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M.Sc. Mathematics (Semester –4th)
FUNCTIONAL ANALYSIS
SUBJECT CODE: MMAT1-417
Paper ID: [19220518]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is Compulsory.
2. Attempt any THREE questions from Section B.
3. Attempt any TWO questions from Section C.

Section – A **(4 marks each)**

Q1. Let X and Y be normal linear spaces over field F and $T \in B\ (X,\ Y)$. Then $T\ =\ \ x \in D(T)\ \sup\ sup\ \left\{\frac{Tx}{x} : x \neq 0\right\} =\ x \in D(T),\ \ x\ =\ 1SUP\{Tx\}\ =\ \ \ \ x \in D(T),\ \ x \leq 1\ sup\{Tx\}$.

Q2. (i) Define equivalent norms, show that two norms $x_1\ =\ \sum_{i=1}^{n}\left|x_i\right|\ and\ x_2 = \left(\sum_{i=1}^{n}\left|x_i\right|^2\right)^{\frac{1}{2}}$ on R^n are equivalent.

(ii) A finite dimensional vector space is algebraically reflexive.

Q3. Let C[0,1] be the space of all continuous functions on [0,1] if C[0,1] is equipped with the L_1 norm $x(t) = \ \int_{t=0}^{1}|x(t)|\ dt$, then C[0,1] is not complete normed linear space.

Q4. Let H be a Hibert space and T_1 , T_2 : H → H be bounded linear operators. Then

i. $\left(T_1 + T_2\right)^* =\ T_1^* +\ T_2^*$

ii. $||T_1^* T_1||||T_1||^2 =\ ||T_1 T_1^*||$.

Section – B **(8 marks each)**

Q5. Let M be a subspace of a normal linear space X such that M ≠ X. Assume that $u \in \frac{X}{M}$ such d = dist (u,M) = $inf_{m \in M} u\ -\ m\ >\ 0.$ Then there is a bounded linear functional $f \in X^*$ Such that

i. $f(x) =\ 0\ for\ all\ x \in M.$

ii. $f(u) =\ d.$

iii. $f\ =\ 1.$

Q6. Let T be a linear transformation from a normed linear space X to a normed linear space Y. Then T is bounded if and only if it is continuous.

Q7. Let, X be a normed linear space over a field K, $f : X \longrightarrow K$ be a linear functional on X. then f is continuous if and only if $kar(f)$ is closed in X.

Q8. A bounded linear operator T on a Hilbert space H into itself is unitary if and only if T is an isometrically isomorphism of H onto itself.

Section – C **(10 marks each)**

Q9. Let H be a complex Hilbert space, $B(H, H)$ be the collection of all bounded linear operator form H onto itself and $T \in B(H, H)$. Then there exists a $T^{*} \in B(H, H)$. Such that $||T|| = ||T^{*}||$.

Q10. State and prove the open mapping theorem.

Q11. Show that the dual/conjugate space of l_p, $(1 < p < \infty)$ is l_p ; here, $1 < p < \infty$ *and* q is the conjugate of p , That is, $\frac{1}{p} + \frac{1}{q} = 1$.