

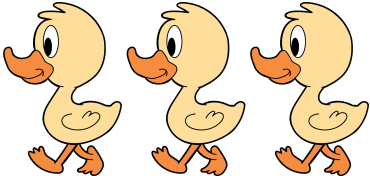
Guide to Instructional and Language Routines adapted from Illustrative Math

Math lessons include language and thinking routines to promote sense-making, and discourse as students engage with learning experiences. Some routines invite students to engage in a brief activity called warm-ups or launch and others to explore new concepts and ideas. Each routine is designed to get students thinking and talking about math in ways that make sense to them. It is important to consider that all students must have opportunities to engage in these routines. Teachers must leverage home languages within instructional routines and foster safe environments where diversity of thought is appreciated.

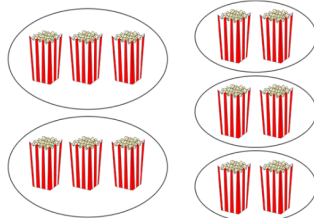
There are 10 different language and thinking routines that your teachers might use throughout the year. This guide can help you know what to expect and how to participate.

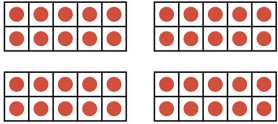
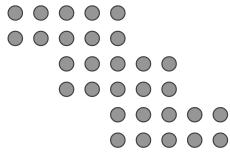
The routines in underlined in blue can work for all grades **K-12**, click to see the routine.

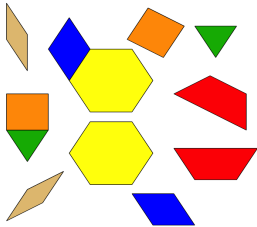
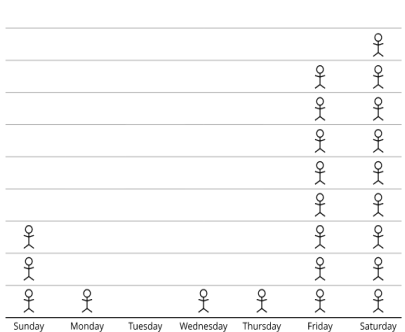
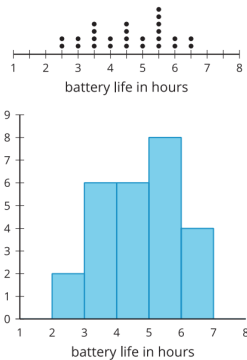
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Act it Out	
Purpose	This routine can be modified and adapted for any grades K-5. It encourages young children to understand the relationship between words and numbers. It provides opportunities for students to make sense of story problems.
Summary	Students listen to a story problem and act it out through movement, using their fingers, or objects to represent the action in the story.
Example	Kindergarten
	 Three (3) little ducks went out one day, Over the hill and far away. Mother duck said, “Quack, quack, quack.” Then the three (3) little ducks came back.
Questions to ask	• What is the story about? • Can you act it out? • Can you use your fingers to show what happened in the story? • Can you use cubes to show what happened in the story?

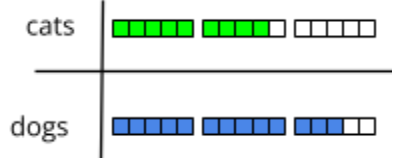
Choral Count		
Purpose	This routine encourages students to make predictions and think about patterns. It also provides opportunities for students to justify their reasoning.	
Summary	Students count aloud starting from a given number. The count might be forwards or backwards. The teacher records the numbers on a chart as students say them. Students then stop and look at the written numbers to make predictions and look for patterns.	
Example	Grade 1	Grade 3
	Count by ones starting at 50. 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79	Count by $\frac{1}{4}$, starting at $\frac{1}{4}$. $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$ $\frac{9}{4}$ $\frac{10}{4}$ $\frac{11}{4}$ $\frac{12}{4}$ $\frac{13}{4}$ $\frac{14}{4}$
Questions to ask	• What do you think will come next? How do you know? • What do you notice? What do you wonder? • What patterns do you see? • What number will be written next in the column/row?	

The Same but Different				
Purpose	This routine encourages students to use their own experiences and what they already know to make good estimates and justify their reasoning.			
Summary	Students reason about differences and similarities between different images that show the same mathematical idea. This protocol allows students to work with partners or individually to come up with a related same but different image allowing students to move towards the generalization of the larger mathematical concept.			
Examples	Early Numeracy	Grade 1-2	Grade 3	Algebra 1
			 Multiplication/Division	$(x+2)(x+3)$ $x^2 + 5x + 6$
Questions to ask	Step 1: Look carefully at the images - How are these two images the same but different? Step 2: Think silently for 1-2 minutes. Step 3: Turn and talk to a partner or to your group. Students can do this orally, or they can record their ideas on a mini whiteboard. Step 4: Share ideas as a whole class – Have students share their ideas of how the two images are the same but different. Consider writing the ideas on a t-chart with one side for same and one side for different. This could also be done using a Venn diagram. It is important that this is done publicly, gathering all ideas into a mathematical forum. Step 5: Summarize the student’s ideas clarifying that we now know that both of the images are the same but different. Restate the students’ ideas and the concepts discussed, calling attention to the important mathematics from the discussion. Step 6: Create one of your own - Ask students to create another same but different image that shows the same mathematical idea. Allow students to work with partners or individually to come up with a related same but different image. In this aspect of creation, students are moving towards the generalization of the larger mathematical concept.			

How Many Do You See?		
Purpose	This routine encourages students to see groups when counting. Being able to see groups of objects in an organized way helps them visualize quantities and improves their ability to do mental computation.	
Summary	Students look at an image, which is typically an arrangement of dots or other shapes. Then students state how many dots or shapes they see. Also included in the discussion will be comments about the way they saw them or determined how many there were. This encourages students to see groups and patterns rather than count each item one by one.	
Examples	Grade 1	Grade 3
		
Questions to ask	- How many do you see and how do you see them? - Could you count them in a different way? - What was helpful in finding the total number of dots?" - How many dots would there be if there was another group of them?	

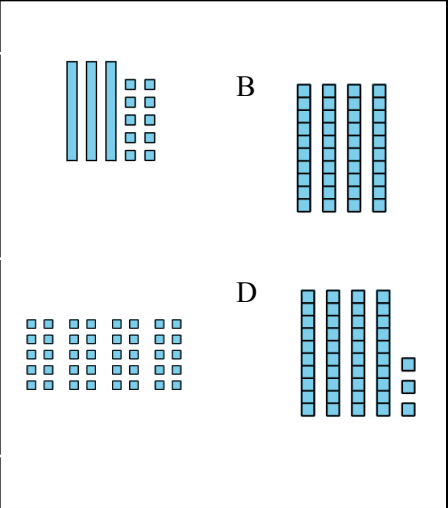
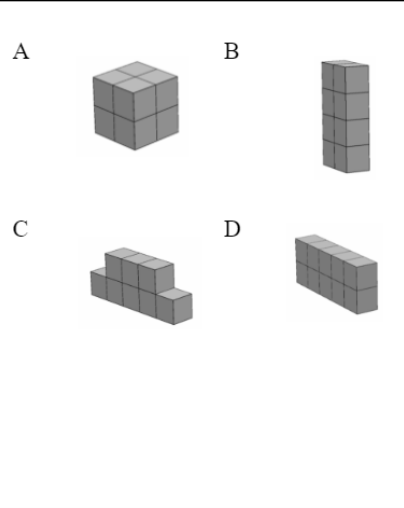
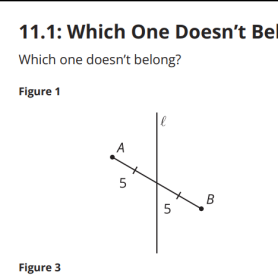
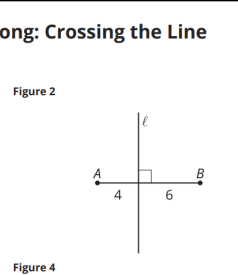
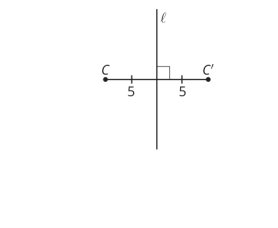
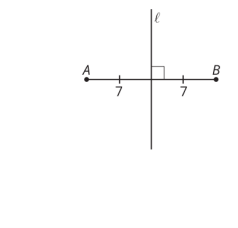
Notice and Wonder			
Purpose	This routine provides an opportunity for students to bring their understandings and experiences to a problem. They share their ideas and ask questions without any pressure to answer or solve a problem. This routine reinforces the importance of making sense of situations before solving a problem.		
Summary	Students look at an image related to the topic of the lesson and are asked, “What do you notice?” The teacher writes all comments on a chart. They are then asked, “What do you wonder?”, and their questions are also recorded on the chart.		
Examples	Kindergarten	Grade 3	Algebra 1
			2.1: Notice and Wonder: Battery Life The dot plot, histogram, and box plot summarize the hours of battery life for 26 cell phones constantly streaming video. What do you notice? What do you wonder? 
Questions to ask	- What do you notice? - What do you wonder? - What questions could you answer using the image? - What other questions do you have?		

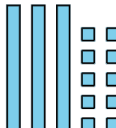
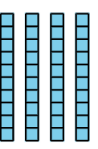
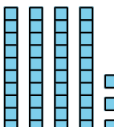
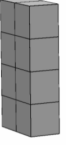
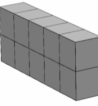
Number Talk			
Purpose	This routine provides an opportunity for students to practice mental math. It helps them solve problems and think about numbers in flexible ways. They not only justify their own reasoning but critique the reasoning of others as they make sense of methods for solving problems.		
Summary	A series of problems are presented one at a time. Students solve the problem in their heads and signal when they have an answer. The teacher takes notes as they justify their answer and explain their method for solving.		
Examples	Grade 1	Grade 5	Grade 7
	$27 + 3$ $27 + 5$ $25 + 7$ $35 + 7$	50×6 50×60 50×600 60×500	9.1: Number Talk: What Percentage? Determine the percentage mentally. 10 is what percentage of 50? 5 is what percentage of 50? 1 is what percentage of 50? 17 is what percentage of 50?
Questions to ask	- How did you solve the problem? - Why did you choose that strategy? - Did one problem help you solve another problem? How? - What other problem would you add to this string? Why?		

Questions About Us		
Purpose	This routine can be used with K-5 students. It provides them opportunities to learn more about their classmates and gives them practice asking questions, organizing quantities, counting, and analyzing data.	
Summary	Students ask their classmates a question with two choices. They keep track of the answers and count the responses. The teacher then asks follow-up questions that students answer using the data that they collected.	
Examples	Would you rather play on the slide or on the swings? 	Which do you like better: cats or dogs? 
Questions to ask	- How can we figure out how many students would rather play on the slide? - How many students would rather play on the slide? - How can we figure out how many students would rather play on the swings?	- How many people like cats better? - How many people like dogs better? - If 1 more person likes cats, how many? - If 1 less person likes dogs, how many? - How many students would rather play on the swings?

True or False?			
Purpose	This routine encourages students to make sense of equations, often without any computation. It provides another opportunity for students to justify their reasoning as they explain to others what they are thinking.		
Summary	Students are presented with a series of equations, one at a time. Some equations may be true, and some may be false. Students use what they know about place value, operations, and number relationships to decide if each is true or false. And then, students explain how they know.		
Examples	Grade 1	Grade 4	Grade 8
	Is each statement true or false? $5 = 5$ $4 + 1 = 5$ $6 = 4 + 1$ $1 + 4 = 4 + 1$	Is each statement true or false? $2 \times 7 = 3 \times 7$ $3 \times 8 = 3 \times 6$ $2 \times 8 = 4 \times 4$ $2 \times 12 = 3 \times 6$	5.1: True or False: Squared Decide if each statement is true or false. $(\sqrt{5})^2 = 5$ $(\sqrt{9})^2 = 3$ $7 = (\sqrt{7})^2$ $(\sqrt{10})^2 = 100$ $(\sqrt{16}) = 2^2$
Questions to ask	- How did you decide if the equation was true or false? - If false, how could you make it true? - Can you prove it is true or false without solving both sides of the equation?		

What Do You Know About ____?		
Purpose	This routine encourages students to share their experiences and understandings about a math topic.	
Summary	Students are presented with a number, expression, or are asked a general question about a math topic. They then list everything they know about that topic. The teacher writes what students say and then references the list later so that students can add more ideas.	
Examples	Grade 2	Grade 5
	What do you know about the number 49?	Write everything you know about 6.497
Questions to ask	- Can you draw a picture or diagram to show what you know about _____. - What experience do you have with _____? - Where might you see _____? - How would you use _____? - When responding to a number: - How could you represent that number using a number line or place value blocks? - What are some numbers that are larger/smaller than that number? - What equations can you write using that number?	

Which One Doesn't Belong?			
Purpose	This routine provides an opportunity for students to reason about characteristics of shapes, math tools, or other images to decide which one doesn't belong. Because any answer is correct, students are able to focus on communicating their reasoning and justifying their choice.		
Summary	Students are shown 4 different images, which may be numbers, equations, shapes, images, or diagrams. They decide which one doesn't belong and explain why.		
Examples	Grade 2	Grade 5	Geometry R
			11.1: Which One Doesn't Belong: Crossing the Line Which one doesn't belong?    
Questions to ask	- Which one doesn't belong? Why? - What is the same and different about the others? - Can you think of a reason why another one doesn't belong?		

3 Reads		
Purpose	The 3 Reads instructional routine is designed to develop students' ability to make sense of problems by deconstructing the process of reading mathematical situations. Over time, students will internalize this process, thereby creating a heuristic for reading and making sense of mathematical story problems, math practice 1.	
Summary	This strategy guides students to negotiate language. A problem is read three times and each time it is read with a different purpose. Read 1: The focus is on comprehending the text. Read 2: The focus is on discerning the question. Read 3: Analyze the important information to solve the question.	
Examples	Grade 2	Grade 5
	A  B  C  D 	A  B  C  D 
Questions to ask	Read 1: What is the situation about Read 2: What are we trying to find? Read 3: What mathematical questions can we ask about the situation?	

- Notice and Wonder
- Number Talk
- Poll the Class
- Take Turns
- Think Pair Share
- True or False
- Which One Doesn't Belong?
- Algebra Talk
- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations

Illustrative Math Routines Grades 6-9

Math Language Routines

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Mathematical Language Routine 1: Stronger and Clearer Each Time

Adapted from Zwiers (2014)

To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014).

Prompt: This routine begins by providing a thought-provoking question or prompt. The prompt should guide students to think about a concept or big idea connected to the content goal of the lesson, and should be answerable in a format that is connected with the activity's primary disciplinary language function.

Response - First Draft: Students draft an initial response to the prompt by writing or drawing their initial thoughts in a first draft. Responses should attempt to align with the activity's primary language function. It is not necessary that students finish this draft before moving to the structured pair meetings step. However, students should be encouraged to write or draw something before meeting with a partner. This encouragement can come over time as class culture is developed, strategies and supports for getting started are shared, and students become more comfortable with the low stakes of this routine. (2–3 mi

Structured Pair Meetings: Next, use a structured pairing strategy to facilitate students having 2–3 meetings with different partners. Each meeting gives each partner an opportunity to be the speaker and an opportunity to be the listener. As the speaker, each student shares their ideas (without looking at their first draft, when possible). As a listener, each student should (a) ask questions for clarity and reasoning, (b) press for details and examples, and (c) give feedback that is relevant for the language goal. (1–2 min each meeting)

Response - Second Draft: Finally, after meeting with 2–3 different partners, students write a second draft. This draft should naturally reflect borrowed ideas from partners, as well as refinement of initial ideas through repeated communication with partners. This second draft will be stronger (with more or better evidence of mathematical content understanding) and clearer (more precision, organization, and features of disciplinary language function). After students are finished, their first and second drafts can be compared. (2–3 min)

Mathematical Language Routine 2: Collect and Display

To capture a variety of students' oral words and phrases into a stable, collective reference.

Collect: During this routine, circulate and listen to student talk during paired, group, or as a whole-class discussion. Jot down the words, phrases, drawings, or writing students use. Capture a variety of uses of language that can be connected to the lesson content goals, as well as the relevant disciplinary language function(s). Collection can happen digitally, or with a clipboard, or directly onto poster paper; capturing on a whiteboard is not recommended due to risk of erasure.

Display: Display the language collected visually for the whole class to use as a reference during further discussions throughout the lesson and unit. Encourage students to suggest revisions, updates, and connections be added to the display as they develop—over time—both new mathematical ideas and new ways of communicating ideas. The display provides an opportunity to showcase connections between student ideas and new vocabulary. It also provides opportunity to highlight examples of students using disciplinary language functions, beyond just vocabulary words.

Mathematical Language Routine 3: Clarify, Critique, Correct

To give students a piece of mathematical writing that is not their own to analyze, reflect on & develop.

Original Statement: Create or curate a written mathematical statement that intentionally includes conceptual (or common) errors in mathematical thinking as well as ambiguities in language. The mathematical errors should be driven by the content goals of the lesson and the language ambiguities should be driven by common or typical challenges with the relevant disciplinary language function. This mathematical text is read by the students and used as the draft, or “original statement,” that students improve. (1–2 min)

Discussion with Partner: Next, students discuss the original statement in pairs. The teacher provides guiding questions for this discussion such as, “What do you think the author means?,” “Is anything unclear?,” or “Are there any reasoning errors?” In addition to these general guiding questions, 1–2 questions can be added that specifically address the content goals and disciplinary language function relevant to the activity. (2–3 min)

Improved Statement: Students individually revise the original statement, drawing on the conversations with their partners, to create an “improved statement.” In addition to resolving any mathematical errors or misconceptions, clarifying ambiguous language, other requirements can be added as parameters for the improved response. These specific requirements should be aligned with the content goals and disciplinary language function of the activity. (3–5 min)

Mathematical Language Routine 4: Information Gap

Adapted from Zwiers 2004

To create a need for students to communicate (Gibbons, 2002).

Problem/Data Cards: Students are paired into Partner A and Partner B. Partner A is given a card with a problem that must be solved, and Partner B has the information needed to solve it on a “data card.” Data cards can also contain diagrams, tables, graphs, etc. Neither partner should read nor show their cards to their partners. Partner A determines what information they need, and prepares to ask Partner B for that specific information. Partner B should not share the information unless Partner A specifically asks for it and justifies the need for the information. Because partners don’t have the same information, Partner A must work to produce clear and specific requests, and Partner B must work to understand more about the problem through Partner A’s requests and justifications.

Bridging the Gap

- Partner B asks “What specific information do you need?” Partner A asks for specific information from Partner B.
- Before sharing the requested information, Partner B asks Partner A for a justification: “Why do you need that information?”
- Partner A explains how they plan to use the information.
- Partner B asks clarifying questions as needed, and then provides the information.
- These four steps are repeated until Partner A is satisfied that they have information they need to solve the problem.

Solving the Problem

- Partner A shares the problem card with Partner B. Partner B does not share the data card.
- Both students solve the problem independently, then discuss their strategies. Partner B can share the data card after discussing their independent strategies.

Mathematical Language Routine 5: Co-craft Questions

To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms and symbols can represent different situations.

Hook: Begin by presenting students with a hook—a context or a stem for a problem, with or without values included. The hook can also be a picture, video, or list of interesting facts.

Students Write Questions: Next, students write down possible mathematical questions that might be asked about the situation. These should be questions that they think are answerable by doing math and could be questions about the situation, information that might be missing, and even about assumptions that they think are important. (1–2 minutes)

Students Compare Questions: Students compare the questions they generated with a partner (1–2 minutes) before sharing questions with the whole class. Demonstrate (or ask students to demonstrate) identifying specific questions that are aligned to the content goals of the lesson as well as the disciplinary language function. If there are no clear examples, teachers can demonstrate adapting a question or ask students to adapt questions to align with specific content or function goals. (2–3 minutes)

Actual Question(s) Revealed/Identified: Finally, the actual questions students are expected to work on are revealed or selected from the list that students generated.

Mathematical Language Routine 6: Three Reads

To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to actively make sense of mathematical situations and information (Kelemanik, Lucenta, & Creighton, 2016).

In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. The intended question or main prompt is intentionally withheld until the third read so that students can concentrate on making sense of what is happening in the text before rushing to a solution or method.

Read #1: The first read focuses on the situation, context, or main idea of the text. After a shared reading, ask students “what is this situation about?” This is the time to identify and resolve any challenges with any non-mathematical vocabulary. (1 minute)

Read #2: After the second read, students list any quantities that can be counted or measured. Students are encouraged not to focus on specific values. Instead they focus on naming what is countable or measurable in the situation. It is not necessary to discuss the relevance of the quantities, just to be specific about them (examples: “number of people in her family” rather than “people,” “number of markers after” instead of “markers”). Some of the quantities will be explicit (example: 32 apples) while others are implicit (example: the time it takes to brush one tooth). Record the quantities as a reference to use when solving the problem after the third read. (3–5 minutes)

Read #3: During the third read, the final question or prompt is revealed. Students discuss possible solution strategies, referencing the relevant quantities recorded after the second read. It may be helpful for students to create diagrams to represent the relationships among quantities identified in the second read, or to represent the situation with a picture (Asturias, 2014). (1–2 minutes).

Mathematical Language Routine 7: Compare and Connect

To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches and representations.

Students Prepare Displays of their Work: Students are given a problem that can be approached and solved using multiple strategies, or a situation that can be modeled using multiple representations. Students are assigned the job of preparing a visual display of how they made sense of the problem and why their solution makes sense. Variation is encouraged and supported among the representations that different students use to show what makes sense.

Compare: Students investigate each others' work by taking a tour of the visual displays. Tours can be self-guided, a "travellers and tellers" format, or the teacher can act as "docent" by providing questions for students to ask of each other, pointing out important mathematical features, and facilitating comparisons. Comparisons focus on the typical structures, purposes, and affordances of the different approaches or representations: what worked well in this or that approach, or what is especially clear in this or that representation. During this discussion, listen for and amplify any comments about what might make this or that approach or representation more complete or easy to understand.

Connect: The discussion then turns to identifying correspondences between different representations. Students are prompted to find correspondences in how specific mathematical relationships, operations, quantities, or values appear in each approach or representation. Guide students to refer to each other's thinking by asking them to make connections between specific features of expressions, tables, graphs, diagrams, words, and other representations of the same mathematical situation. During the discussion, amplify language students use to communicate about mathematical features that are important for solving the problem or modeling the situation. Call attention to the similarities and differences between the ways those features appear.

Mathematical Language Routine 8: Discussion Supports

To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009).

Unlike the other routines, this support is a collection of strategies and moves that can be combined and used to support discussion during almost any activity.

Examples of possible strategies:

- Revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
- Press for details in students' explanations by requesting for students to challenge an idea, elaborate on an idea, or give an example.
- Practice phrases or words through choral response.
- Think aloud by talking through thinking about a mathematical concept while solving a related problem or doing a task.
- Give students time to make sure that everyone in the group can explain or justify each step or part of the problem. Then make sure to vary who is called on to represent the work of the group so students get accustomed to preparing each other to fill that role.

Sentence Frames

Sentence frames can support student language production by providing a structure to communicate about a topic. Helpful sentence frames are open-ended, so as to amplify language production, not constrain it. The table shows examples of generic sentence frames that can support common disciplinary language functions across a variety of content topics. Some of the lessons in these materials include suggestions of additional sentence frames that could support the specific content and language functions of that lesson.

language function	sample sentence frames	language function	sample sentence frames
describe	• It looks like... • I notice that... • I wonder if... • Let's try... • A quantity that varies is _____. • What do you notice? • What other details are important?	justify	• I know _____ because... • I predict _____ because... • If _____ then _____ because... • Why did you...? • How do you know...? • Can you give an example?
explain	• First, I _____ because... • Then/Next, I... • I noticed _____ so I... • I tried _____ and what happened was... • How did you get...? • What else could we do?	generalize	• _____ reminds me of _____ because... • _____ will always _____ because... • _____ will never _____ because... • Is it always true that...? • Is _____ a special case?
critique	• That could/couldn't be true because...` • This method works/doesn't work because... • We can agree that... • _____'s idea reminds me of... • Another strategy would be _____ because... • Is there another way to say/do...?	compare and contrast	• Both _____ and _____ are alike because... • _____ and _____ are different because... • One thing that is the same is... • One thing that is different is... • How are _____ and _____ different? • What do _____ and _____ have in common?
represent	• _____ represents _____. • _____ stands for _____. • _____ corresponds to _____. • Another way to show _____ is... • How else could we show this?	interpret	• We are trying to... • We will need to know... • We already know... • It looks like _____ represents... • Another way to look at it is... • What does this part of _____ mean? • Where does _____ show...?