

10/20/23

Second Order Equations - Constant Coefficients
Homogeneous Complex: Problem 5

$$y'' + 16y = 0$$

(a) $r^2 + 16 = 0$

$$\begin{array}{r} r^2 + 16 = 0 \\ \underline{-16 \quad -16} \\ r^2 = -16 \end{array}$$

$$\sqrt{r^2} = \sqrt{-16}$$

$$r = \pm 4i \quad \text{roots} = 4i, -4i$$

general solution
 $A \cos(4t) + B \sin(4t)$

(b) find the particular solution with $y(0) = 9$, $y'(0) = 32$

$$y = A \cos(4t) + B \sin(4t)$$

$$y' = -4A \sin(4t) + 4B \cos(4t)$$

$$\text{so, } 9 = A \cos(0) + B \sin(0)$$

$$9 = A + 0$$

$$A = 9$$

now find B

$$32 = -4A \sin(0) + 4B \cos(0)$$

$$32 = -0 + 4B$$

$$B = \frac{32}{4} = 8, B = 8$$

Particular solution:
 $9 \cos(4t) + 8 \sin(4t)$