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M.Sc. Mathematics (Semester –4th)
FUNCTIONAL ANALYSIS
SUBJECT CODE: MMAT1-417
Paper ID: [19220518]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is Compulsory.
2. Attempt any THREE questions from Section B.
3. Attempt any TWO questions from Section C.

Section – A **(4 marks each)**

Q1. Let X and Y be normal linear spaces over field F and $T \in B(X, Y)$. Then
$T = \sup_{x \in D(T)} sup \left\{ \frac{Tx}{x} : x \neq 0 \right\} = \sup_{x \in D(T),\ x = 1} SUP\{Tx\} = \sup_{x \in D(T),\ x \leq 1} sup\{Tx\}$
.

Q2. (i) Define equivalent norms, show that two norms $x_1 = \sum_{i=1}^{n} |x_i|$ and $x_2 = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{\frac{1}{2}}$ on R^n are equivalent.
(ii) A finite dimensional vector space is algebraically reflexive.

Q3. Let C[0,1] be the space of all continuous functions on [0,1] if C[0,1] is equipped with the L_1 norm $x(t) = \int_{t=0}^{1} |x(t)|\, dt$, then C[0,1] is not complete normed linear space.

Q4. Let H be a Hibert space and $T_1, T_2 : H \to H$ be bounded linear operators. Then
i. $(T_1 + T_2)^* = T_1^* + T_2^*$
ii. $\|T_1^* T_1\| \|T_1\|^2 = \|T_1 T_1^*\|$.

Section – B **(8 marks each)**

Q5. Let M be a subspace of a normal linear space X such that M ≠ X. Assume that $u \in \frac{X}{M}$ such d = dist (u,M) = $inf_{m \in M} u - m > 0$. Then there is a bounded linear functional $f \in X^*$ Such that
i. $f(x) = 0\ for\ all\ x \in M.$
ii. $f(u) = d.$
iii. $f = 1.$

Q6. Let T be a linear transformation from a normed linear space X to a normed linear space Y. Then T is bounded if and only if it is continuous.

Q7. Let, X be a normed linear space over a field K, $f : X \to K$ be a linear functional on X. then f is continuous if and only if $kar(f)$ is closed in X.

Q8. A bounded linear operator T on a Hilbert space H into itself is unitary if and only if T is an isometrically isomorphism of H onto itself.

Section – C **(10 marks each)**

Q9. Let H be a complex Hilbert space, $B(H, H)$ be the collection of all bounded linear operator form H onto itself and $T \in B(H, H)$. Then there exists a $T^* \in B(H, H)$. Such that ||T|| = $||T^*||$.

Q10. State and prove the open mapping theorem.

Q11. Show that the dual/conjugate space of l_p, $(1 < p < \infty)$ is l_p ; here, $1 < p < \infty \; and \; q$ is the conjugate of p , That is, $\frac{1}{p} + \frac{1}{q} = 1$.