## The North Carolina High School Collaborative Instructional Framework

# NC Math 1 **Unit 1: Equations & Introduction to Functions**

10 Days Block Schedule

September 2017 Update

20 Days Traditional Schedule

RESEARCH BRIEF: Unit 1: Equations & Introduction to Functions

### **Essential Questions:**

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables change?
- How can equations and inequalities be used to model real world situations?

## **Student Objectives Learning Outcomes** Given an equation students will solve and justify their method • I will **solve** an equation or inequality and **justify** my steps. NC.M1.A-REI.3. NC.M1.A-REI.1. NC.M1.A.REI.12 and steps of solving. Students will be able to interpret key features of expressions,

- equations, graphs, tables, and verbal descriptions in context. Create an equation or inequality and interpret reasonable solutions in context.
- Given a formula students will solve for a specified variable.
- Given a function students will determine domain and range.
- Given a function create an equation from various representations and use them to solve problems.
- Given a function in function notation students will evaluate and interpret results in context.
- Understand what it takes to be a function in categorical, numerical, and graphical scenarios.
- Students should be able to understand functions as a correspondence between inputs and outputs.

- I will be able to **interpret** key features and solutions. NC.M1.A-SSE.1a.
- NC.M1.F-IF.4
- I will be able to **create** an equation or inequality from a word problem. NC.M1.A-CED.1
- I will manipulate a formula to solve for a specific variable.
- I will be able to recognize domain and range values in a function. NC.M1.F-IF.1
- I will evaluate a function for a given value. NC.M1.F-IF.2
- I will understand what a function is. NC.M1.F-IF.1
- I will identify the relationship between input and output. NC.M1.F-IF1

## Standards Addressed in this Unit

# Construct expressions, equations, and inequalities from a given context and determine the appropriateness of the solution.

- NC.M1.A-SSE.1a: Interpret expressions that represent a quantity in terms of its context. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.
- NC.M1.A-REI.3: Solve linear equations and inequalities in one variable.
- NC.M1.A-REI.1: Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.
- NC.M1.A-REI.12: Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.
- NC.M1.A-CED.1: Create equations that describe numbers or relationships. Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.
- NC.M1.A-CED.4: Create equations that describe numbers or relationships. Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

## Distinguish key features of a function given multiple representations.

- NC.M1.F-IF.1: Understand the concept of a function and use function notation. Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that:
  - $\circ$  if f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x
  - $\circ$  the graph of f is the graph of the equation y = f(x).
- NC.M1.F-IF.2: Understand the concepts of a functions and use function notation. Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- NC.M1.F-IF.4: Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.
- NC.M1.F-IF.6: Interpret functions that arise in applications in terms of the context. Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

## **Implementing the Standards for Mathematical Practice**

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.

- 5. Use appropriate tools strategically.
- 6. Attend to precision.

- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

# **Aligned Resources for this Unit**

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## The Math Resource for Instruction - Customized for the Content of this Unit

#### NC.M1.A-SSE.1a

## Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

#### **Concepts and Skills**

## Pre-requisite

- Identify parts of an expression using precise vocabulary (6.EE.2b)
- Interpret numerical expressions written in scientific notation (8.EE.4)
- For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)

#### **Connections**

- Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3)
- Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9)
- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

#### The Standards for Mathematical Practices

#### Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

- 2 Reason abstractly and quantitatively.
- 4 Model with mathematics
- 7 Look for and make use of structure.

## **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Quadratic term, exponential term

## Mastering the Standard for this Unit

## **Comprehending the Standard**

This set of standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

For this part of the standards, students recognize that the linear expression mx + b has two terms, m is a coefficient, and b is a constant.

Students extend beyond simplifying an expression and address interpretation of the components in an algebraic expression.

## Assessing for Understanding

Students should recognize that in the expression 2x + 1, "2" is the coefficient, "2" and "x" are factors, and "1" is a constant, as well as "2x" and "1" being terms of the binomial expression. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

**Example**: The height (in feet) of a balloon filled with helium can be expressed by 5 + 6.3s where s is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression.

#### NC.M1.A-REI.3

Solve equations and inequalities in one variable.

Solve linear equations and inequalities in one variable.

## **Concepts and Skills**

## Pre-requisite

- Solving multi-step equations (8.EE.7)
- Solving two-step inequalities (7.EE.4)

#### Connections

- Create one variable linear equations and inequalities (NC.M1.A-CED.1)
- Justify a solution methods and the steps in the solving process (NC.M3.A-REI.1)
- Solve systems of linear equations (NC.M1.A-REI.6)

#### The Standards for Mathematical Practices

#### **Connections**

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

### **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context.

# Comprehending the Standard

Students are taught to solve multi-step equations in 8<sup>th</sup> grade. Students should become fluent solving multi-step equations in Math 1.

Students were taught to solve two-step inequalities in 7<sup>th</sup> grade. In Math 1 students extend this skill to multi-step inequalities.

This should be taught with the mathematical reasoning found in NC.M1.A-REI.1. Students should <u>not</u> be presented with a list steps to solve a linear equation/inequalities. Like many purely procedural practices, such steps are only effective for linear equations. It is more effective for students to be taught the mathematical reasoning for the solving process as these concepts can be applied to all types of equations.

# Mastering the Standard for this Unit Assessing for Understanding

Students should be able to solve multistep linear equations and inequalities.

**Example:** Solve:

a) 
$$\frac{7}{2}y - 8 = 111$$

b) 
$$3x - 2 > 9 + 5x$$

c) 
$$\frac{3+x}{7} = \frac{x-9}{4}$$

d) 
$$\frac{2}{3}x + 9 < 8(\frac{1}{3}x - 2)$$

e) 
$$\frac{1}{5}(10 - 20x) \le -14$$

#### NC.M1.A-REI.1

## Understand solving equations as a process of reasoning and explain the reasoning.

Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

#### **Concepts and Skills**

## **Pre-requisite**

- Students have been using properties of operations and equality throughout middle school. (6.EE.3, 7.EE.1, 7.EE.4). This is the first time that justification is required by a content standard.
- Solve multi-step equations (8.EE.7)

#### **Connections**

- Understand the relationship between factors of a quadratic equation and the solution of the equation (NC.M1.A-APR.3)
- Create and solve one variable linear and quadratic equations (NC.M1.A-CED.1)
- Solve for a quantity of interest in a formula (NC.M1.A-CED.4)
- Solve linear and quadratic equations and systems of linear equations (NC.M1.A-REI.3, NC.M1.A-REI.4, NC.M1.A-REI.5, NC.M1.A-REI.6)

#### The Standards for Mathematical Practices

#### **Connections**

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

#### **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to defend their method of solving an equation and each step of the solving process.

New Vocabulary: quadratic equation

## Comprehending the Standard

When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method

Properties of operations can be used to change expressions on either side of the equation to equivalent expressions.

In the properties of equality, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions.

Students do not have to name the property, but can describe property using mathematical reasoning.

## **Mastering the Standard for this Unit**

## **Assessing for Understanding**

Students should be able to justify a chosen solution method and justify each step in the process. This would be a good opportunity to discuss efficiency.

**Example:** To the right are two methods to solve the same equation. Justify each step in the solving process. Which method do you prefer? Why?

#### Method 1:

$$5(x+3) - 3x = 55$$

$$5x + 15 - 3x = 55$$

$$2x + 15 = 55$$

$$2x + 15 - 15 = 55 - 15$$

$$2x = 40$$

$$\frac{2x}{2} = \frac{40}{2}$$

$$x = 20$$

#### Method 2:

$$5 5 5 5$$

$$x + 3 - \frac{3}{5}x = 11$$

$$\frac{2}{5}x + 3 = 11$$

$$\frac{2}{5}x + 3 - 3 = 11 - 3$$

$$\frac{2}{5}x = 8$$

$$\frac{5}{2}(\frac{2}{5})x = \frac{5}{2}(8)$$

$$x = 20$$

5(x+3) - 3x = 55

For example: Transforming 2x - 5 = 7 to 2x = 12 is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well.

Students should be able to critique the solving process of others, recognize incorrect steps and provide corrective action to the process.

**Example:** The following is a student solution to the inequality 
$$\frac{5}{18} - \frac{x-2}{9} \le \frac{x-4}{6}$$
.

- a) There are two mathematical errors in this work. Identify at what step each mathematical error occurred and explain why it is mathematically incorrect.
- b) How would you help the student understand his mistakes?
- c) Solve the inequality correctly.

$$\begin{split} &\frac{5}{18} - \frac{x-2}{9} \leq \frac{x-4}{6} \\ &\frac{5}{18} - \binom{2}{2} \frac{x-2}{9} \leq \binom{3}{3} \frac{x-4}{6} \\ &\frac{5}{18} - \frac{2x-2}{18} \leq \frac{3x-4}{18} \\ &5 - (2x-2) \leq 3x-4 \\ &5 - 2x + 2 \leq 3x-4 \\ &7 - 2x \leq 3x-4 \\ &-5x \leq -11 \\ &x \leq \frac{11}{5} \end{split}$$

(https://www.illustrativemathematics.org/content-standards/HSA/REI/A/1/tasks/807)

Note: While this standard does not cover inequalities, this could be a good extension.

#### NC.M1.A-REI.12

## Represent and solve equations and inequalities graphically

Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

#### **Concepts and Skills**

## Pre-requisite

- Solve two-step linear inequalities (7.EE.4b)
- Solve linear inequalities in one variable (NC.M1.A-REI.3)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

#### Connections

- Create one variable linear inequalities and use the inequality to solve problems (NC.M1.A-CED.1)
- Create a system of linear inequalities to model a situation in context (NC.M1.A-CED.3)

#### The Standards for Mathematical Practices

#### Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

## **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain the reasoning behind their graphical representation of an inequality or system of inequalities.

#### Mastering the Standard for this Unit

## **Comprehending the Standard**

Students should understand that since there is no way to list every solution to a linear inequality in two variables, the solutions must be represented graphically.

It is an American tradition to shade the region that represent the solutions of the inequality. In other countries, they shade regions of the plane that do <u>not</u> contain solutions, marking that region out. This results in an unmarked solution region making it easier to identify and work with points in the solution region. This means that it is important for students to understand what the shaded region represents according to the context of the problem.

#### **Assessing for Understanding**

Students should be able to represent and interpret solutions to one variable inequalities on a number line.

**Example:** Elvira, the cafeteria manager, has to be careful with her spending and manages the cafeteria so that they can serve the best food at the lowest cost. To do this, Elvira keeps good records and analyzes all of her budgets. Elvira's cafeteria has those cute little cartons of milk that are typical of school lunch. The milk supplier charges \$0.35 per carton of milk, in addition to a delivery charge of \$75. What is the maximum number of milk cartons that Elvira can buy if she has budgeted \$500 for milk?

- a) Write and solve an inequality that models this situation, then graph the solution on a number line.
- b) Describe in words the quantities that would work in this situation.

(www.mathematicsvisionproject.org)

**Example:** Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 900 pounds of weight for safety reasons.

- a) Let p represent the total number of people. Write an inequality to describe the number of people that a boat can hold. Draw a number line diagram that shows all possible solutions.
- b) Let w represent the total weight of a group of people wishing to rent a boat. Write an inequality that describes all total weights allowed in a boat. Draw a number line diagram that shows all possible solutions.

(https://www.illustrativemathematics.org/content-standards/tasks/642)

#### NC.M1.A-CED.1

## Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

## **Concepts and Skills**

#### Pre-requisite

• Create two-step linear equations and inequalities from a context (7.EE.4)

#### **Connections**

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)
- Solve linear and quadratic equations and linear inequalities (NC.M1.A-REI.3, NC.M1.A-REI.4)
- Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)
- Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12)

## **The Standards for Mathematical Practices**

#### Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

#### **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function

## Mastering the Standard for this Unit

### **Comprehending the Standard**

Students create equations and inequalities in one-variable and use them to solve the problems.

In Math I, focus on linear, quadratic, and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can also be created from an associated function.

After the students have created an equation, they can use other representations to solve problems, such as graphs and tables
Students in Math I are not responsible for interval notation as a solution. They are to write answers to these inequalities using inequality notation.

## **Assessing for Understanding**

Students should be able to create an equation from a function and use the equation to solve problems.

**Example:** A government buys x fighter planes at z dollars each, and y tons of wheat at w dollars each. It spends a total of B dollars, where B = xz + yw. In (a)–(c), write an equation whose solution is the given quantity.

- a) The number of tons of wheat the government can afford to buy if it spends a total of \$100 million, wheat costs \$300 per ton, and it must buy 5 fighter planes at \$15 million each.
- b) The price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at \$500 a ton, for a total of \$50 million.
- c) The price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of \$90 million.

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/580)

Students should be able to create equations and inequalities from various representations, such as verbal descriptions, and use them to solve problems.

**Example:** Mary and Jeff both have jobs at a baseball park selling bags of peanuts. They get paid \$12 per game and \$1.75 for each bag of peanuts they sell. Create equations, that when solved, would answer the following questions:

- a) How many bags of peanuts does Jeff need to sell to earn \$54?
- b) How much will Mary earn if she sells 70 bags of peanuts at a game?
- c) How many bags of peanuts does Jeff need to sell to earn at least \$68?

#### NC.M1.A-CED.4

## Create equations that describe numbers or relationships.

Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

## **Concepts and Skills**

## Pre-requisite

- Solve linear equations in one variable (8.EE.7 and NC.M1.A-REI.3)
- Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$  where p is a positive rational number (8.EE.2)
- Justify a solution method and each step in the solving process (NC.M1.A-REI.1)

#### Connections

- Create an equation in two variables that represent a relationship between quantities (NC.M1.A-CED.2)
- Justify a solving method and each step in the solving process (NC. M1.A-REI.1)

## The Standards for Mathematical Practices

#### Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

- 4 Model with mathematics
- 7 Look for and make use of structure

### **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify the steps in their solving process.

## Mastering the Standard for this Unit

## Comprehending the Standard

Students should be able to solve an equation for a given variable. In Math 1, focus on real mathematical and scientific formulas. This may be a good opportunity to talk with the science teachers and ask them for formulas that they use often.

This standard also covers solve for variables in mathematical forms as well as formulas.

(Students are <u>not</u> expected to write linear equation into "proper" standard form.)

This standards should be taught in conjunction with NC.M1.A-REI.1 in which students have to justify each step of the solving process and justify a particular solving method.

## **Assessing for Understanding**

Students should be able to solve for variables in mathematical forms as well as formulas.

**Example:** Solve 
$$(y - y_1) = m(x - x_1)$$
 for  $m$ .

Students should be able to solve for variable in science and math formula.

**Example:** (NCDPI Math I released EOC #18) Energy and mass are related by the formula  $E = mc^2$ .

- m is the mass of the object
- c is the speed of light

Which equation finds m, given E and c?

C) 
$$m = \frac{c^2}{F}$$

B) 
$$m = Ec^2$$

D) 
$$m = \frac{E}{c^2}$$

**Example:** In each of the equations below, rewrite the equation, solving for the indicated variable.

- a) If F denotes a temperature in degrees Fahrenheit and C is the same temperature measured in degrees Celsius, then F and C are related by the equation, F = 95C + 32. Rewrite this equation to solve for C in terms of F.
- b) The surface area S of a sphere of radius r is given by  $S = 4\pi r^2$ . Solve for r in terms of S.

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/4/tasks/1828)

**Example:** The equation for an object that is launched from the ground is given by  $h(t) = -16t^2 + v_0 t$  where h is the height, t is the time, and  $v_0$  is the initial velocity. What is the initial velocity of an object that is one-hundred feet off the ground four seconds after it is launched?

#### NC.M1.F-IF.2

## Understand the concept of a function and use function notation.

Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context

## **Concepts and Skills**

## Pre-requisite

- Use substitution to determine if a number if a solution (6.EE.5)
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Define a function and use functions notation (NC.M1.F-IF.1)

#### **Connections**

- Creating and solving one variable equations (NC.M1.A-CED.1)
- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Function standards that relate domain and range (NC.M1.F-IF.3, NC.M1.F-IF.4, NC.M1.F-IF.5, NC.M1.F-IF.7)
- Comparing the end behavior of functions (NC.M1.F-LE.3)

### The Standards for Mathematical Practices

#### Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

## **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the domain, range, input, output and the relationship between the variables of a function in context.

New Vocabulary: exponential function, quadratic function

## Mastering the Standard for this Unit

#### Comprehending the Standard

Students should be fluent in using function notation to evaluate a linear, quadratic, and exponential function.

Students should be able to interpret statements in function notation in contextual situations.

## **Assessing for Understanding**

Students should be able to use evaluate functions written in function notation.

**Example:** Evaluate f(2) for the function f(x) = 5(x - 3) + 17.

Evaluate f(2) for the function  $f(x) = 1200(1 + .04)^x$ .

Evaluate f(2) for the function  $f(x) = 3x^2 + 2x - 5$ .

Students should be able to evaluate functions and interpret the result in a context.

**Example:** You placed a yam in the oven and, after 45 minutes, you take it out. Let *f* be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.

a) 
$$f(0) = 65$$

- b) f(5) < f(10)
- c) f(40) = f(45)
- d) f(45) > f(60)

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/625)

**Example:** The rule  $f(x) = 50(0.85)^x$  represents the amount of a drug in milligrams, f(x), which remains in the bloodstream after x hours. Evaluate and interpret each of the following:

- a) f(0)
- b)  $f(2) = k \cdot f(1)$ . What is the value of k?

**Example:** Suppose that the function f(x) = 2x + 12 represents the cost to rent x movies a month from an internet movie club. Makayla now has \$10. How many more dollars does Makayla need to rent 7 movies next month?

(NCDPI Math 1 released EOC #12)

**Example:** Let f(t) be the number of people, in millions, who own cell phones t years after 1990. Explain the meaning of the following statements.

- a) f(10) = 100.3
- b) f(a) = 20
- c) f(20) = b
- d) n = f(t)

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/634)

#### NC.M1.F-IF.1

## Understand the concept of a function and use function notation.

Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that:

- if f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x.
- the graph of f is the graph of the equation y = f(x).

#### **Concepts and Skills**

## Pre-requisite

- Understand that a function is a rule that assigns to each input exactly one output (8.F.1)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)

#### Connections

- Create and graph two variable equations (NC.M1.A-CED.2)
- All other function standards

#### **The Standards for Mathematical Practices**

#### Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

#### **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to accurate describe a function in their own terms.

New Vocabulary: notation

## Comprehending the Standard

Students should understand the definition of a function. It is deeper than just "x" cannot repeat or the vertical line test. Students should understand what it takes to be a function in categorical, numerical, and graphical scenarios.

In 8<sup>th</sup> grade, students studied the definition of a function. In Math 1, function notation is introduced. While this standard places a focus of the definition of a function on the correspondence of input and output values, a function can also be defined by how one variable changes in relation to another variable. This view of a function is highlighted in other standards throughout Math 1 when students are asked to identify, interpret, and use the rate of change.

For this unit, domain and range may be given in

# Mastering the Standard for this Unit

## **Assessing for Understanding**

Students should be able to understand functions in categorical scenarios.

**Example:** A certain business keeps a database of information about its customers.

- a) Let *C* be the rule which assigns to each customer shown in the table his or her home phone number. Is *C* a function? Explain your reasoning.
- b) Let *P* be the rule which assigns to each phone number in the table above, the customer name(s) associated with it. Is *P* a function? Explain your reasoning.
- c) Explain why a business would want to use a person's social security number as a way to identify a particular customer instead of their phone number.

Customer Name	Home Phone Number
Heather Baker	3105100091
Mike London	3105200256
Sue Green	3234132598
Bruce Swift	3234132598
Michelle Metz	2138061124

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624)

inequality notation. Students in Math I are not responsible for interval or set notation as a solution. They are to write answers to these inequalities using inequality notation.

Students should be able to understand functions as a correspondence between inputs and outputs.

**Example:** A pack of pencils cost \$0.75. If n number of packs are purchased, then the total purchase price is represented by the function t(n) = 0.75n.

- a) Explain why *t* is a function.
- b) What is a reasonable domain and range for the function *t*?

**Example:** Suppose f is a function.

- a) If 10 = f(-4), give the coordinates of a point on the graph of f.
- b) If 6 is a solution of the equation f(w) = 1, give a point on the graph of f.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/630)

#### NC.M1.F-IF.4

## Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

## **Concepts and Skills**

#### Pre-requisite

- Describe quantitatively the functional relationship between two quantities by analyzing a graph (8.F.5)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

#### **Connections**

- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

#### The Standards for Mathematical Practices

#### **Connections**

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

#### **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identification of key features and interpret those key features in context.

New Vocabulary: maximum, minimum

## **Mastering the Standard for this Unit**

#### **Comprehending the Standard**

Students should understand the key features of any contextual situation. For example, plots over time represent functions as do some scatterplots. These are often functions that "tell a story" hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables, graphs, and verbal descriptions.

## **Assessing for Understanding**

Students should be able to identify and interpret key features of functions.

**Example:** An epidemic of influenza spreads through a city. The figure below is the graph of I = f(w), where I is the number of individuals (in thousands) infected w weeks after the epidemic begins.

- a) Estimate f(2) and explain its meaning in terms of the epidemic.
- b) Approximately how many people were infected at the height of the epidemic? When did that occur? Write your answer in the form f(a) = b.
- c) For approximately which w is f(w) = 4.5; explain what the estimates mean in terms of the epidemic.
- d) An equation for the function used to plot the image above is  $f(w) = 6w(1.3)^{-w}$ . Use the graph to estimate the solution of the inequality  $6w(1.3)^{-w} \ge 6$ . Explain what the solution means in terms of the epidemic. (This would make a great Honors level extension to this standard)

8 6 4 2 0 2 4 6 8 10 12 14 16 w

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/637)

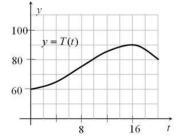
Students should understand the concept behind the key features (intercepts, increasing/decreasing, positive/negative, and maximum/minimum) for any given graph, not just "function families". This means that students should be asked to work with graphical and tabular representations of functions that the student could not solve or manipulation algebraically.

By contrast, NC.M1.F-IF.7, has students work with specific functions in which students have the ability to use algebraic manipulation to identify additional key features.

**Example:** The figure shows the graph of T, the temperature (in degrees Fahrenheit) over one particular 20-hour period in Santa Elena as a function of time t.

- a) Estimate T(14).
- b) If t = 0 corresponds to midnight, interpret what we mean by T(14) in words.
- c) Estimate the highest temperature during this period from the graph.
- d) When was the temperature decreasing?
- e) If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, when should she leave?

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/639)



**Example:** Eliana observed her dog, Lola, running around the yard and recorded the time and distance that Lola was away from her dog house in the table below.

- a) Sketch a graph of Lola's play time away from her dog house.
- b) Describe what is happening between minutes 2 & 3.

Time	Distance				
(minutes)	(feet)				
0	0				
1	5				
2	30				
3	15				
4	25				
5	50				

#### NC.M1.F-IF.6

## Interpret functions that arise in applications in terms of the context.

Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

## **Concepts and Skills**

## Pre-requisite

- Determine and interpret the rate of change of a linear function (8.F.4)
- Describe qualitatively the functional relationship between two quantities and sketch a graph from a verbal description (8.F.5)

#### **Connections**

- Interpret key features of graphs and tables (NC.M1.F-IF.4)
- Analyze linear, quadratic and exponential functions by generating different representations (NC.M1.F-IF.7)

#### **The Standards for Mathematical Practices**

#### Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

## **Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: average rate of change

## Mastering the Standard in this Unit

## Comprehending the Standard

Students calculate the average rate of change of a function given a graph, table, and/or equation.

The average rate of change of a function y = f(x) over an interval  $a \le x \le b$  is  $\frac{change in y}{change in x} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$ .

This standard is more than just slope. It is asking students to find the average rate of change of any function over any given interval. Be sure to include multiple representations (numerically, graphically, or symbolically) of functions for students to work with.

It is an important connection for further courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and exponentials do not have constant rates of change due to their curvature

#### Assessing for Understanding

Students should be able to find the average rate of change over a specified interval.

**Example:** Find the average rate of change of each of the following functions over the interval  $1 \le x \le 5$ .

a) 
$$f(x) = 3x - 7$$

b) 
$$g(x) = x^2 + 2x - 5$$

c) 
$$h(x) = 3(2)^x$$

**Example:** The table below shows the average weight of a type of plankton after several weeks.

Time(weeks)	Weight (ounces)				
8	0.04				
9	0.07				
10	0.14				
11	0.25				
12	0.49				

(NCDPI Math 1 released EOC #21)

What is the average rate of change in weight of the plankton from week 8 to week 12?

- a) 0.0265 ounce per week
- b) 0.0375 ounce per week
- c) 0.055 ounce per week
- d) 0.1125 ounce per week

**Example:** The table below shows the temperature, T, in Tucson, Arizona t hours after midnight. When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

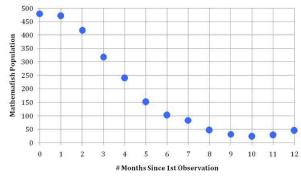
t (hours after midnight)	0	3	4		
$T$ (temp. in $\circ$ F)	85	76	70		

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500)

**Example:** You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathematish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46

## **Mathemafish Population**



Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project.

What to include in your summary report:

- a) Calculate the average rate of change of the mathematish population over specific intervals. Indicate how and why you chose the intervals you chose.
- b) When was the population decreasing the fastest?
- c) During what month did you notice the largest effects of the EPA intervention?
- d) Explain the overall effects of the intervention.
- e) Remember to justify all your conclusions using supporting evidence.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/686)