

Chapter 10 Project: Pollution in a Chain of Lakes

Name _____ Name _____

Background:

There are a wide variety of water pollutants, including organic compounds (sewage, for example), inorganic compounds such as radioactive waste, biologic pollutants that include disease-causing and non-native organisms, hot-water discharges (thermal pollution), and suspended sediment (silt and clay). Water pollutants can have many different effects. Pollutants can be toxic, carcinogenic, or disease causing. Some pollutants alter the physical nature of the habitat or ecosystem and create less-than-ideal living conditions.

The physics and chemistry of water pollution is not simple. A pollutant dumped into a lake or stream will mix into the water if the water is well stirred. Some pollutants can travel downstream for great distances without much mixing and diluting. Water-soluble pollutants may remain dissolved in the water column or may drop out, depending upon the local water chemistry. Suspended particles may stay in suspension if turbulence is high enough or if the particle is small enough, but will eventually settle out in quiet waters such as lakes and reservoirs.

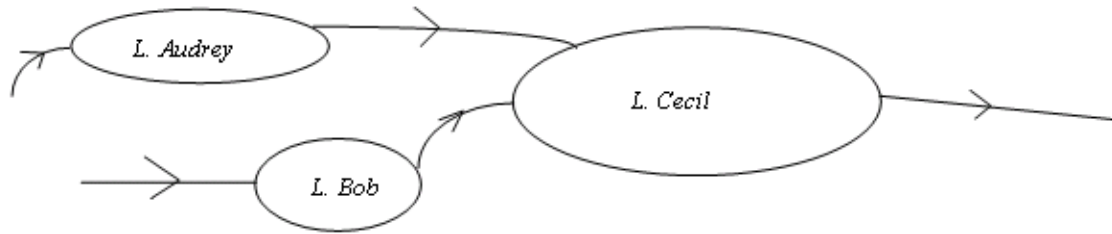
Water flows downhill in narrow channels called **rivers** or streams. The size of the river is usually measured by the amount of water flowing through the river in a certain amount of time. The volume of water per time is **discharge**, usually given in cubic feet or cubic meters per second. A small river might have a discharge of 100 cubic feet per second, equivalent to about 3 cubic meters per second. The bigger the channel (wider and deeper) and the faster the water flows, the higher the discharge. Lakes can be thought of as wide points in the river. That makes sense because water in lakes is moving, albeit very slowly.

This project examines several hypothetical scenarios involving water pollution in a chain of lakes. In the project we neglect the contribution from groundwater and overland flow, and consider a very simple system where water that flows into the lakes comes directly from the rivers, whose sources are not specified. We also neglect loss of water from the system due to evaporation under the sun, or due to recharge of gravels beneath the rivers and lakes.

Before you work through these problems, you might want to review the example in Chapter 10 in which we modeled the water pollution dynamics in lakes Erie and Ontario.

Problem #1

Lakes Audrey and Bob account for nearly 100% of the inflow of Lake Cecil. See diagram below.



Basic Problem: Scientists estimate that there are currently 5,000 kilograms, 12,000 kilograms and 20,000 kilograms of pollutants in Lakes Audrey, Bob and Cecil. They want to know how the pollution levels will change if no new pollution is dumped into the lakes. In this case, the new water inflow for Lakes Audrey and Bob will be clean. For Lake Cecil, inflow water comes from Lakes Audrey and Bob, which is currently polluted. It is known that the percentage of water replaced each year in Audrey, Bob, and Cecil is 25%, 30%, and 16%, respectively.

Modeling the lakes (assumptions made to make the problem simpler):

1. The annual flow of water for each lake can be modeled in a three stage process:
 - stage 1:** Polluted water flows out
 - stage 2:** New water flows in.
 - stage 3:** The new water from stage 2 mixes completely with the water remaining after stage 1.
2. All other water losses and gains equal zero and the volume of each lake is constant.
3. The pollutant in the model is thoroughly mixed in the water, and does not degrade, disappear or even settle out of the water column (does not leave the system).

a) Which pollutants might show these characteristics?

Mathematical Representation: Let $A(n)$, $B(n)$, and $C(n)$ represent the amount of pollution in Lakes Audrey, Bob, and Cecil, respectively, in year n .

The current pollution amounts are $A(0) = 5,000$, $B(0) = 12,000$, and $C(0) = 20,000$ kilograms.

b) Write the difference equation for each lake:

$$A(n) = \underline{\hspace{4cm}}$$

$$B(n) = \underline{\hspace{4cm}}$$

$$C(n) = \underline{\hspace{4cm}}$$

c) Determine the pollution amounts for years $n=1$ through $n=5$ and record below. Use 1 decimal place of precision.

n	$A(n)$	$B(n)$	$C(n)$
0	5,000	12,000	20,000
1			
2			
3			
4			
5			

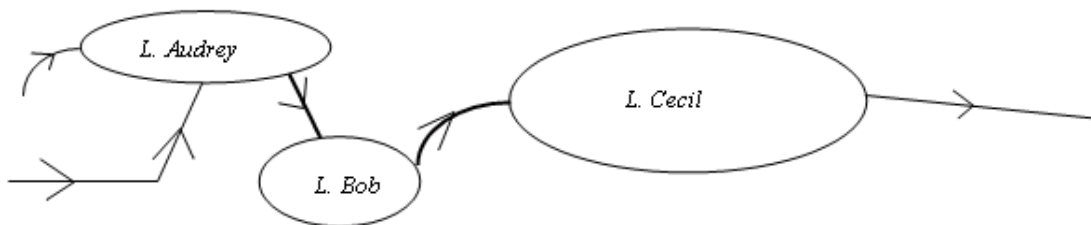
d) Write the solution equation for Lake Bob. Then determine the number of years until Lake Bob has 10 kg of pollution remaining. Use algebra and logs to solve this problem.

e) Return to page 1 and review the “modeling the lakes” paragraph. List what you think are the three greatest shortcomings of the modeling process used in this problem (3 things oversimplified or left out). Focus on environmental factors.

f) You are head of the TLCP (Three Lakes Cleanup Project). After the first year angry media and environmentalists attack your program because the pollution level in Lake Cecil has *increased*. What is your public response? How can you explain in lay person's terms what is going on, defend your program, and your career?

Problem #2

In an attempt to reduce the amount of pollution flowing into Lake Cecil, the Army Corps of Engineers comes up with an idea. They bring in the heavy machinery and reroute the inflow of Lake Bob into Lake Audrey. The total outflow from Lake Audrey is then routed through a canal to Lake Bob. Lake Bob empties into Lake Cecil as before. To increase the capacity of the Bob-Cecil stream, the Corps builds levees.



After the Corps does its work, the lake and river system looks like the diagram above. Because of the new configuration, 62.5% of Lake Audrey and 50% of Lake Bob are replaced every year. Lake Cecil maintains its 16% flushing rate.

Let's restart the clock and assume that the initial pollution levels are at 5,000 kilograms, 12,000 kilograms and 20,000 kilograms as before. Assume all other modeling assumptions stay the same as in problem #1.

a) Write the difference equation for each lake.

$$A(n) = \underline{\hspace{2cm}}$$

$$B(n) = \underline{\hspace{2cm}}$$

$$C(n) = \underline{\hspace{2cm}}$$

b) Determine the pollution amounts for years $n=1$ through $n=5$ and record below. Use 1 decimal place of precision.

n	$A(n)$	$B(n)$	$C(n)$
0	5,000	12,000	20,000
1			
2			
3			
4			
5			

c) How do these numbers compare with those from the previous problem, before bulldozing? How did rerouting the lakes affect the pollution levels and their change with time?

d) If the volume of Lake Cecil is $V_C = 31.25 \times 10^6$ liters, then the volume of Lake Bob can be found as well. Because 16% of Lake Cecil is flushed each year, we know that $(0.16)(31.25 \times 10^6 \text{ liters}) = 5.0 \times 10^6$ liters flow into and out of Lake Cecil. But the flow into Lake Cecil is the same as the flow out of Lake Bob, so 5.0×10^6 liters flows out of Lake Bob each year.

The flow out of Lake Bob is equal to the 50% of its flushed volume, so

$$0.50 V_B = 5.0 \times 10^6 \text{ liters}$$

Dividing each side of the equation by 0.50 results in the volume of Lake Bob:

$$V_B = 10.0 \times 10^6 \text{ liters}$$

Use a similar thought process to determine the volume of Lake Audrey, V_A . Show work below.

e) Make a table of the concentration of pollution for each lake in years $n = 0$ through $n = 5$. Express the concentration in ppm (parts per million) by weight using 1 decimal place of precision. *Note: 1 liter of water weighs 1 kg. Use 1 decimal place of precision.*

n	$Conc_A$	$Conc_B$	$Conc_C$
0			
1			
2			
3			
4			

5			
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Problem #3

Let's assume that everything stays the same as in problem #2, but with one change: Each year 200 kg of pollution is dumped directly into Lake Audrey.

- a) Write the difference equation for each lake describing the pollution levels from one year to the next.

$$A(n) = \underline{\hspace{2cm}}$$

$$B(n) = \underline{\hspace{2cm}}$$

$$C(n) = \underline{\hspace{2cm}}$$

- b) Determine the pollution amounts for years $n=1$ through $n=5$ and record below. Use 1 decimal place of precision.

n	$A(n)$	$B(n)$	$C(n)$
0	5,000	12,000	20,000
1			
2			
3			
4			
5			

- c) Write the solution equation for Lake Audrey. Then predict the number of years until Lake Audrey has 10 kg of pollution remaining. Use algebra and logs to solve this problem.

d) In Problems #2 and #3 the difference equations and initial conditions for Lakes Bob and Cecil are identical. At the same time the pollution amounts over the years $n = 1$ to $n = 5$ differ. Explain this contradiction.

e) Analyzing the long term pollution levels in the three lakes.

(i) What is the equilibrium value for Lake Audrey? Is it a stable or unstable equilibrium? Explain what this tells you about the long term pollution levels for that lake.

(ii) Once the pollution equilibrium level has been reached in Lake Audrey, a constant amount of pollution will flow into Lake Bob. Calculate this amount.

(iii) Rewrite the difference equation for Lake Bob to reflect the constant pollution inflow from Lake Audrey.

(iv) Based on your answer to part (iii), determine the pollution equilibrium level for Lake Bob.

(v) Repeat parts (ii), (iii), and (iv) to determine the pollution equilibrium level for Lake Cecil.

(vi) **Conclusion** In the long term, the pollution levels will reach what equilibrium values?

Lake Audrey _____ Lake Bob _____ Lake Cecil _____

Problem #4

As you've learned, streams and lakes change over time. Suppose that many years from now Lakes Audrey, Bob, and Cecil have the same geographical layout as in problems #2 and #3, but their environments have changed. In particular, it is determined that

- No more pollution is dumped into any of the lakes.
- Lake Audrey's inflow is unpolluted.
- 80% of Lake Audrey is replaced each year; 65% ends up in Lake Bob and 15% ends up in Lake Cecil.
- 100% of Lake Bob flows into Lake Cecil each year. This amount, along with 15% of Lake Audrey, constitute the inflow of Lake Cecil.
- 15% of Lake Cecil flows out each year.
- All other gains and losses for each lake are zero.
- $A(0) = 5,000$, $B(0) = 12,000$, and $C(0) = 20,000$ kilograms of pollution.

a) Write the difference equation for each lake.

$$A(n) = \underline{\hspace{4cm}}$$

$$B(n) = \underline{\hspace{4cm}}$$

$$C(n) = \underline{\hspace{4cm}}$$

b) Determine the pollution amounts for years $n=1$ through $n=5$ and record below.

n	$A(n)$	$B(n)$	$C(n)$
0	5,000	12,000	20,000
1			
2			
3			
4			
5			

c) Suppose that the volume of Lake Cecil is 20×10^6 liters. Determine the volumes of Lakes Bob and Audrey based upon inflow and outflow percentages. *For simplicity, let V_A , V_B , and V_C represent the volumes of Lakes Audrey, Bob, and Cecil.*

d) As head of the TLCP you face a problem: Environmental scientists have determined that a pollution level in Lake Cecil greater than 28,000 kg will kill off most fish. Thus the predicted, temporary "spike" in pollution in year $n = 1$ is unacceptable.

Note that the pollution in the lakes is in suspension; thus when water flows out of one lake, pollution flows with it. If you could remove pollution from Lake Audrey and Lake Bob, less pollution would flow into Lake Cecil.

Your staff offers a solution: treat lakes Audrey and Bob with a chemical that will cause some of their pollution to settle to the bottom. Technically speaking, the pollution will **precipitate** or **flocculate** out. This chemical precipitator can cause undesirable effects in lake ecosystems, and should be used at the *minimum level necessary*.

Without getting into the details of the chemical precipitator, determine the best combination of pollution removal from the two lakes so that fish will survive in Lake Cecil, and minimum environmental harm is done. Justify your solution.