Willingboro
Public Schools

"Where Excellence is the Expectation"

Willingboro Public Schools
WPS Mathematics Honors Pre-Calculus Curriculum

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Mathematics | High School

(From New Jersey Student Learning Standards for Mathematics)

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Standard	Mathematically Proficient Students …
1. Make sense of problems and persevere in solving them	start by explaining to themselves the meaning of a problem and looking for entry points to its solution. check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"
2. Reason abstractly and quantitatively	make sense of quantities and their relationships in problem situations.
3. Construct viable arguments and critique the reasoning of others	understand and use stated assumptions, definitions, and previously established results in constructing arguments.
4. Model with mathematics	apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
5. Use appropriate tools strategically	are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.

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6. Attend to precision	try to communicate precisely to others.
7. Look for and make use of structure	look closely to discern a pattern or structure
8. Look for and express regularity in repeated reasoning	notice if calculations are repeated, and look both for general methods and for shortcuts.

(NJ DOE, 2024)

New Jersey Legislative Statutes and Administrative Code
Mathematics

Amistad Law: *N.J.S.A. 18A 52:16A-88*
Radical Math: includes lessons on topics for social justice in mathematics African American Mathematician Bios

Holocaust Law: *N.J.S.A. 18A:35-28*
https://www.bbc.com/future/article/20191031-hilda-geiringer-mathematician-who-fled-the-nazis

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*
https://prideinstem.org/ LGBTA+ Mathematician Bios https://www.ngpf.org/blog/math/math-monday-celebrating-disabled-mathematicians/ Mathematicians with Disabilities

Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*
http://www.ams.org/about-us/diversity

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https://mathematicallygiftedandblack.com/

Standards in Action: *Climate Change*

New Jersey became the first state in the nation to include climate change across content areas with the adoption of the 2020 New Jersey Student Learning Standards (NJSLS). The goal of inclusion of climate change education implementation is to foster generations of New Jersey students that can analyze, question, interpret, to think independently, and bring critical deduction to fulfill, and to lead in jobs created by burgeoning industries of the future green economy.

Suggestions for how to incorporate climate change examples into math instruction will be noted in green text throughout the units.

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards without a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

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Course Sequence/Table of Contents:

Click here for the Honors Pre-Calculus Mathematics Pacing Guide

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Overview	Standards for Mathematical Content		Unit Focus	Standards for Mathematical Practice
Unit 1 **Complex Numbers and Transformations**	• **N.CN.A.1** • **N.CN.A.2** • **N.CN.A.3** • **N.CN.B.4** • **N.CN.B.5** • **N.CN.B.6** • **A.REI.B.4** • **F.LE.A.2**	• **F.LE.B.5** • **N.VM.C.8** • **N.VM.C.10** • **N.VM.C.11** • **N.VM.C.12** • **F.IF.C.8** • **F.LE.A.4**	• Perform arithmetic operations with complex numbers • Use complex numbers in polynomial identities and equations • Represent complex numbers on a complex plane as rectangular and polar planes • Solve problems involving vector quantities • Write expressions in matrix form and solve them • Use matrices to transform figures on a coordinate plane • Express matrix multiplication in left and right notation	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 1: Suggested Open Educational Resources	N.CN.A.1 Complex number patterns N.CN.A.2 Powers of a complex number N.CN.A.3 Conjugate of a complex number N.CN.B.4 Graphing complex numbers N.CN.B.5 Operations with complex numbers N.CN.B.6 Complex distance A.REI.B.4 Completing the square		F.LE.B.5, F.LE.A.2 Exponential Parameters N.VM.C.8 Working with matrices N.VM.C.10 Laws of matrices N.VM.C.11 Using matrices to do translations N.VM.C.12 Determinant of a matrix F.IF.C.8b Carbon 14 dating in practice I F.LE.A.4 Carbon 14 dating	
Unit 2 **Vectors and Matrices**	• **N.VM.A.1** • **N.VM.A.2** • **N.VM.A.3** • **N.VM.B.4** • **N.VM.B.5** • **N.VM.C.6** • **N.VM.C.7** • **N.VM.C.8** • **N.VM.C.9** • **N.VM.D.11** • **A.REI.A.1** • **N.VM.C.10**		• Understand incidence relationships in networks and encode information about them via high-dimensional matrices • Study matrices and the role of the zero and identity matrices • Use matrices to solve higher-order systems of linear equations • Understand the difference between a vector and a scalar • Apply vector properties to real-world problems in which directionality manifests mathematically such as velocity • Use multiple approaches for performing vector operations • Solve equations involving linear transformations of the coordinate space	

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			• Convert between parametric equations and the slope-intercept form of a line • Understand left- and right-handed matrix notation • Perform matrix operations while understanding which traditional properties of equality do or do not apply • Design a computer game using vectors and matrices	
Unit 2: Suggested Open Educational Resources	N.VM.A.1 Magnitude and Direction of Vectors N.VM.A.2 Components of a Vector N.VM.A.3 Velocity Vectors N.VM.B.4 Vector Addition N.VM.B.5 Multiplying Vectors by Scalars N.VM.C.6 Using Matrices to Represent Data N.VM.C.7 Scaling Matrices F.IF.B.4 Quadratic and Polynomial Functions		N.VM.C.8 Matrix Operations N.VM.C.9 Properties of Equality in Matrix Operations N.VM.D.11 Linear Transformations using Matrices A.REI.A.1 Products and Reciprocals N.VM.C.10 Determinant and Matrix Inverse A.REI.C.5 Solving Systems of Equations A.REI.C.6 2x2 Systems of Linear Equations	
Unit 3 **Rational and Exponential Functions**	• **A.APR.C.5** • **A.APR.D.7** • **F.IF.C.7**	• **N.CN.C.8** • **N.CN.C.9** • **F.BF.A.1** • **F.IF.B.4** • **F.BF.B.4** • **F.BF.B.5**	• Use various methods to find the roots of polynomials and the roots of unity • Apply polynomial identities to find square roots of complex numbers • Interpret functions that arise in applications in terms of the context • Model phenomena with logarithmic or exponential functions • Use Cavalieri's Principle to derive the formulas for the volume of a sphere and other geometric solids • Demonstrate the relationships between rational expressions and mathematical operations that do (and do not) apply to them • Compose functions and describe their domains and ranges • Determine inverse functions and explain their domains and ranges, particularly with respect to the original function	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.
Unit 3: Suggested Open Educational	A.APR.C.5 Binomial Theorem - Powers of 11 A.APR.D.7 Operations of Rational Expressions F.IF.C.7 Graphs of Power Functions G.GM.A.2 Using Cavalieri's Principle		N.CN.C.8 Complex Cubes and Roots of 1 N.CN.C.9 Fundamental Theorem of Algebra F.BF.A.1 Crude Oil and Gas Mileage F.BF.B.4 Inverse Functions - Rainfall F.BF.B.5 Exponentials and Logarithms	

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Resources	G.GM.A.3 Volume of a Water Centerpiece			
Unit 4 Trigonometric Functions/Inverses and Probability/Statistics	• **S.ID.A.4** • **S.IC.A.1** • **S.IC.A.2** • **S.IC.B.3** • **S.CP.B.8** • **S.CP.B.9**	• **S.CP.A.4** • **S.CP.A.5**	• Explore the periodicity and relationship between sine, cosine, and tangent functions • Derive formulas for non-right triangles • Study the graphs of trigonometric functions and their inverses • Use inverse trigonometric functions to solve for angles of elevation for inclined surfaces • Discover the three techniques for counting statistical outcomes including the multiplication rule • Analyze information related to random variables and discrete probability distributions	MP.8 Look for and express regularity in repeated reasoning.
Unit 4: Suggested Open Educational Resources	S.ID.A.4 Do You Fit in This Car? S.IC.A.1 School Advisory Panel S.IC.A.2 Sarah, the chimpanzee S.IC.B.3 Strict Parents S.CP.B.8 Multiplication Rule S.CP.B.9 Margin of Error for Estimating a Population Mean		S.M.A.3 Probability Distribution S.M.A.4 Random Variables S.CP.A.4 Two-Way Tables and Probability S.CP.A.5 Breakfast Before School S.M.B.5 Weighing Outcomes S.M.B.6 Making Fair Decisions S.M.B.7 Fred's Fun Factory	

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Unit 1 Pre-Calculus			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**

• **N.CN.A.1**. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
• **N.CN.A.2**. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers
  • **N.CN.A.3**. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
  • **N.CN.B.4**. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
  • **N.CN.B.5**. Represent addition,
	n/a	MP.2 Reason abstractly and quantitatively.
		MP.6 Attend to precision.
		MP.7 Look for and make use of structure.
			Concepts:
			• Complex number i is defined such that $i^2 = -1$.
			• Every complex number has the form $a + bi$ with a and b real.
			• Complex number operations exist as they do for real numbers.
			• Complex numbers can be represented graphically.
			Students are able to:
			• understand $i^2 = -1$ and the commutative, associative properties to add and subtract complex numbers are to be used.
			• determine that $i^2 = -1$ and the commutative, associative, and distributive properties to multiply complex numbers.
			• graph complex numbers in rectangular and polar form and explain how those forms can represent the same number.
			• represent complex number operations geometrically on the complex plane
			• calculate and graph distance between complex numbers
			
			Learning Goal 1: Add, subtract, and multiply complex numbers using the commutative, associative and distributive properties. Graph complex numbers using rectangular and polar coordinates, and represent operations with complex numbers using graphs.

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Unit 1 Pre-Calculus			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1+\sqrt{3}i)$ has modulus 2 and argument 120°. • **N.CN.B.6**. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.			

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Unit 1 Pre-Calculus			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• **A.REI.B.4.** Solve quadratic equations in one variable. **A.REI.B.4b**. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	**8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. We have learned to/that… • use square root and cube root symbols to represent solutions to equations in the form $x^2 = p$ and $x^3 = p$ • evaluate square roots of small perfect squares and cube roots of small perfect cubes • $\sqrt{2}$ is an irrational number **8.EE.C.7** Solve linear equations in one variable.	MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concepts: • As with real solutions, complex solutions to quadratic equations may be determined by taking square roots, factoring, and completing the square. Students are able to: • solve quadratic equations in one variable that have complex solutions by taking square roots. • solve a quadratic equations in one variable that have complex solutions by completing the square. • solve a quadratic equations in one variable that have complex solutions by factoring. • write complex solutions in $a \pm bi$ form. Learning Goal 2: Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring.

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Unit 1 Pre-Calculus			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
	b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. We have learned to/that… ● solve linear equations in one variable with rational number coefficients, including equations that require expanding expressions using the distributive property and combining like terms		
● **A.REI.C.7**. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.	n/a	MP.1 Make sense of problems and persevere in solving them.	Concepts: ● Solutions of linear systems contain different function types. Students are able to: ● solve a system containing one linear equation and one quadratic equation algebraically. ● graph a system containing one linear equation and one quadratic equation to determine a solution. Learning Goal 3: Solve simple systems consisting of a linear and quadratic equation in two variables algebraically and graphically.
● **A.REI.C.6.** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on	**8.EE.C.8** Analyze and solve pairs of simultaneous linear equations.	MP.1 Make sense of problems and persevere in solving them.	Concepts: ● Solving a system of linear equations containing *n* variables requires *n* equations. Students are able to:

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Unit 1 Pre-Calculus			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
pairs of linear equations in two variables.	b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6 We are learning to/that… ● solve systems of two linear equations in two variables algebraically. ● estimate solutions of two linear equations in two variables by graphing the equations.		● use the substitution method and/or elimination method to find the solution of a system containing three linear equations. Learning Goal 4: Solve algebraically a system of three linear equations.
● **F.LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ● **F.LE.B.5.** Interpret the parameters in a linear or exponential function in terms of a context.	**8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP 4. Model with mathematics MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	Concepts: ● Recursion and recursive series and sequences Students are able to: ● distinguish between recursive and explicit formulas. ● represent geometric and arithmetic sequences recursively. ● represent geometric and arithmetic sequences with explicit formulas. ● translate between recursive form and explicit form of geometric and arithmetic sequences. ● recognize explicit formula for geometric sequences as exponential functions containing a domain in the integers only. ● interpret the parameters of an exponential function representing a geometric sequence. ● interpret the parameters of a linear function representing an arithmetic sequence.

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Unit 1 Pre-Calculus			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
	We have learned to/that… ● construct a function to model a linear relationship between two quantities. ● determine the rate of change and initial value of a function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. ● interpret the rate of change and initial value of a function in terms of the situation it models		Learning Goal 5: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
● **N.VM.C.8** Add, subtract, and multiply matrices of appropriate dimensions. ● **N.VM.C.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. ● **N.VM.C.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable	n/a	MP.1 Make sense of problems and persevere in solving them. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	Concepts: ● Use matrices to represent and manipulate data and equations. ● Vector quantities can be used and operated on by using matrices, but not all algebraic properties of equality apply to matrices. ● 2 x 2 matrics can be used to represent transformations of a figure on a coordinate plane. The absolute value of the determinant may be interpreted in terms of area. Students are able to: ● manipulate matrices to represent both scalar and vector quantities as necessary ● understand that matrix multiplication is not a commutative operation, but does satisfy the associative and distributive properties

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Unit 1 Pre-Calculus			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
dimensions to produce another vector. Work with matrices as transformations of vectors. • **N.VM.C.12** Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.			• understand and state the role of the zero and the identity matrices as analogous to the role of 0 and 1 for real numbers. Learning Goal 6: Use properties of matricies and vectors to perform operations and use matrices in applications.
• **F.IF.C.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function **F.IF.C.8b:** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	**8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example*, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. We have learned to/that… • know the properties of integer exponents • determine whether two numerical expressions involving integer exponents are equivalent • generate equivalent expressions using the properties of exponents	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concepts: • Alternate, equivalent forms of an exponential expression containing rational exponents may reveal specific attributes of the function that it defines. Students are able to: • use properties of exponent transform/rewrite an exponential expression for an exponential function. • explain the properties of the quantity or the function. Learning Goal 7: Use the properties of exponents to transform expressions for exponential functions, explain properties of the quantity revealed in the transformed expression or different properties of the function.
• **F.LE.A.4**. Understand the inverse relationship between exponents and logarithms. For exponential models,	n/a	MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics.	Concepts: • Exponents and logarithms have an inverse relationship. • Solutions to an exponential equation in one variable can be written as a logarithm.

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Unit 1 Pre-Calculus			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.			Students are able to: ● transform an exponential model represented by $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e. ● write the solution to $ab^{ct} = d$ as a logarithm. ● use technology to evaluate logarithms having base 2, 10, or e. Learning Goal 8: Express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

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Unit 1 - Complex Numbers and Transformations

Overview

In this first unit, learners will become facile with performing arithmetic operations with complex numbers, using complex numbers in polynomial identities and equations, and representing complex numbers on a complex plane as both rectangular and polar planes. Next, learners will review the difference between vector and scalar quantities, solve problems involving vector quantities, and learn to use matrices to represent both vectors and scalars. Learners will write and solve expressions in matrix form, use matrices to transform figures on a coordinate plane, and learn to express quantities in right notation. They will also learn the purpose of the zero matrix and the identity matrix and how they are analogous to 0 and 1 in the set of real numbers. Learners will then understand how a vector may be represented by a one-column matrix, understand how multiplying a vector represented as a matrix by another matrix of suitable dimensions to produce another vector and/or demonstrate the geometric transformation of vectors. Finally, learners will discover how a 2 x 2 matrix can represent transformations of figures of the plane and how the absolute value of the determinant of such a matrix may be interpreted graphically in terms of area.

Essential Questions	Enduring Understandings
• What is the difference between a real number, an imaginary number, and a complex number? • What is the significance of a complex solution to a quadratic or exponential function? • How does a complex or imaginary solution manifest graphically? • How are complex and imaginary numbers manipulated algebraically? • How can complex numbers be represented on the complex plane? • How can complex number operations be represented geometrically? • For which real-world scenarios can complex numbers be used as a model? • What is a matrix and what can it be used to represent? • What is the role of the zero and identity matrices in terms of matrix operations? • How can one work with matrices to perform vector transformations? • What is the significance of the absolute value of the determinant of a 2 x 2 matrix, and how does it allow for geometric translations of various figures? • How does a complex number appear? • Where do we use Complex numbers in real life? • How does Matrices help solve problems?	• Understand the difference between real, imaginary, and complex numbers • Perform arithmetic operations with complex numbers • Use complex numbers in polynomial identities and equations • Explain how looking at a graph can inform the viewer how many solutions to a quadratic or polynomial expression could be real, complex, and/or imaginary • Complex numbers may be graphed on the complex plane • Complex number operations can be represented geometrically • A matrix is one way to visually interpret an expression and there are many algebraic operations that can be performed on a matrix similar to any other form of an expression. • Matrices can be used to represent scalar quantities, but have more utility as a method of representing vector quantities • Certain matrices can be used to transform other geometric figures as well • Analyze functions using different representations • Apply complex number operations as Transformations • Implement the right notation of Matrices that encompasses new transformation • Persevices in handling a new transformation.
District/School Formative Assessment Plan	**District/School Summative Assessment Plan**
Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards. Teachers are encouraged to incorporate Formative Assessments into all lessons. During instruction, teachers will collect ongoing information on students' mastery of content through a variety of methods:	*Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.* **Benchmark Assessments:** • Assessment 11.1.1: Mid-Unit Assessment • Assessment 11.1.2: End of Unit Assessment

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• Questioning: using Socratic method, probing questions, a hierarchical system in complexity (Bloom's Taxonomy) • Exit tickets, rotational activities (stations), quizzes, and small group activities • Classwork, homework, group work (formative assessment) • Pre-Assessment, teacher's observation, class discussion, and journal	• Assessment 11.1.3: End of Unit Performance Assessment **Standardized Assessments:** • NJSLA **Other Summative Assessments:** Teachers are encouraged to design and their own assessments (topic/module tests and quizzes) individually and/or with their department or grade-level partners, as per Uniform Grading Profile..

Focus Mathematical Concepts

Vocabulary	Common Misconceptions	Fluency
• Complex number • Real number • Imaginary number • Complex conjugate • Modulus • Quotient • Rectangular form • Polar form • Complex plane • Matrix • Dimensions • Zero matrix • Identity matrix • Scalar • Vector • Magnitude • Direction • Determinant	Students may: • Confuse irrational numbers with non-real or complex numbers. • Misrepresent an imaginary unit i as `-1 instead of` $\sqrt{-1}$. • Ignore the negative solutions in the cases of quadratic equations, when the use of quadratic formula is not critical. For example, for the equation $x^2 = 9$, students may mention 3 and forget about (– 3), or mention $3i$ and forget about (- $3i$) for the equation $x^2 = -9$. • Not understand how i raised to various powers will produce 1, -1, $\sqrt{-1}$, and $-\sqrt{-1}$. • Not grasp how to graphically represent complex numbers and how typical Cartesian coordinates system are not appropriate. • Have trouble understanding what makes a quantity a scalar versus a vector • Not see how some previously prime binomials may now be solved with complex solutions • Look at the graph of a quadratic or polynomial function and not figure out whether or not the function must have complex roots • Confuse rows and columns in matrices, have difficulty with the terminology, and	**N.CN.A.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. **N.CN.A.2** Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N.CN.C.7** Solve quadratic equations with real coefficients that have complex solutions. **A.SSE.A.2** Use the structure of an expression to identify ways to rewrite it. **A.SSE.B.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. Math Fluency Support for Grade 11 Targeted Fact reinforcement **Fact Fluency** **Dr. Riccomini's Math Facts – Basic Facts Link** https://www.pattan.net/Videos/Facilitating-Fluency-and-Automaticity-through-Purp **Math Fact Practice** http://www.xtramath.org **Differentiation problems** https://www.openmiddle.com/

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	understand the significance of the zero and identity matrices	

District/School Tasks	District/School Primary and Supplementary Resources
• NJDOE Digital Item Library • NJSLA Released Items in Mathematics • NJSLA Mathematics Practice Tests • NJSLA Mathematics Evidence Statements • Progression Tables • GUES Strategy + ECR problems	**District-Mandated Resources** • **Eureka Mathematics** • **i-Ready** • **Zearn** **Assessment Resources:** • N.CN.A.1 What are imaginary numbers? • N.CN.A.2 Operations of complex numbers • N.CN.A.3 Complex conjugates • N.CN.B.4 Cartesian, polar, and complex coordinate planes • N.CN.B.5 Representing complex number operations on the complex plane • N.CN.B.6 Determining the modulus of complex numbers • N.VM.C.8 Matrix Operations • N.VM.C.10 Zero and identity matrices • N.VM.C.11 Transformation matrix • N.VM.C.12 Area representation of the determinant **Project Ideas:** • **Math in the News: The Search for Missing Flight MH370** • **Math Goes to the Movies - Complex Numbers in CGI**

Instructional Best Practices and Exemplars

Instructional Best Practices
- Use explicit modeling techniques
- Connect visual representations
- Differentiate Instruction
- Incorporate vocabulary development
- Pose meaningful questions
- Present tasks that require reasoning and problem solving
- Support productive struggle
- Encourage mathematical discourse
- Collect and analyze student evidence of understanding

Exemplars and Explanations:
Pre-Calculus Appendix B

Integrated Accommodations and Modifications

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

Special Education Students	English Language Learners	At Risk
• Utilize modifications & accommodations delineated in the student's IEP • Provide additional manipulatives to support instruction • Allow for alternative strategies to solve algorithms or tasks • Provide the steps needed to complete the task • Model frequently • Provide repetition and practice. • Use visuals to demonstrate/model the processes • Restate, reread, and clarify directions/questions • Ask students to restate information, directions, and assignments. • Provide copy of class notes • Distribute study guide for classroom tests. • Provide preferential seating to be mutually determined by the student and teacher • Provide extra textbooks for home. • Provide regular parent/ school communication • Allow extended time to complete assignment • Establish procedures for accommodations / modifications for assessments • Allow student to take/complete tests in an alternate setting as needed Appendix A: Special Education Accommodations and Modifications	• WIDA Can Do Descriptors https://wida.wisc.edu/teach/can-do/descriptors • Modify Assignments • Use testing and portfolio assessment • Utilize Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary) • Repeat, rephrase, paraphrase key concepts and directions • Allow for extended time for assignment completion as needed • Highlight key vocabulary • Define essential vocabulary in context • Use graphic organizers, visuals, manipulatives and other concrete materials • Use gestures, facial expressions and body language • Read aloud • Build on what students already know and prior experience	• Pair visual prompts with verbal presentations • Ask students to restate information, directions, and assignments. • Provide repetition and and practice • Model skills / techniques to be mastered. • Provide extended time to complete class work • Provide copy of class notes • Provide preferential seating to be mutually determined by the student and teacher • Allow the use of a computer to complete assignments. • Establish expectations for correct spelling on assignments • Provide extra textbooks for home. • Provide Peer Support • Increase one on one time

Gifted and Talented Students	504 Plan
• Utilize advanced, accelerated, or compacted content • Provide assignments that emphasize higher- level thinking skills. • Allow for individual student interest • Gear assignments to development in areas of affect, creativity, cognition, and research skills	• Pair visual prompts with verbal presentations • Ask students to restate information, directions, and assignments. • Provide repetition and and practice • Model skills / techniques to be mastered. • Provide extended time to complete class work

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• Allow for a variety in types of resources • Provide problem-based assignments with planned scope and sequence • Utilize inquiry-based instruction • Adjust the pace of lessons • Utilize Choice Boards • Provide Problem-Based Learning • Establish flexible Grouping	• Provide copy of class notes • Break long assignments into smaller parts • Assist student in setting short term goals • Allow for preferential seating to be mutually determined by the student and teacher • Provide extra textbooks for home. • Model and reinforce organizational systems (i.e. color-coding) • Write out homework assignments, check student's recording of assignments
Interdisciplinary Connections	**Computer Science and Design Thinking**
English Language Arts • SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. • L.11-12.6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression. • W.11-12.10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.	**Computer Science and Design Thinking Practices** • Fostering an Inclusive Computing and Design Culture • Collaborating Around Computing and Design • Recognizing and Defining Computational Problems • Developing and Using Abstractions • Creating Computational Artifacts • Testing and Refining Computational Artifacts • Communicating About Computing and Design **Computer Science and Design Thinking Standards** • 8.1.12.NI.3: Explain how the needs of users and the sensitivity of data determine the level of security implemented. • 8.1.12.NI.4: Explain how decisions on methods to protect data are influenced by whether the data is at rest, in transit, or in use. • 8.1.12.IC.1: Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices. • 8.1.12.IC.3: Predict the potential impacts and implications of emerging technologies on larger social, economic, and political structures, using evidence from credible sources.
Career Readiness, Life Literacies and Key Skills	
Career Readiness, Life Literacies and Key Skills Practices • Act as a responsible and contributing community member and employee. • Attend to financial well-being. • Consider the environmental, social and economic impacts of decisions. • Demonstrate creativity and innovation. • Utilize critical thinking to make sense of problems and persevere in solving them. • Model integrity, ethical leadership and effective management. • Plan education and career paths aligned to personal goals. • Use technology to enhance productivity, increase collaboration, and communicate effectively.	

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- Work productively in teams while using cultural/global competence.

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- 9.1.12.CP.1: Summarize how one's credit history can affect finances, including loan terms, employment, and qualifying for loans.
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- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
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- **Self - Management**
- **Social Awareness**
- **Responsible Decision Making**
- **Relationship Skills**

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https://www.nj.gov/education/safety/wellness/selearning/index.shtml
Pacing Guide
Pre-Calculus Unit 1 Pacing Guide

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Unit 2 Vectors and Matrices			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• **N.VM.A.1**. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). 2. • **N.VM.A.2**. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. • **N.VM.A.3**. Solve problems involving velocity and other quantities that can be represented by vectors	**6.NS.B.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision.	Concepts: • Unlike scalars, vector quantities have both a magnitude and a direction, which manifest mathematically when performing vector operations – $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$ – $(x – a)$ is a factor of $p(x)$ if and only if $p(a) = 0$ Students are able to: • represent vectors using rays and appropriate symbols for their magnitudes • find the components of a vector • solve problems involving velocity and other vector quantities Learning Goal 1: Understand how to calculate, graph, and symbolically display the magnitude and direction of a vector.
• **N.VM.B.4**. Add and subtract vectors. • **N.VM.B.5**. Multiply a vector by a scalar. • **N.VM.C.7**. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. • **N.VM.C.8**. Add, subtract, and multiply matrices of appropriate dimensions.	**7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. We have learned to/that… • apply the properties of operations as strategies to **add, subtract, factor**, and **expand** linear expressions with rational coefficients	MP.7 Look for and make use of structure.	Concepts: • Vector operations, including multiplying vectors by scalars, follow specific steps and rely on several key properties. • Matrices with one column may be used to represent a vector quantity and multiplied by another single-column matrix to produce a new vector. • Matrix multiplication is not commutative and proper, consistently applied notation must be used throughout such a problem to ensure it is done correctly. Students are able to: • perform operations using vectors • understand what multiplying a vector and a scalar creates

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

Unit 2 Vectors and Matrices			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• **N.VM.C.9**. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. • **N.VM.C.11**. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	**6.EE.A.3** Apply the properties of operations to generate equivalent expressions. *For example*, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. We have learned to/that… • generate equivalent expressions using the properties of operations		• multiply matrices using consistent notation • use matrices to solve vector operation problems Learning Goal 2: Use appropriate technique to add, subtract, and multiply vector quantities taking their magnitude and direction (and that of the resultant) into account. Learning Goal 3: Understand how to represent a vector quantity using a matrix of suitable dimensions and how to use matrix operations as an alternate method of vector operations.
• **N.VM.C.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	n/a	MP.1 Make sense of problems and persevere in solving them. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concepts: • Just as the numbers 0 and 1 have specific significance and play a role when performing algebraic manipulations of real numbers, so too do the zero and identity matrices play a significant role in matrix addition and multiplication. • In the event that a square matrix has a multiplicative inverse, the determinant of that matrix is nonzero. Students are able to: • graph a polynomial function given its equation. • identify zeros from the graph and using an appropriate factoring technique. • show key features of the graph, including end behavior. • use technology to graph and describe key features of the graph for complicated cases.

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Unit 2 Vectors and Matrices			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
			Learning Goal 4: Show how the zero and identity matrices are used analogously to the real numbers 0 and 1. Discuss multiplicative inverses in matrices.
• **A.REI.A.1**. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.6 Attend to precision.	Concepts: • Solve an equation using requisite properties of equality. • Recognize which traditional algebraic properties of equality apply to matrix operations (and which do not). Students are able to: • Solve matrix operations problems using properties of equality. • Demonstrate why matrix multiplication is not commutative. Learning Goal 5: Solve problems involving matrices using only the properties of equality that are valid for matrices.
• **N.VM.C.6**. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	n/a	MP.1 Make sense of problems and persevere in solving them. MP 4. Model with mathematics MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure	Concepts: • Certain types of matrices are commonly used to represent scenarios in probability simulations as well as computer and video games. • Incidence matrices show the relationship between two sets and may be used to display the sets graphically. Students are able to: • Understand and apply vocabulary terms specific to these applications of matrices (e.g., nodes, branches). • Create directed graphs from incidence matrices and vice versa. • Explain and demonstrate why the entry values of an incidence matrix are limited to -1, 0, and 1. • Write reduced incidence matrices given the original matrices. • Take the sum of a column of an incidence matrix and explain the relevance.

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Unit 2 Vectors and Matrices			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
			Learning Goal 6: Create an incidence matrix that clearly displays nodes and branches, then rewrite as a reduced incidence matrix. Explain the significance of the sum of each column of the original and reduced forms of the matrix.

Unit 2 Vectors and Matrices

Overview

In this second unit of Pre-Calculus, learners will understand the main difference between vector and scalar quantities along with how to perform vector operations such as addition, subtraction, and geometric transformations. They will learn about the various methods by which vectors may be added or subtracted. Learners will also look at incidence relationships in networks and encode information about them via high-dimensional matrices. Matrix properties are studied as well as the role of the zero and identity matrices in matrix manipulations. Learners then are shown how to solve 3 x 3 systems of linear equations using previous methods (substitution and elimination) before being taught how matrices to study and solve higher order systems of equations such as 3 x 3 systems are substantially easier and less time-consuming. Learners will also perform linear transformations of the coordinate plane and space, along with conversions between parametric equations and the slope-intercept form of a line. Finally, the unit ends as students program video games using matrices and vectors.

Essential Questions	**Enduring Understandings**
• What is the difference between a vector and a scalar? • How may a vector be used to represent real-life quantities? • By which methods can equations involving linear transformations of the coordinate space be solved? • How can vectors be used to solve problems describing the motion of an object? • What is a matrix and what can it be used for? • What is the inverse of a matrix? • What are the zero and identity matrices? • What is right- and left-handed matrix notation? • What is a parametric equation? • How can vectors and matrices be used to design a simple computer game? • How can a system of linear equations be represented and solved using matrices and/or vectors? • How is a Vector represented and used in real life examples? • How can we apply Algebraic Operations on Matrices? • How do we Solve Equations Involving Linear Transformations of the	• Systems of linear equations can be represented as a single matrix equation in a vector variable. • Certain quantities (such as velocity) are best expressed as vectors rather than scalars as their directionality manifests mathematically when performing operations and solving problems involving those quantities. • Matrices provide an alternative method to elimination and substitution for solving systems of linear equations, and are often the preferred approach for solving systems that involve more than two variables. • The inverse of a matrix often has useful properties just like the matrix itself. • Matrix notation must be correctly applied throughout a matrix multiplication problem as it is not a commutative process (as is the case with real numbers). • Vectors can be represented by matrices with a single column. • Video/computer game coding and commands may be written by using special matrices known as incidence matrices. • Incidence matrices have special components, including nodes and branches, which are represented by either -1, 0, or 1. • Recognize vector quantities as having both magnitude and direction

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Coordinate Space? ● When are vectors useful?	● Vectors can be used to solve problems involving velocity ● Design computer game using Vectors and Matrices

District/School Formative Assessment Plan	District/School Summative Assessment Plan
Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards. Teachers are encouraged to incorporate Formative Assessments into all lessons. During instruction, teachers will collect ongoing information on students' mastery of content through a variety of methods: ● Questioning: using Socratic method, probing questions, a hierarchical system in complexity (Bloom's Taxonomy) ● Exit tickets, rotational activities (stations), quizzes, and small group activities ● Classwork, homework, group work (formative assessment) ● Pre-Assessment, teacher's observation, class discussion, and journal	*Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.* **Benchmark Assessments:** ● Assessment 11.2.1: Mid-Unit Assessment ● Assessment 11.2.2: End of Unit Assessment ● Assessment 11.2.3: End of Unit Performance Assessment **Standardized Assessments:** ● NJSLA **Other Summative Assessments:** Teachers are encouraged to design and their own assessments (topic/module tests and quizzes) individually and/or with their department or grade-level partners, as per Uniform Grading Profile.

Focus Mathematical Concepts

Vocabulary	Common Misconceptions	Fluency
● Incidence ● Networks ● Encode ● Matrix ● Zero matrix ● Identity matrix ● Vector ● Scalar ● Velocity ● Linear transformation ● Parametric equation ● Left and right notation ● Magnitude ● Direction ● Vector components ● Dimension ● Commutative ● Coordinate plane ● Inverse ● Incident matrix ● Payoffs	Students may: ● Not recall that matrix multiplication is commutative and ensure that notation is correctly applied throughout the problem-solving process. ● Fail to apply directionality of vectors correctly from a mathematics perspective (not assigning positive and negative directions). ● Not grasp how the zero and identity matrices are analogous to numbers 0 and 1 ● Understand the difference between speed (a scalar quantity) and velocity (a vector quantity) ● Not understand the relationship between matrices and video/computer game coding commands	**A.SSE.A.2** Use the structure of an expression to identify ways to rewrite it. **F-BF.B.3** Identify the effect on a graph when changing f(x). **A-REI.B.4** Fluently solve quadratic equations in one variable. Math Fluency Support for Grade 11 Targeted Fact reinforcement **Fact Fluency** **Dr. Riccomini's Math Facts – Basic Facts Link** https://www.pattan.net/Videos/Facilitating-Fluency-and-Automaticity-through-Purp **Math Fact Practice** http://www.xtramath.org **Differentiation problems** https://www.openmiddle.com/

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Instructional Best Practices and Exemplars

Instructional Best Practices

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Exemplars and Explanations:

Pre-Calculus Appendix B

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- **Relationship Skills**

https://www.nj.gov/education/safety/wellness/selearning/index.shtml

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Pacing Guide
Pre-Calculus Unit 2 Pacing Guide

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Unit 3 Rational and Exponential Functions			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• **A.APR.C.5** Know and apply the Binomial Theorem for the expansion of (x + y) n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. • **A.APR.D.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	n/a	MP.1 Make sense of problems and persevere in solving them. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concepts: • Understand how applying the Binomial Theorem for the expansion of binomial expressions raised to a certain power is more efficient than FOILing each adjacent pair of expressions • Use Pascal's Triangle as an additional method of binomial expansion where FOIL would be extremely time-consuming • Demonstrate how rational expressions are manipulated and which general properties of equality and algebraic maneuvers apply Students are able to: • apply multiple methods of binomial expansions • understand when using a certain method of expansion is preferred by looking at an analyzing the expression • correctly interpret Pascal's triangle • work with rational expressions to either solve, simplify, or perform standard mathematical operations • demonstrate why certain limitations exist in terms of how a rational expression may be manipulated Learning Goal 1: Apply the Binomial Theorem and/or Pascal's Triangle in order to perform binomial expansions in which the power is sufficiently large to warrant its use Learning Goal 2: Show how to work with rational expressions via simplification, expansion, addition, subtraction, multiplication and division. Show why certain mathematical operations are invalid when working with rational expressions.
• **F.IF.C.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	**8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically.	Concepts: • A function's inverse can be graphed when provided with the original function in tabular, graphical, and algebraic form • Exponential and logarithmic functions are inverses of each other. • Domain and range restrictions in a function often create domain and range restrictions in the inverse of that function

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Unit 3 Rational and Exponential Functions			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• **F.IF.B.4**. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.	that exhibits the qualitative features of a function that has been described verbally. We have learned to/that… • describe qualitatively the functional relationships between two quantities by analyzing a graph • sketch a graph that exhibits the qualitative features of a function given a verbal description **8.F.A.3** Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example*, the function A = s² giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. We have learned to/that… • the equation y = mx + b defines a linear function	MP.6 Attend to precision. MP.7 Look for and make use of structure.	• Domain and range restrictions exist for many rational, exponential, and logarithmic expressions, which often correlate to real-world scenarios that create these limitations Students are able to: • Graph rational, exponential and logarithmic functions, as well as their inverses. • Explain any domain and range restrictions that exist for a given rational, exponential and/or logarithmic function and its inverse. • Algebraically determine the inverse of a given function. • Use the graph of a given function to determine the graph of its inverse. Learning Goal 3: Graph rational, exponential, and logarithmic functions given their equations, and predict the equation of such a function when provided in graphical or tabular form. Learning Goal 4: Determine and/or graph the inverse of a function given key features about the original function.

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Unit 3 Rational and Exponential Functions			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
	• graph linear equations		
	n/a	MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics.	Concepts: • Cavalieri's principle can be used to create a generalized equation for the volume of a square or circular pyramid as well as a sphere Students are able to: • apply Cavalieri's principle to calculate the volume of a 3-D figure given information such as its radius, base, and/or height. • make an informal argument for the formulas of circle area and circumference, and volume of conic sections citing Cavalieri's principle as necessary Learning Goal 5: Explain volume formulas and use them to solve problems.
• **N.CN.C.8.** Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. • **N.CN.C.9.** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	n/a	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.8 Look for and express regularity in repeated reasoning.	Concepts: • Although most polynomials have real solutions, there are many that will have complex solutions, which is detailed in the Fundamental Theorem of Algebra. • In some cases, as the degree of the polynomial gets larger, there will be both real and complex solutions. • If a complex solution of a polynomial is shown to exist, then its conjugate must also be a solution. • Some seemingly prime polynomials in fact have complex solutions. • Complex zeros cannot be graphed on a Cartesian coordinate plane, but may be graphed on a complex plane. Students are able to: • understand and apply the Fundamental Theorem of Algebra, particularly for quadratic polynomials • manipulate polynomials to solve for non-real solutions Learning Goal 6: Use the Fundamental Theorem of Algebra to demonstrate that when the number of real solutions to

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Unit 3 Rational and Exponential Functions			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
			a polynomial function does not match its degree, that there must be some complex solutions. Complex solutions always come in pairs (one solution and its complex conjugate).
• F.BF.A.1. Write a function that describes a relationship between two quantities. F.BF.A.1b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	**8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. We have learned to/that… • construct a function to model a linear relationship between two quantities. interpret the rate of • change and initial value of a function in terms of the situation it models	MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concepts: • Functions of various types can be combined to model real world situations. Students are able to: • use arithmetic operations to combine functions of varying types in order to model relationships between quantities. Learning Goal 7: Construct a function that combines, using arithmetic operations, standard function types to model a relationship between two quantities.

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Unit 3 Rational and Exponential Functions			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• **F.BF.B.4** Find inverse functions. F.BF.B.4a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠ 1.* **F.BF.B.4.b** *Verify by composition that one function is the inverse of another.* **F.BF.B.4.c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.* **F.BF.B.4.d** *Produce an invertible function from a non-invertible function by restricting the domain.* • **F.BF.B.5** Use the inverse relationship between exponents and logarithms to solve problems involving	n/a	MP.1 Make sense of problems and persevere in solving them. MP.6 Attend to precision. MP.8 Look for and express regularity in repeated reasoning.	Concepts: • For a function f(x) that has an inverse, the domain/input for f(x) is the inverse function's range/output and that the range/output for f(x) is the inverse function's domain/input. • Solving for the inverse of a function is algebraically possible. • Using a composite function containing a given function and its purported inverse can prove their inverse relationship as they will cancel each other out. • The graph or table of an invertible function may be used to determine its inverse. • Restricting the domain of an otherwise non-invertible function may produce an inverse that would otherwise not be applicable. • Exponential and logarithmic functions are inverses of each other. Students are able to: • use function notation to represent the inverse of a function – $f^{-1}(x)$ • transform an equation in order to isolate the independent variable recognizing that the domain/input for f(x) is the inverse function's range/output and that the range/output for f(x) is the inverse function's domain/input • apply composite function algebra to prove that two functions are inverses of each other • produce an invertible function by restricting the domain of a non-invertible function • use the inverse relationship between exponents and logarithms in order to solve problems, many of which can have real-world applications Learning Goal 8: Determine the inverse function for a given function (which may be presented graphically, in tabular form, or as an expression).

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Unit 3 Rational and Exponential Functions			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
logarithms and exponents.			

Unit 3 Rational and Exponential Functions	
Overview	
Learners will be introduced to a variety of methods for finding the roots of polynomials beyond synthetic division and long division, both of which were covered in previous courses. These new methods will include applying the Binomial Theorem, Pascal's triangle, and in large part to a conceptual extent, the Fundamental Theorem of Algebra, which will activate and re-contextualize prior knowledge on complex solutions to quadratic and polynomial functions. Learners will also be able to predict or analyze the end behavior of graphs of polynomial functions based on key factors, which in turn allows them to predict the number of roots of each function. Learners will also use common polynomial identities for finding the square roots of complex numbers. Learners will also be tasked with demonstrating the relationships between rational expressions and the set of mathematical operations that specifically apply to them (and how to explain under which conditions some operations do not). Next, learners will begin to apply Cavalieri's principle to derive the formulas for the volume of a sphere and other 3-D geometric solids. A refresher on inverse functions will follow, with function composition, domain, and range being the most important characteristics of interest. Learners will recall the relationship between the domains and ranges of inverse functions. Learners will review how to graph inverse functions in general; finally, learners will use logarithmic and exponential functions (themselves being inverses of each other) to graph and mathematically model real-world phenomena.	
Essential Questions	**Enduring Understandings**
• Does every complex number have a square root? • How does identifying the end behavior of rational functions help solve problems? • How can using the inverse relationship between exponential and logarithmic functions be used to solve problems? • What is the Fundamental Theorem of Algebra? • By which methods can it be determined that a polynomial has complex solutions? • How can complex numbers be graphed? • Does every complex number have a square root? • How does identifying the end behavior of rational functions help solve problems? • How finding Inverse functions of Logarithmic and Exponential functions help solve problems?	• Understand the implications of the Fundamental Theorem of Algebra to apply for complex numbers. • When a complex solution to a polynomial is shown to exist, its complex conjugate must also be a solution for that polynomial. • Creating a composite function of a given function and its supposed inverse may be used to prove (or disprove) their inverse relationship. • Determining the domain restrictions of rational functions can help solve problems and inform what algebraic methods may be used to manipulate the rational function. • Polynomial identities can be used to find the square roots of complex numbers. • Cavalieri's principle is a step towards integral calculus as it is used to derive the formulas for the volume of a sphere and other geometric solids. • Understand the implications of the fundamental theorem of algebra to apply for complex numbers • Compose functions to verify that one fucntion is inverse of another

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	function ● Determining the domian of ratioal functions help solve problems
District/School Formative Assessment Plan	**District/School Summative Assessment Plan**
Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards. Teachers are encouraged to incorporate Formative Assessments into all lessons. During instruction, teachers will collect ongoing information on students' mastery of content through a variety of methods: ● Questioning: using Socratic method, probing questions, a hierarchical system in complexity (Bloom's Taxonomy) ● Exit tickets, rotational activities (stations), quizzes, and small group activities ● Classwork, homework, group work (formative assessment) ● Pre-Assessment, teacher's observation, class discussion, and journal	*Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.* **Benchmark Assessments:** ● Assessment 11.3.1: Mid-Unit Assessment ● Assessment 11.3.2: End of Unit Assessment ● Assessment 11.3.3: End of Unit Performance Assessment **Standardized Assessments:** ● NJSLA **Other Summative Assessments:** Teachers are encouraged to design and their own assessments (topic/module tests and quizzes) individually and/or with their department or grade-level partners, as per Uniform Grading Profile.

Focus Mathematical Concepts

Vocabulary	Common Misconceptions	Fluency
● Root ● Zero ● Solution ● Polynomial ● Unity ● Identity ● Volume ● Cavalieri's principle ● Rational expression ● Complex number ● Complex conjugate ● Composite function ● Inverse function ● Rational function ● Domain ● Range ● Binomial Theorem ● FOIL ● Expansion ● Fundamental Theorem of Algebra ● Exponential ● Logarithmic	Students may: ● See a polynomial that has non-real solutions and assume that it must be prime. ● Not grasp that all complex numbers have square roots even if they don't look calculable. ● Neglect to recall that when a complex root of a polynomial is discovered that its complex conjugate must also be a root of that same polynomial. ● Improperly apply domain restrictions to rational functions (e.g., just setting the denominator equal to zero every time). ● Assume that simplifying rational expressions results in their graphs being identical to the original expressions instead of applying discontinuities. ● Conflate the three types of discontinuities. ● Believe that there are no circumstances under which complex numbers may be graphed since they cannot be graphed on a	**A.SSE.A.2** Use the structure of an expression to identify ways to rewrite it. **F-BF.A.1** Write a function that describes a relationship between two quantities. **F-BF.B.3** Identify the effect on a graph when changing f(x). **F-IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context Math Fluency Support for Grade 11 Targeted Fact reinforcement **Fact Fluency** **Dr. Riccomini's Math Facts – Basic Facts Link** https://www.pattan.net/Videos/Facilitating-Fluency-and-Automaticity-through-Purp **Math Fact Practice**

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• Complex plane	Cartesian coordinate plane alongside real numbers. • Believe that all functions have inverses and need to see counter examples, as well as examples in which a non-invertible function can be made into an invertible function by restricting the domain. For example, $f(x) = x^2$ has an inverse $f^{-1}(x) = \sqrt{x}$ provided that the domain is restricted to $x \geq 0$.	http://www.xtramath.org **Differentiation problems** https://www.openmiddle.com/

District/School Tasks	District/School Primary and Supplementary Resources
• NJDOE Digital Item Library • NJSLA Released Items in Mathematics • NJSLA Mathematics Practice Tests • NJSLA Mathematics Evidence Statements • Progression Tables • GUES Strategy + ECR problems	**District-Mandated Resources** • **Eureka Mathematics** • **i-Ready** • **Zearn** **Assessment Resources:** • A.APR.C.5 Binomial Theorem - Powers of 11 • A.APR.D.7 Operations of Rational Expressions • F.IF.C.7 Graphs of Power Functions • G.GM.A.2 Using Cavalieri's Principle • G.GM.A.3 Volume of a Water Centerpiece • N.CN.C.8 Complex Cubes and Roots of 1 • N.CN.C.9 Fundamental Theorem of Algebra • F.BF.A.1 Crude Oil and Gas Mileage • F.BF.B.4 Inverse Functions - Rainfall • F.BF.B.5 Exponentials and Logarithms **Project Ideas:** • **Teaching Mathematics with Art - Pascal's Triangle** • **Use Cavalieri's Principle to Compare Aquarium Volumes**

Instructional Best Practices and Exemplars

Instructional Best Practices
- Use explicit modeling techniques
- Connect visual representations
- Differentiate Instruction
- Incorporate vocabulary development
- Pose meaningful questions

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- Present tasks that require reasoning and problem solving
- Support productive struggle
- Encourage mathematical discourse
- Collect and analyze student evidence of understanding

Exemplars and Explanations:
Pre-Calculus Appendix B

Integrated Accommodations and Modifications

Special Education	English Language Learners	At Risk
• Utilize modifications & accommodations delineated in the student's IEP • Provide additional manipulatives to support instruction • Allow for alternative strategies to solve algorithms or tasks • Provide the steps needed to complete the task • Model frequently • Provide repetition and practice. • Use visuals to demonstrate/model the processes • Restate, reread, and clarify directions/questions • Ask students to restate information, directions, and assignments. • Provide copy of class notes • Distribute study guide for classroom tests. • Provide preferential seating to be mutually determined by the student and teacher • Provide extra textbooks for home. • Provide regular parent/ school communication • Allow extended time to complete assignment • Establish procedures for accommodations / modifications for assessments • Allow student to take/complete tests in an alternate setting as needed	• WIDA Can Do Descriptors https://wida.wisc.edu/teach/can-do/descriptors • Modify Assignments • Use testing and portfolio assessment • Utilize Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary) • Repeat, rephrase, paraphrase key concepts and directions • Allow for extended time for assignment completion as needed • Highlight key vocabulary • Define essential vocabulary in context • Use graphic organizers, visuals, manipulatives and other concrete materials • Use gestures, facial expressions and body language • Read aloud • Build on what students already know and prior experience	• Pair visual prompts with verbal presentations • Ask students to restate information, directions, and assignments. • Provide repetition and and practice • Model skills / techniques to be mastered. • Provide extended time to complete class work • Provide copy of class notes • Provide preferential seating to be mutually determined by the student and teacher • Allow the use of a computer to complete assignments. • Establish expectations for correct spelling on assignments • Provide extra textbooks for home. • Provide Peer Support • Increase one on one time

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Appendix A: Special Education Accommodations and Modifications		

Gifted and Talented	504 Plan
• Utilize advanced, accelerated, or compacted content • Provide assignments that emphasize higher- level thinking skills. • Allow for individual student interest • Gear assignments to development in areas of affect, creativity, cognition, and research skills • Allow for a variety in types of resources • Provide problem-based assignments with planned scope and sequence • Utilize inquiry-based instruction • Adjust the pace of lessons • Utilize Choice Boards • Provide Problem-Based Learning • Establish flexible Grouping	• Pair visual prompts with verbal presentations • Ask students to restate information, directions, and assignments. • Provide repetition and and practice • Model skills / techniques to be mastered. • Provide extended time to complete class work • Provide copy of class notes • Break long assignments into smaller parts • Assist student in setting short term goals • Allow for preferential seating to be mutually determined by the student and teacher • Provide extra textbooks for home. • Model and reinforce organizational systems (i.e. color-coding) • Write out homework assignments, check student's recording of assignments
Interdisciplinary Connections	**Computer Science and Design Thinking**
English Language Arts • SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. • L.11-12.6. Acquire and use accurate general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression. • W.11-12.10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.	**Computer Science and Design Thinking Practices** • Fostering an Inclusive Computing and Design Culture • Collaborating Around Computing and Design • Recognizing and Defining Computational Problems • Developing and Using Abstractions • Creating Computational Artifacts • Testing and Refining Computational Artifacts • Communicating About Computing and Design **Computer Science and Design Thinking Standards** • 8.1.12.NI.3: Explain how the needs of users and the sensitivity of data determine the level of security implemented. • 8.1.12.NI.4: Explain how decisions on methods to protect data are influenced by whether the data is at rest, in transit, or in use. • 8.1.12.IC.1: Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices. • 8.1.12.IC.3: Predict the potential impacts and implications of emerging technologies on larger social, economic, and political structures, using evidence from credible sources.
Career Readiness, Life Literacies and Key Skills	

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Career Readiness, Life Literacies and Key Skills Practices

- Act as a responsible and contributing community members and employee.
- Attend to financial well-being.
- Consider the environmental, social and economic impacts of decisions.
- Demonstrate creativity and innovation.
- Utilize critical thinking to make sense of problems and persevere in solving them.
- Model integrity, ethical leadership and effective management.
- Plan education and career paths aligned to personal goals.
- Use technology to enhance productivity, increase collaboration and communicate effectively.
- Work productively in teams while using cultural/global competence.

Career Readiness, Life Literacies and Key Skills Standards

9.1 Personal Financial Literacy

- 9.1.12.CFR.1: Compare and contrast the role of philanthropy, volunteer service, and charities in community development and quality of life in a variety of cultures.
- 9.1.12.CFR.2: Summarize causes important to you and compare organizations you seek to support to other organizations with similar missions.
- 9.1.12.CP.1: Summarize how one's credit history can affect finances, including loan terms, employment, and qualifying for loans.
- 9.1.12.CP.2: Identify the advantages of maintaining a positive credit history.
- 9.1.12.CP.3: Summarize factors that affect a positive credit rating, including on-time payments, debt versus available credit, length of open credit, and how often you apply for credit.
- 9.1.12.FP.3: Relate the concept of delayed gratification (i.e., psychological distance) to meeting financial goals, investing and building wealth over time.
- 9.1.12.FP.5: Evaluate how behavioral bias (e.g., overconfidence, confirmation, recency, loss aversion, etc.) affects decision-making.

9.2 Career Awareness, Exploration, Preparation, and Training

- 9.2.12.CAP.1: Analyze unemployment rates for workers with different levels of education and how the economic, social, and political conditions of a time period are affected by a recession.
- 9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.
- 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.

9.4 Life Literacies and Key Skills

21st Century Skills

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.GCA.1: Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others (e.g., SL.11-12.1., HS-ETS1-1, HS-ETS1-2, HS-ETS1-4, 6.3.12.GeoGI.1, 7.1.IH.IPERS.6, 7.1.IL.IPERS.7, 8.2.12.ETW.3).
- 9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

Technology Integration

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- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).

SEL Competencies

- **Self - Awareness**
- **Self - Management**
- **Social Awareness**
- **Responsible Decision Making**
- **Relationship Skills**

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Pacing Guide

Pre-Calculus Unit 3 Pacing Guide

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Unit 4 Trigonometric Functions/Inverses and Probability/Statistics			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• **F.TF.B.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. • **F.TF.B.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. • **F.TF.B.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	n/a	MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concepts: • Periodic functions may model real-world scenarios. Students are able to: • use characteristics of real world phenomena to select a trigonometric model. • identify amplitude, frequency and midline appropriate for the model. Learning Goal 4: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
• **G.SRT.D.9** Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. • **G.SRT.D.10** Prove the Laws of Sines and Cosines and use them to solve problems	n/a	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.6 Attend to precision.	Concepts: No new concept(s) introduced Students are able to: • prove the Pythagorean identity: $\sin^2(\theta) + \cos^2(\theta) = 1$. • use the Pythagorean identity to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ when given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. Learning Goal 5: Use the Pythagorean identity $(\sin\theta)^2 + (\cos\theta)^2 = 1$ to find $\sin\theta$, $\cos\theta$, or $\tan\theta$, given $\sin\theta$, $\cos\theta$, or $\tan\theta$, and the quadrant of the angle.

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Unit 4 Trigonometric Functions/Inverses and Probability/Statistics			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• **G.SRT.D.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		MP.7 Look for and make use of structure.	
• **S.IC.A.1**. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	**6.SP.B.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. We have learned to/that… • display numerical data in plots on a number line, including dot plots, histograms, and box plots	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics.	Concepts: • Statistics is a process for making inferences about a population based on analysis of a random sample from the population. Students are able to: • identify and evaluate random sampling methods. • explain the importance of randomness to sampling and inference making. • explain the difference between values that describe a population and a sample, in context. Learning Goal 2: Identify and evaluate random sampling methods.
• **S.IC.A.2**. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	**6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **6.SP.B.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **6.SP.B.5** Summarize numerical data sets in	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics.	Concepts: • Random processes can be described mathematically by using a model: a list or description of possible outcomes. Students are able to: • determine whether a given model is consistent with results from and experiment. • know the difference between experimental and theoretical modeling. • know how far predictions can be projected based on sample size. • design simulations of random sampling. Learning Goal 3: Determine if the outcomes and properties of a specified model are consistent with results from a given data-generating process (e.g. using simulation).

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Unit 4 Trigonometric Functions/Inverses and Probability/Statistics			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
	relation to their context, such as by giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.		
• S.IC.B.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how	n/a	MP.4 Model with mathematics.	Concepts: • Collecting data from a random sample of a population makes it possible to draw conclusions about the whole population. • Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. • Sample surveys, experiments, and observational studies serve different statistical purposes allowing for different statistical analyses.

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

Unit 4 Trigonometric Functions/Inverses and Probability/Statistics			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
randomization relates to each.			Students are able to: ● distinguish between sample surveys, experiments, and observational studies. ● explain the importance of randomization in each of these processes. ● identify voluntary response samples and convenience samples. ● describe simple random samples, stratified random samples, and cluster samples. ● explain how under coverage, nonresponse, and question wording can lead to bias in a sample survey. Learning Goal 4: Identify the differences among and purposes of sample surveys, experiments, and observational studies, explaining how randomization relates to each.
● **S.CP.A.4.** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. ● **S.CP.A.5**. Recognize and explain the NEW Concepts of conditional probability and independence in everyday language and everyday situations.	n/a	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concepts: ● Two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities. ● Independence of event *A* and event *B* means that the conditional probability of *A* given *B* is the same as the probability of, and the conditional probability of *B* given *A* is the same as the probability of *B*. Students are able to: ● identify events as independent or dependent. ● interpret the conditional probability of A given B as answering the question 'now that B *has* occurred, what is the probability that event A will occur?'. ● determine the conditional probability of *A* given *B* using *P*(*A* and *B*)/*P*(*B*). ● represent conditional probability of *A* given *B* as P(A|B). ● calculate conditional probabilities. ● construct two-way frequency tables for two categorical variables. ● calculate probabilities from the two-way frequency table. ● use the probabilities to assess independence of two variables.

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

Unit 4 Trigonometric Functions/Inverses and Probability/Statistics			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
			Learning Goal 8: Use two-way frequency tables to determine if events are independent and to calculate conditional probability. Use everyday language to explain independence and conditional probability in real-world situations.
• S.CP.B.8 Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. • S.CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.	**7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **7.SP.C.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	Concepts: • Mutually exclusive events exist. Students are able to: • analyze event B's outcomes to determine the proportion of B's outcomes that also belong to event A. • interpret this proportion as conditional probability of A given B. • identify two events as mutually exclusive (disjoint). • calculate probabilities using the Addition rule P(A or B) = P(A) + P(B) – P(A and B). Learning Goal 9: Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* and apply the Addition Rule [*P(A* or *B)* = *P(A)* + *P(B)* – *P(A* and *B)*].
	7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concepts: • Mutually exclusive events exist • Conditional probabilities of two events occurring, whether compound and or mutually exclusive, may be calculated using some form of equation • Real-world scenarios, when modelable and for which probabilities may be assigned empirically, may be analyzed • Using probabilities is conducive to making good, well-informed decisions Students are able to:

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

Unit 4 Trigonometric Functions/Inverses and Probability/Statistics			
Content Standards	**Prerequisite Standards (through Grade 8)**	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
			• analyze event B's outcomes to determine the proportion of B's outcomes that also belong to event A. • interpret this proportion as conditional probability of A given B. • identify two events as mutually exclusive (disjoint). • calculate probabilities using the Addition rule P(A or B) = P(A) + P(B) – P(A and B) • calculate probabilities using the Multiplication Rule P(A and B) = P(A)P(B|A) = P(B)P(A|B) • use probability permutations and combinations to determine the probability of compound events, including solving problems modeling real-world scenarios • Calculate expected value • Create a probability distribution for a random variable and use it to calculate theoretical probabilities and expected value • Make decisions by assigning probabilities to each payoff values and finding expected values • Weigh the possible outcomes assigning • Using probabilities, make fair decisions such as using a random number generator • Analyze decisions and strategies using concepts of probability Learning Goal 9: Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* and apply the Addition Rule [*P(A* or *B) = P(A) + P(B) – P(A* and *B)*]. Learning Goal 10: Find the conditional probability of both *A* and *B* occurring by applying the Addition Rule P(A and B) = P(A)P(B|A) = P(B)P(A|B) . Learning Goal 11: Solve problems, including real-world scenarios like rolling the same number with two separate dice on the same roll, by using probability permutations and combinations.

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Unit 4 Trigonometric Functions/Inverses and Probability/Statistics

Overview

Learners will explore sine, cosine, and tangent functions and their periodicity, using that information to solve problems when a trigonometric function serves as an appropriate model. Studying the graphs of those functions, along with their inverses, is also to be expected of learners in this unit. Various forms of inverse trigonometric functions can be applied to solve real-world problems, such as solving for the angle of elevation or line of sight, and learners will be expected to properly superimpose a right triangle on a sketch or diagram of the scenario in order to achieve this. Learners will also be able to prove the subtraction and addition formulas for sine, cosine, and tangent (and use them to solve problems) by the conclusion of this unit. Lastly, learners will gain knowledge of how to use a select few trigonometric functions that may be applied to non-right triangles, such as the Law of Sines, along with the various applications therein.

The second part of the unit focuses on specific applications of statistics and probability. Learners will be introduced to the three techniques for counting outcomes namely the multiplication rule for independent events, information related to random variables and discrete probability solutions, analyze strategies and make decisions in a variety of contents (one principle example of which is how to make informed fair decisions during gameplay). In addition, learners will apply the Multiplication Rule in a uniform probability model and be able to interpret the answer in terms of the model. Learners will also understand how to calculate and interpret the expected value of a certain event, determine discrete probability distributions, and estimate probability distributions empirically.

Essential Questions	Enduring Understandings
• How do trigonometric functions help find the area of a triangle? • What is an inverse trigonometric function and what purposes can it serve? • How does analyzing the graphs of trigonometric functions and their inverses help solve problems? • How can the addition and subtraction formulas of sine, cosine, and tangent be proven and used to work through problems? • Which trigonometric concepts may be applied to non-right triangles? • What types of probability are there for multiple potentially related events? • What are the three techniques for counting outcomes? • What is a discrete probability distribution? • What is the expected value, how is it calculated, and what can its interpretation potentially be used for? • What is a random variable and how is its probability distribution determined? • What is the Multiplication Rule of uniform probability? • How can probability be used to inform good decisions during gameplay? • How can probability functions be used to model real-world scenarios? • How does Trigonometric functions help find the area of a triangle? • How does graphs of Trigonometric and Inverses functions helpe solve problems? • How deos Determining Discrete Probability Distributions help solve	• Trigonometric functions have periodicity, making them uniquely applicable for a certain subset of problems that can be modeled as such. • Trigonometric functions can be used to find the area of a triangle. • Graphs of trigonometric functions can help model and solve problems. • Inverse trigonometric functions have utility as well; one of the most common uses of inverse trigonometric functions is to calculate an unknown angle in a right triangle given other necessary information. • There are trigonometric identities, as well as addition and subtraction formulas of sine, cosine, and tangent, which can be proven and may help with solving certain types of problems. • Certain trigonometric concepts and formulas, such as the Law of Sines, may be applied to non-right triangles. • Statistics is a process for making inferences about population parameters based on a random population sample. • There are three techniques for counting statistical outcomes, and there are a variety of ways to present this information in graphical, tabular, or other forms. • A discrete probability distribution is one in which there are not infinite possible outcomes to an event, but instead a finite number of outcomes. • There exists a Multiplication Rule for uniform probability that mathematically determines the probability of two events both occurring. • Expected value represents the likely outcome of a given scenario; it must be calculated by summing the multiples of all outcomes and their respective probabilities in a discrete probability distribution.

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

problems?	• Interpreting the expected value of a given set can be helpful in making good, informed decisions during gameplay that involves chance, such as rolling dice or playing a card game. • Probability functions are used to model certain real-world scenarios. • Students determine viewing angle, line of sight, height of objects, and angle of elevation for inclined surfaces using inverse trigonometric functions and their periodic phenomena. • Survey problems and elevation problems can be solved using vectors • For situations where the probabilities associated with a discrete random variable can be calculated given a description of the random variable, students determine the probability distribution.
District/School Formative Assessment Plan	**District/School Summative Assessment Plan**
Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards. Teachers are encouraged to incorporate Formative Assessments into all lessons. During instruction, teachers will collect ongoing information on students' mastery of content through a variety of methods: • Questioning: using Socratic method, probing questions, a hierarchical system in complexity (Bloom's Taxonomy) • Exit tickets, rotational activities (stations), quizzes, and small group activities • Classwork, homework, group work (formative assessment) • Pre-Assessment, teacher's observation, class discussion, and journal	*Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.* **Benchmark Assessments:** • Assessment 11.4.1: Mid-Unit Assessment • Assessment 11.4.2: End of Unit Assessment • Assessment 11.4.3: End of Unit Performance Assessment **Standardized Assessments:** • NJSLA **Other Summative Assessments:** Teachers are encouraged to design and their own assessments (topic/module tests and quizzes) individually and/or with their department or grade-level partners, as per Uniform Grading Profile.

Focus Mathematical Concepts

Vocabulary	Common Misconceptions	Fluency
• Trigonometric function • Inverse trigonometric functions • Periodicity • Angle of elevation • Viewing angle • Line of sight • Survey • Elevation • Unit circle • Law of Sines • Law of Cosines • Right triangles	Students may: • Not understand the notation of inverse trigonometric functions • Incorrectly apply inverse trigonometric functions in order to solve problems • Assume that trigonometric functions can only ever apply to right triangles • Not understand how the periodicity of trigonometric functions and their inverses manifests graphically • Have trouble correctly determining the period, midline, amplitude, and/or phase shift	**A.SSE.A.2** Use the structure of an expression to identify ways to rewrite it. **F-BF.B.3** Identify the effect on a graph when changing f(x). **F-TF.B.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

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• Non-right triangles • Fundamental Counting Principle • Permutations • Combinations • Multiplication Rule • Discrete variable • Discrete probability distribution • Random variable • Expected value • Empirical estimations	of a trigonometric function given its graph or equation. • Believe making decisions is simply comparing the value of one observation of a sample statistic to the value of a population parameter, not realizing that a distribution of the sample statistic needs to be created. • Try to match expected value with actual discrete values in a discrete probability distribution, even if they don't match (e.g., rounding the expected value of a 6-sided die roll from 3.5 to 4). • Believe that causal effect can be drawn in surveys and observational studies, instead of understanding that causality is in fact a property of experiments. • Believe that independence of events and mutually exclusive events are the same thing. • Believe that the probability of *A* or *B* is always the sum of the two events individually, or that the probability of *A* and *B* is the product of the two events individually, not realizing that one of the probabilities may be conditional.	**S-CP.B.9** Use permutations and combinations to compute probabilities of compound events and solve problems. **S-M.B.5** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. Math Fluency Support for Grade 11 Targeted Fact reinforcement **Fact Fluency** **Dr. Riccomini's Math Facts – Basic Facts Link** https://www.pattan.net/Videos/Facilitating-Fluency-and-Automaticity-through-Purp **Math Fact Practice** http://www.xtramath.org **Differentiation problems** https://www.openmiddle.com/

District/School Tasks	District/School Primary and Supplementary Resources
• NJDOE Digital Item Library • NJSLA Released Items in Mathematics • NJSLA Mathematics Practice Tests • NJSLA Mathematics Evidence Statements • Progression Tables • GUES Strategy + ECR problems	**District-Mandated Resources** • **Eureka Mathematics** • **i-Ready** • **Zearn** **Assessment Resources:** • S.ID.A.4 Using Mean, Standard Deviation to Fit a Normal Distribution • S.IC.A.1 Understanding Uses of Statistics • S.IC.A.2 Comparing Models to Real-World Probability Distributions • S.IC.B.3 Surveys, Experiments, Observational Studies • S.CP.B.8 Multiplication Rule • S.CP.B.9 Margin of Error for Estimating a Population Mean • S.M.A.3 Probability of Independent Events • S.M.A.4 Random Variables

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	• S.CP.A.3 Modeling Probability • S.CP.A.4 Two-Way Frequency Tables • S.M.B.5 Weighing Outcomes • S.M.B.6 Making Fair Decisions • S.M.B.7 Bob's Bagel Shop **Project Ideas:** • **Casino Royale** • **Creating a Random Number Table**

Instructional Best Practices and Exemplars

Instructional Best Practices

- Use explicit modeling techniques
- Connect visual representations
- Differentiate Instruction
- Incorporate vocabulary development
- Pose meaningful questions
- Present tasks that require reasoning and problem solving
- Support productive struggle
- Encourage mathematical discourse
- Collect and analyze student evidence of understanding

Exemplars and Explanations:

Pre-Calculus Appendix B

Integrated Accommodations and Modifications

Special Education	English Language Learners	At Risk
• Utilize modifications & accommodations delineated in the student's IEP • Provide additional manipulatives to support instruction • Allow for alternative strategies to solve algorithms or tasks • Provide the steps needed to complete the task • Model frequently • Provide repetition and practice. • Use visuals to demonstrate/model the processes • Restate, reread, and clarify directions/questions	• WIDA Can Do Descriptors https://wida.wisc.edu/teach/can-do/descriptors • Modify Assignments • Use testing and portfolio assessment • Utilize Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary) • Repeat, rephrase, paraphrase key concepts and directions • Allow for extended time for assignment completion as needed • Highlight key vocabulary • Define essential vocabulary in context	• Pair visual prompts with verbal presentations • Ask students to restate information, directions, and assignments. • Provide repetition and and practice • Model skills / techniques to be mastered. • Provide extended time to complete class work • Provide copy of class notes • Provide preferential seating to be mutually determined by the student and teacher • Allow the use of a computer to complete assignments.

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

• Ask students to restate information, directions, and assignments. • Provide copy of class notes • Distribute study guide for classroom tests. • Provide preferential seating to be mutually determined by the student and teacher • Provide extra textbooks for home. • Provide regular parent/ school communication • Allow extended time to complete assignment • Establish procedures for accommodations / modifications for assessments • Allow student to take/complete tests in an alternate setting as needed Appendix A: Special Education Accommodations and Modifications	• Use graphic organizers, visuals, manipulatives and other concrete materials • Use gestures, facial expressions and body language • Read aloud • Build on what students already know and prior experience	• Establish expectations for correct spelling on assignments • Provide extra textbooks for home. • Provide Peer Support • Increase one on one time

Gifted and Talented	504 Plan
• Utilize advanced, accelerated, or compacted content • Provide assignments that emphasize higher- level thinking skills. • Allow for individual student interest • Gear assignments to development in areas of affect, creativity, cognition, and research skills • Allow for a variety in types of resources • Provide problem-based assignments with planned scope and sequence • Utilize inquiry-based instruction • Adjust the pace of lessons • Utilize Choice Boards • Provide Problem-Based Learning • Establish flexible Grouping	• Pair visual prompts with verbal presentations • Ask students to restate information, directions, and assignments. • Provide repetition and and practice • Model skills / techniques to be mastered. • Provide extended time to complete class work • Provide copy of class notes • Break long assignments into smaller parts • Assist student in setting short term goals • Allow for preferential seating to be mutually determined by the student and teacher • Provide extra textbooks for home. • Model and reinforce organizational systems (i.e. color-coding) • Write out homework assignments, check student's recording of assignments

Interdisciplinary Connections	Computer Science and Design Thinking
English Language Arts • SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.	**Computer Science and Design Thinking Practices** • Fostering an Inclusive Computing and Design Culture • Collaborating Around Computing and Design • Recognizing and Defining Computational Problems • Developing and Using Abstractions

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

• L.11-12.6. Acquire and use accurate general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression. • W.11-12.10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.	• Creating Computational Artifacts • Testing and Refining Computational Artifacts • Communicating About Computing and Design **Computer Science and Design Thinking Standards** • 8.1.12.NI.3: Explain how the needs of users and the sensitivity of data determine the level of security implemented. • 8.1.12.NI.4: Explain how decisions on methods to protect data are influenced by whether the data is at rest, in transit, or in use. • 8.1.12.IC.1: Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices. • 8.1.12.IC.3: Predict the potential impacts and implications of emerging technologies on larger social, economic, and political structures, using evidence from credible sources.

Career Readiness, Life Literacies and Key Skills

Career Readiness, Life Literacies and Key Skills Practices

- Act as a responsible and contributing community members and employee.
- Attend to financial well-being.
- Consider the environmental, social and economic impacts of decisions.
- Demonstrate creativity and innovation.
- Utilize critical thinking to make sense of problems and persevere in solving them.
- Model integrity, ethical leadership and effective management.
- Plan education and career paths aligned to personal goals.
- Use technology to enhance productivity, increase collaboration and communicate effectively.
- Work productively in teams while using cultural/global competence.

Career Readiness, Life Literacies and Key Skills Standards

9.1 Personal Financial Literacy

- 9.1.12.CFR.1: Compare and contrast the role of philanthropy, volunteer service, and charities in community development and quality of life in a variety of cultures.
- 9.1.12.CFR.2: Summarize causes important to you and compare organizations you seek to support to other organizations with similar missions.
- 9.1.12.CP.1: Summarize how one's credit history can affect finances, including loan terms, employment, and qualifying for loans.
- 9.1.12.CP.2: Identify the advantages of maintaining a positive credit history.
- 9.1.12.CP.3: Summarize factors that affect a positive credit rating, including on-time payments, debt versus available credit, length of open credit, and how often you apply for credit.
- 9.1.12.FP.3: Relate the concept of delayed gratification (i.e., psychological distance) to meeting financial goals, investing and building wealth over time.
- 9.1.12.FP.5: Evaluate how behavioral bias (e.g., overconfidence, confirmation, recency, loss aversion, etc.) affects decision-making.

9.2 Career Awareness, Exploration, Preparation, and Training

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

- 9.2.12.CAP.1: Analyze unemployment rates for workers with different levels of education and how the economic, social, and political conditions of a time period are affected by a recession.
- 9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.
- 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.

9.4 Life Literacies and Key Skills

21st Century Skills

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.GCA.1: Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others (e.g., SL.11-12.1., HS-ETS1-1, HS-ETS1-2, HS-ETS1-4, 6.3.12.GeoGI.1, 7.1.IH.IPERS.6, 7.1.IL.IPERS.7, 8.2.12.ETW.3).
- 9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

Technology Integration

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).

SEL Competencies

- **Self - Awareness**
- **Self - Management**
- **Social Awareness**
- **Responsible Decision Making**
- **Relationship Skills**

https://www.nj.gov/education/safety/wellness/selearning/index.shtml

Pacing Guide

Pre-Calculus Unit 4 Pacing Guide

Appendix A: Accommodations and Modifications

Appendix A: Accommodations and Modifications: Unit 1

Accomodation:

- Visual Supports: Use visual aids such as diagrams of the complex plane, matrix grids, and geometric transformations to support comprehension.

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

- Step-by-Step Instructions: Provide written and oral step-by-step guides for algebraic operations with complex numbers and matrix calculations.
- Extended Time: Allow extra time for problem-solving and completing assignments or assessments.
- Manipulatives and Technology: Incorporate graphing calculators, interactive software (like GeoGebra or Desmos), and physical manipulatives (e.g., colored tiles for matrices).
- Simplified Language: Use simplified language or glossaries for mathematical terminology (e.g., “imaginary number,” “determinant”).
- Frequent Checks for Understanding: Pause regularly during lessons to ask questions or summarize key points.
- Peer Support or Small Group Instruction: Offer collaborative opportunities with peers or small group sessions for personalized help.
- Formula Sheets: Provide reference sheets containing key formulas for matrix operations and complex numbers.
- Audio and Video Resources: Use videos or audio recordings that explain complex numbers and matrices visually and verbally.
- Break Tasks into Smaller Steps: Scaffold learning by breaking complex problems into manageable parts.

Modifications:

- Reduced Complexity: Limit the number of steps required in multi-step problems (e.g., focus on addition and subtraction of complex numbers before multiplication or division).
- Lowered Cognitive Load: Assign simpler problems focusing on basic concepts rather than advanced applications or proofs.
- Use of Calculators or Software: Allow use of calculators or software for complex matrix multiplications or complex number operations.
- Alternative Assessments: Permit oral explanations, projects, or visual presentations instead of written tests.
- Focus on Conceptual Understanding: Emphasize understanding of graphical representation and real-life applications rather than rigorous algebraic proofs.
- Selective Content Coverage: Prioritize key essential questions (e.g., fundamental operations and interpretations) and defer advanced topics like determinants and transformations if needed.

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

- Provide Partially Completed Problems: Give students problems with some steps completed to guide them through the process.
- Use Real-Life Contexts: Adapt problems to familiar contexts or practical applications to increase relevance and engagement.
- Allow for Formula Usage: Provide formula sheets or allow formula reference during assessments to reduce memorization demands.
- Extended Scaffolding: Offer additional guided practice and simplified examples before moving to independent work.

Appendix A: Accommodations and Modification: Unit 2

Accommodations:

- Visual Representations: Use diagrams, graphs, and animations to illustrate vectors, matrices, and transformations.
- Step-by-Step Guidance: Provide detailed written and verbal instructions for matrix operations and vector calculations.
- Manipulatives and Technology: Incorporate graphing calculators, software tools (e.g., GeoGebra, Desmos), and physical models for hands-on learning.
- Simplified Terminology: Provide glossaries or simplified definitions of terms like “parametric equation,” “identity matrix,” and “linear transformation.”
- Chunking Content: Break lessons into smaller segments with frequent pauses for review and questions.
- Extra Time: Allow additional time for completing tasks and assessments.
- Frequent Check-ins: Use formative assessments, quizzes, or quick checks to ensure understanding.
- Collaborative Learning: Use peer tutoring or small group discussions to reinforce concepts.
- Formula and Reference Sheets: Provide sheets with key formulas and matrix operation rules.
- Multimodal Instruction: Use videos, audio explanations, and interactive simulations to cater to different learning styles.
- Alternative Response Modes: Allow students to explain reasoning orally, use drawings, or demonstrate understanding through projects.

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

Modifications:

- Simplify Problems: Focus on basic vector and matrix operations before introducing more complex concepts like inverse matrices or parametric equations.
- Reduce Scope: Limit the number of concepts per lesson; for example, teach vector addition and scalar multiplication separately before combined applications.
- Use Calculator/Software for Computations: Allow use of graphing calculators or software for matrix multiplication or solving systems.
- Lower Expectations for Complexity: Assign simpler problems with fewer steps or components.
- Provide Partially Completed Examples: Give problems with initial steps done to guide students.
- Alternative Assessments: Replace written tests with projects, presentations, or oral explanations.
- Focus on Conceptual Understanding: Prioritize real-life applications and visual understanding over detailed algebraic proofs.
- Limit Abstract Content: Postpone or minimize focus on notation distinctions (e.g., right- and left-handed matrix notation) if challenging.
- Provide Additional Scaffolding: Use guided notes or graphic organizers to structure learning.
- Real-World Context Emphasis: Adapt tasks to familiar or practical scenarios like simple computer games or motion problems.

Appendix A: Accommodations and Modification: Unit 3

Accommodations:

- Visual Aids and Graphing Tools: Use graphing calculators, software (Desmos, GeoGebra), or interactive simulations to visualize complex numbers and rational function behavior.
- Step-by-Step Instruction: Break down problem-solving processes, such as finding inverses or identifying end behavior, into clear, manageable steps.
- Glossaries and Concept Maps: Provide key vocabulary lists and concept maps highlighting relationships between complex numbers, polynomials, and functions.

Key: Major Clusters Supporting Additional Clusters *Benchmarked Standard

- Use of Manipulatives: Incorporate algebra tiles or color-coded diagrams to represent complex roots or polynomial factors.
- Extra Time: Allow extended time for processing abstract concepts and completing assignments.
- Frequent Checks for Understanding: Use formative questions and quick assessments to clarify misconceptions early.
- Multimodal Learning: Supplement lectures with videos, written notes, and hands-on activities.
- Calculator/Technology Use: Permit use of calculators or software for computations involving complex numbers or graphing rational functions.
- Provide Worked Examples: Give students fully worked-out problems that model solving for inverses, graphing, and identifying roots.
- Small Group Instruction: Provide opportunities for targeted small group or peer tutoring focusing on challenging concepts.
- Alternative Expression: Allow students to explain answers orally, use drawings, or present solutions using technology.

Modifications:

- Simplify Content Scope: Focus initially on identifying square roots of simple complex numbers before progressing to general proofs or abstract concepts.
- Reduce Problem Complexity: Assign problems with fewer terms or less complicated polynomials and functions.
- Limit Abstract Definitions: Postpone or minimize detailed proof requirements, focusing instead on application and intuition.
- Use Concrete Examples: Emphasize real-world or visual examples rather than purely symbolic or theoretical problems.
- Provide Partially Completed Problems: Supply partially worked problems to reduce cognitive load.
- Alternative Assessment: Replace lengthy tests with project-based or oral presentations emphasizing conceptual understanding.
- Allow Calculator/Software Use: Encourage reliance on technology for complex computations or graphing tasks.
- Adjust Expectations: Focus on mastery of foundational ideas rather than all procedural steps.
- Focus on Conceptual Connections: Emphasize relationships (e.g., inverse functions undo each other) rather than exhaustive symbolic manipulation.
- Provide Graphic Organizers: Use charts or organizers to visually connect concepts like polynomial roots, the Fundamental Theorem of Algebra, and inverse functions.

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Appendix A: Accommodations and Modification: Unit 4

Accommodations:

- Visual Supports: Use diagrams, unit circle charts, and graphing tools to illustrate triangles, trig functions, and their graphs.
- Step-by-Step Guides: Provide detailed instructions for using formulas like addition/subtraction identities and probability calculations.
- Technology Integration: Allow use of graphing calculators, apps, or software (e.g., Desmos) to explore trig graphs and probability distributions.
- Vocabulary Supports: Offer glossaries or word banks for key terms like “inverse function,” “random variable,” and “expected value.”
- Hands-on Activities: Use physical models (triangle manipulatives) or card/coin games to demonstrate probability concepts and counting methods.
- Extra Processing Time: Give extended time for solving multi-step trig or probability problems.
- Chunked Information: Break lessons into smaller parts, focusing separately on trigonometry, probability, and their applications.
- Frequent Checks: Use formative questions to ensure understanding before moving forward.
- Multiple Modalities: Supplement lectures with videos, visual aids, and verbal explanations.
- Small Group Support: Provide guided practice or tutoring for students who need reinforcement.

Modifications:

- Simplify Problem Complexity: Use problems with fewer steps or simpler numbers (e.g., common angles, smaller sample spaces).
- Reduce Content Depth: Focus on understanding core concepts (like what inverse trig functions represent) without requiring full proofs of formulas.
- Limit Abstract Reasoning: De-emphasize formal derivations, instead using applied examples and visual reasoning.

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- Use Concrete Examples: Prioritize real-life applications (e.g., using trig to find distances or angles in practical settings) over abstract problems.
- Provide Partially Completed Work: Give students partial solutions or formula sheets to reduce cognitive load.
- Allow Technology Use: Encourage calculators or software to perform calculations or graphing.
- Alternative Assessments: Use projects, oral explanations, or visual presentations instead of formal tests.
- Focus on Conceptual Understanding: Emphasize how trig and probability models solve real problems rather than requiring mastery of every formula.
- Scaffold Language: Simplify language in instructions and explanations to support comprehension.
- Visual Organizers: Use charts or diagrams to connect probability rules, distributions, and trig function relationships.

Appendix B: Instructional Best Practices and Exemplars:

Appendix B: Instructional Best Practices and Exemplars: Unit 1

Sample Activities:

- **Complex Number Exploration**
  Students graph complex numbers on the complex plane and practice performing arithmetic operations (addition, subtraction, multiplication, division) using both algebraic and geometric representations.
- **Polar and Exponential Form Conversion**
  Students convert complex numbers between rectangular form and polar/exponential form, then use De Moivre's Theorem to find powers and roots of complex numbers.
- **Modeling with Exponential and Logarithmic Functions**
  Students analyze and create real-world models using exponential growth and decay functions, interpreting parameters and making predictions based on function behavior.
- **Vector Operations and Applications**
  Students perform operations on vectors (addition, scalar multiplication, dot product), then apply these to solve geometric problems and model

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situations such as forces or velocity.

- **Analyzing and Interpreting Function Behavior**
  Students study families of functions, including logarithmic, exponential, and trigonometric, focusing on transformations, end behavior, and inverse functions, using graphs and tables.

Sample Exemplar:

- **Graphing and Interpreting Complex Numbers**
  Represent complex numbers on the complex plane and use graphical methods to perform operations like addition, subtraction, multiplication, and division.

- **Using De Moivre's Theorem for Roots and Powers**
  Apply De Moivre's Theorem to compute powers and roots of complex numbers expressed in polar form and interpret the results geometrically.

- **Modeling Real-World Exponential Growth and Decay**
  Construct and analyze exponential models to describe phenomena such as population growth or radioactive decay, including interpreting parameters and predicting future values.

- **Solving Systems of Equations Involving Quadratics and Exponentials**
  Solve systems algebraically and graphically where one or more equations are quadratic or exponential, demonstrating connections between algebraic solutions and graphical representations.

- **Analyzing Vector Operations and Parametric Equations**
  Perform operations on vectors and use parametric equations to model and analyze motion or other applications in the plane, interpreting the results in context.

Appendix B: Instructional Exemplars and Explanations: Unit 2

Sample Activities:

- **Vector Addition and Scalar Multiplication Exploration**
  Students use physical or digital tools to add and scale vectors graphically, then verify results algebraically.

- **Dot Product Applications**
  Calculate the dot product of vectors to determine angles between them and explore projections in geometric contexts.

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- **Modeling Motion Using Vectors**
  Analyze real-world motion problems by representing velocity and displacement as vectors, and solve for resultant vectors.

- **Solving Systems Involving Vectors Algebraically**
  Use systems of equations to solve problems involving vectors, such as finding unknown components or points of intersection.

- **Geometric Interpretation of Vector Equations**
  Graph vector equations in the coordinate plane and interpret their geometric meaning, including lines and planes represented by vectors.

Sample Exemplar:

- Demonstrate vector addition and scalar multiplication both graphically and algebraically to solve geometric problems.
- Use the dot product to find the angle between two vectors and determine orthogonality in applied contexts.
- Model real-world scenarios involving motion and forces by representing quantities as vectors and performing operations on them.
- Solve systems of linear equations involving vectors and interpret solutions in terms of geometric intersections.
- Analyze and graph vector-valued functions, including parametric equations, to explore motion along a plane or in space.

Appendix B: Instructional Best Practices and Exemplars: Unit 3

Sample Activities:

- Practice polynomial division and factorization through synthetic division and long division to simplify complex expressions.
- Investigate the zeros of polynomial functions by graphing and analyzing their multiplicities and behaviors at roots.
- Explore transformations of functions by composing, decomposing, and finding inverses, focusing on function operations and their effects on graphs.
- Solve problems involving complex numbers, including powers and roots of complex numbers expressed in trigonometric form.
- Model real-world scenarios using exponential and logarithmic functions, including solving equations involving these functions and interpreting their solutions.

Sample Exemplar:

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- Demonstrate the use of polynomial division (long and synthetic) to factor higher-degree polynomials and find zeros, including interpreting remainders.
- Apply De Moivre's Theorem to find powers and roots of complex numbers in trigonometric form, and connect these to graphing transformations.
- Analyze functions by interpreting average rates of change over intervals and connecting this to function behavior and graph shapes.
- Model real-world phenomena using exponential, logarithmic, and other function types, including composing and decomposing functions to interpret results.
- Prove and apply function properties such as inverses and compositions, and investigate how transformations affect function graphs.

Appendix B: Instructional Best Practices and Exemplars: Unit 4

Sample Activities:

- **Data Collection and Simulation:** Students design and conduct a simple experiment or survey, then use simulation techniques to model the probability of outcomes and analyze results.
- **Conditional Probability Exploration:** Using real-world scenarios, students calculate conditional probabilities and create tree diagrams to visualize and solve problems involving dependent events.
- **Confidence Interval Construction:** Students collect sample data, calculate sample statistics, and build confidence intervals to estimate population parameters, discussing margin of error and interpretation.
- **Hypothesis Testing Practice:** Students perform basic hypothesis tests using simulated data sets to determine if results support or refute claims, applying significance levels.
- **Compound Event Analysis:** Students investigate combined events involving unions and intersections, using Venn diagrams and probability rules to compute probabilities of complex events.

Sample Exemplar:

- Construct and interpret a two-way frequency table to analyze the relationship between two categorical variables.
- Design and conduct a simulation to estimate probabilities for a complex event involving multiple dependent or independent events.
- Use sampling methods to collect data and develop confidence intervals to estimate population parameters with a given level of confidence.

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- Perform and interpret hypothesis tests for population proportions, including calculating p-values and making decisions based on significance levels.
- Analyze compound events using probability rules, including conditional probability and independence, supported by tree diagrams and Venn diagrams.

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