a different surface. Compare the results. Consider using a motion detector to see how close to a constant velocity you were moving.

Force & Motion: (Dual range force sensor and CBR)

Spring - If you F=ma, exploring this will help you see the relationship between the position and acceleration.

Spring & Hooke's law: F vs. x graph

What does the period of a spring depend upon? Explore, graph, and discover/verify the relationship.

Pull a box - Try to maintain a constant force? Graph force vs. acceleration. What is the meaning of the slope? (mass)

Exploration of Circuits - what models exists in parallel and series?

Linear

Slinky electromagnetic lab - Magnetic field sensor and Current probe. Change the current through the slinky and measure the magnetic field strength in the middle of the slinky. (Pictures on my blog.) It is so exciting getting fundamental constant of nature in the analysis. Percent error? (Attend to precision.)

Temperature
Warning. Newton's law of cooling cannot be obtained by a simple exponential regression.

Resistor and Capacitor Circuit
Exponential growth and decay. See calculus activity Charged Up.tns

Light Intensity and Distance
Right triangle
Trig
Area

Let's challenge our students with inquiry.

Inquiry lab instructions

a) Describe your experimental design with a labeled diagram. Include how you vary and measure the variables. For example, how did you vary the pressure and temperature.
b) What assumptions are you making in your design?
c) Mention what the independent and dependent variables are.
d) What are sources of experimental uncertainty and how can you minimize them?
e) Record your observations in an appropriate format. Make a table and a graph if necessary.
f) Analyze and find a function for your data. Can you identify a curve that best fits the data? Is there a way to plot 'creative' variable on the access, so the graph is linear and the slope is meaningful.
g) What pattern(s) did you find from your observations? Write a verbal description and a mathematical relationship.
h) How might you design your lab differently to mitigate errors or uncertainty in data collection or explore another question?

Challenge: Given a tennis ball and various ways to measure height,
predict the necessary drop height that will cause the ball to rebound back up exactly to a set level. Demonstrate your prediction using an equation derived from a graph of your data.

Challenge: Given two battery operated cars (or robots, or things that move at approximately a constant velocity), meter sticks, stopwatches,
predict the distance at which two cars will collide when they start at opposite ends separated by a distance of 1.5 meters. One car leaves 3 seconds after the first car is released.
(Vary the parameters based on your vehicles)

Make a poster with motion diagrams, derivation, graphs of position and velocity versus time graphs.

Challenge: GIVEN: Pullback Cart
TASK: A company makes the following claim about their pullback cars:
"Our cars demonstrate nearly constant acceleration".
Is their claim valid?
POSTER: Present your case using items from each of the Science Practices 4,5,6.1-2 (see page 12 of this link.)

[Page 6 of that document](#) shows:

> Scientific inquiry experiences in AP courses should be designed and implemented with increasing student involvement to help enhance inquiry learning and the development of critical-thinking and problem solving skills and abilities. Adaptations of Herron's approach (1971) [Herron, M.D. (1971). The nature of scientific enquiry. School Review, 79(2), 171- 212] and that of Rezba, Auldridge, and Rhea (1999) [Rezba, R.J., T. Auldridge, and L. Rhea. 1999. Teaching & learning the basic science skills.]define inquiry instruction for investigations in four incremental ways: