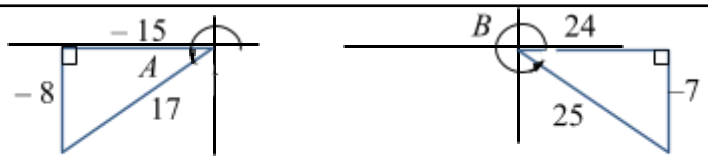
`Secondary 4 Express/ 5 Normal Academic Preliminary Examination 2020
[Additional Mathematics] Paper 1
Marking Scheme

No	Solution	Remark / Notes
1 (a) (i)	 $\tan(-B) = -\tan B$ $= -\left(\frac{-7}{24}\right)$ $= \frac{7}{24}$	
1 (a) (ii)	$\cos(90° - B) = \sin B$ $= -\frac{7}{25}$	
1 (a) (iii)	$\cos A = 2\cos^2\left(\frac{A}{2}\right) - 1$ $-\frac{15}{17} = 2\cos^2\left(\frac{A}{2}\right) - 1$ $2\cos^2\left(\frac{A}{2}\right) = 1 - \frac{15}{17}$ $\cos^2\left(\frac{A}{2}\right) = \frac{1}{17}$ $\cos\left(\frac{A}{2}\right) = \pm\frac{\sqrt{17}}{17}$ Since $180° < \cos A < 270°$, $\square$ $90° < \cos\left(\frac{A}{2}\right) < 135°$, $\frac{A}{2}$ is in the 2nd Quadrant $\square$ $\cos\left(\frac{A}{2}\right) < 0$ $\cos\left(\frac{A}{2}\right) = -\frac{\sqrt{17}}{17}$ **(reject positive answer)**	

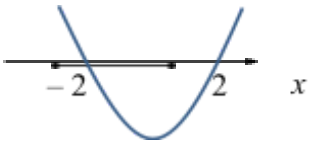
1 (b)	$\frac{\cos(A+B)}{\cos(A-B)} = \frac{2}{3}$ $\frac{\cos A\cos B - \sin A\sin B}{\cos A\cos B + \sin A\sin B} = \frac{2}{3}$ $3\cos A\cos B - 3\sin A\sin B = 2\cos A\cos B + 2\sin A\sin B$ $\cos A\cos B = 5\sin A\sin B$ $\frac{1}{5} = \frac{\sin A\sin B}{\cos A\cos B}$ $\tan A\tan B = \frac{1}{5}$	
2 (a)	$\lvert 2x^2 - 15 \rvert = -x$ $2x^2 - 15 = -x$ or $2x^2 - 15 = -(-x)$ $2x^2 + x - 15 = 0$ $\quad 2x^2 - x - 15 = 0$ $(x+3)(2x-5) = 0$ $\quad (x-3)(2x+5) = 0$ $x = -3$ or $2\frac{1}{2}$ $\quad x = 3$ or $-2\frac{1}{2}$ (reject) $\quad$ (reject) $\therefore x = -3$ or $-2\frac{1}{2}$	
2 (b)	$3^{3x-2} \times 12^{x-1} \div 2^{-2x+1} = 5$ $3^{3x-2} \times (3\times 2^2)^{x-1} \div 2^{-2x+1} = 5$ $3^{3x-2} \times 3^{x-1} \times 2^{2x-2} \div 2^{-2x+1} = 5$ $3^{4x-3} \times 2^{4x-3} = 5$ $6^{4x-3} = 5$ Taking lg on both sides, $(4x-3)\lg 6 = \lg 5$ $4x - 3 = \frac{\lg 5}{\lg 6}$ $4x = 3.89824$ $x = 0.975\ (3sf)$	
2 (c) (i)	$x^2 = 3x - 8$ Let $x = \alpha$, $\alpha^2 = 3\alpha - 8$ $\quad$ ---- (1) $\times\alpha$, $\quad \alpha^3 = 3\alpha^2 - 8\alpha$ sub (1), $\quad \alpha^3 = 3(3\alpha - 8) - 8\alpha$ $\alpha^3 = 9\alpha - 24 - 8\alpha$ $\alpha^3 = \alpha - 24$ $\quad$ (shown)	
2 (c) (ii)	$\alpha + \beta = 3$ $\quad$ and $\quad \alpha\beta = 8$	

	$\frac{\alpha-1}{\beta}+\frac{\beta-1}{\alpha}$ $=\frac{\alpha^2-\alpha+\beta^2-\beta}{\alpha\beta}$ $=\frac{\alpha^2+\beta^2-(\alpha+\beta)}{\alpha\beta}$ $=\frac{\left[(\alpha+\beta)^2-2\alpha\beta\right]-(\alpha+\beta)}{\alpha\beta}$ $=\frac{\left[(3)^2-2(8)\right]-(3)}{8}$ $=-\frac{5}{4}$ $\left(\frac{\alpha-1}{\beta}\right)\left(\frac{\beta-1}{\alpha}\right)$ $=\frac{\alpha\beta-\alpha-\beta+1}{\alpha\beta}$ $=\frac{8-(3)+1}{8}$ $=\frac{3}{4}$ Equation : $x^2+\frac{5}{4}x+\frac{3}{4}=0$ or $4x^2+5x+3=0$	
3 (a)	$\int_0^5 f(x)\,dx$ $=\int_0^2 f(x)\,dx+\int_2^5 f(x)\,dx$ $=4+12$ $=16$	
3 (b)	$\int_0^2\left[f(x)+mx^2\right]dx=\int_5^2 f(x)\,dx$ $\int_0^2 f(x)\,dx+\int_0^2 mx^2\,dx=-\int_2^5 f(x)\,dx$ $4+\left[\frac{mx^3}{3}\right]_0^2=-12$ $4+\left(\frac{8m}{3}-0\right)=-12$ $\frac{8}{3}m=-16$ $m=-6$	

4 (a)	$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$	
4 (b)	$a = \frac{8-(-2)}{2} = 5$ $b = \frac{2\pi}{8\pi} = \frac{1}{4}$ $c = 8 - 5 = 3$	
(i)	LHS $= \frac{1-\sin x\cos x+2\cos^2 x}{\sin^2 x}$ $= \frac{1}{\sin^2 x} - \frac{\cos x}{\sin x} + \frac{2\cos^2 x}{\sin^2 x}$ $= \cos ec^2 x - \cot x + 2\cot^2 x$ $= (1+\cot^2 x) - \cot x + 2\cot^2 x$ $= 3\cot^2 x - \cot x + 1$ $= RHS$	
OR	RHS $= 3\cot^2 x - \cot x + 1$ $= \frac{3\cos^2 x}{\sin^2 x} - \frac{\cos x}{\sin x} + 1$ $= \frac{3\cos^2 x - \cos x\sin x + \sin^2 x}{\sin^2 x}$ $= \frac{2\cos^2 x + \cos^2 x + \sin^2 x - \cos x\sin x}{\sin^2 x}$ $= \frac{2\cos^2 x + 1 - \cos x\sin x}{\sin^2 x}$ $= LHS$	
5 (ii)	$\frac{1-\sin x\cos x+2\cos^2 x}{\sin^2 x} = 3$ $3\cot^2 x - \cot x - 2 = 0$ $(\cot x - 1)(3\cot x + 2) = 0$ $\cot x = 1$ or $\cot x = -\frac{2}{3}$ $\tan x = 1$ $\quad\quad$ $\tan x = -\frac{3}{2}$ $\alpha = \frac{\pi}{4}$ $\quad\quad$ $\alpha = 0.98279$	

	$x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$ $\qquad\qquad$ $x = 2.16$ or 5.30 (3sf) $\therefore x = \frac{\pi}{4}$, 2.16, $\frac{5\pi}{4}$ or 5.30	

6 (a)	$f(x) = 3x^3 + ax^2 - 10x + b$ $f(4) = 0$ $3(4)^3 + a(4)^2 - 10(4) + b = 0$ $16a + b = -152$ ---- (1) $f(-1) = 5$ $3(-1)^3 + a(-1)^2 - 10(-1) + b = 5$ $a + b = -2$ ---- (2) (1) – (2), $15a = -150$ $a = -10$ sub. into (2), $-10 + b = -2$ $b = 8$	
6 (b)	$x - 4$) $3x^2 + 2x - 2$ $3x^3 - 10x^2 - 10x + 8$ $-(3x^3 - 12x^2)$ $2x^2 - 10x$ $-(2x^2 - 8x)$ $-2x + 8$ $-(-2x + 8)$ 0 $\therefore 3x^3 - 10x^2 - 10x + 8 = 0$ $(x - 4)(3x^2 + 2x - 2) = 0$ $\therefore x = 4$ or $x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-2)}}{2(3)}$ $= \frac{-1 \pm \sqrt{7}}{3}$ $= 0.549$ or -1.22 (3sf) $\therefore x = 4$, 0.549 or –1.22	$3x^3 - 10x^2 - 10x + 8 = 0$ $(x - 4)(3x^2 + Ax - 2) = 0$ By comparing coeff of x^2, $(x - 4)(3x^2 + 2x - 2) = 0$

7 (a) (i)	$\frac{dy}{dx} = 4x - 5$ Since y is increasing, $\frac{dy}{dx} > 0$ $4x - 5 > 0$ $x > 1\frac{1}{4}$	
7 (a) (ii)	For the curve to be always positive, D < 0, $(-5)^2 - 4(2)(13 + k) < 0$ $-79 - 8k < 0$ $k > -9\frac{7}{8}\ or\ -\frac{79}{8}$ $\therefore k = -9$	
7 (b)	$(2x + 1)(x - 4) \leq x(x - 7)$ $x^2 - 4 \leq 0$ $(x + 2)(x - 2) \leq 0$ $\therefore -2 \leq x \leq 2$ 	
8 (a)	$y = \ln\left(\frac{1-x}{x^2}\right)$ $y = \ln(1 - x) - 2\ln x$ $\frac{dy}{dx} = \frac{-1}{1-x} - \frac{2}{x}$	
8 (b)	When $y = \ln 2$, $\ln\left(\frac{1-x}{x^2}\right) = \ln 2$ $\frac{1-x}{x^2} = 2$ $1 - x = 2x^2$ $2x^2 + x - 1 = 0$ $(x+1)(2x-1) = 0$ $x = -1$ or $\frac{1}{2}$ When $x = -1$, $\frac{dy}{dt} = 9$,	

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$9 = \left(\frac{-1}{1-(-1)} - \frac{2}{(-1)} \right) \times \left. \frac{dx}{dt} \right|_{x=-1}$$

$$9 = \left(\frac{3}{2} \right) \times \left. \frac{dx}{dt} \right|_{x=-1}$$

$$\left. \frac{dx}{dt} \right|_{x=-1} = 6$$

When $x = \frac{1}{2}$, $\frac{dy}{dt} = 9$,

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$9 = \left(\frac{-1}{1-(\frac{1}{2})} - \frac{2}{(\frac{1}{2})} \right) \times \left. \frac{dx}{dt} \right|_{x=-1}$$

$$9 = (-6) \times \left. \frac{dx}{dt} \right|_{x=-1}$$

$$\left. \frac{dx}{dt} \right|_{x=-1} = -\frac{3}{2} \text{ (rejected)}$$

9 (i)

$y = 9x^6 - 3$

$$\frac{dy}{dx} = 54x^5$$

To find stationary point, $\frac{dy}{dx} = 0$

$54x^5 = 0$

$x = 0$

sub. into y, $y = -3$

$\therefore$ Coordinates of stationary point = $(0, -3)$

9 (ii)

x	0^-	0	0^+
$\frac{dy}{dx}$	–	0	+

By First Derivative Test, $(0, -3)$ is a minimum point.

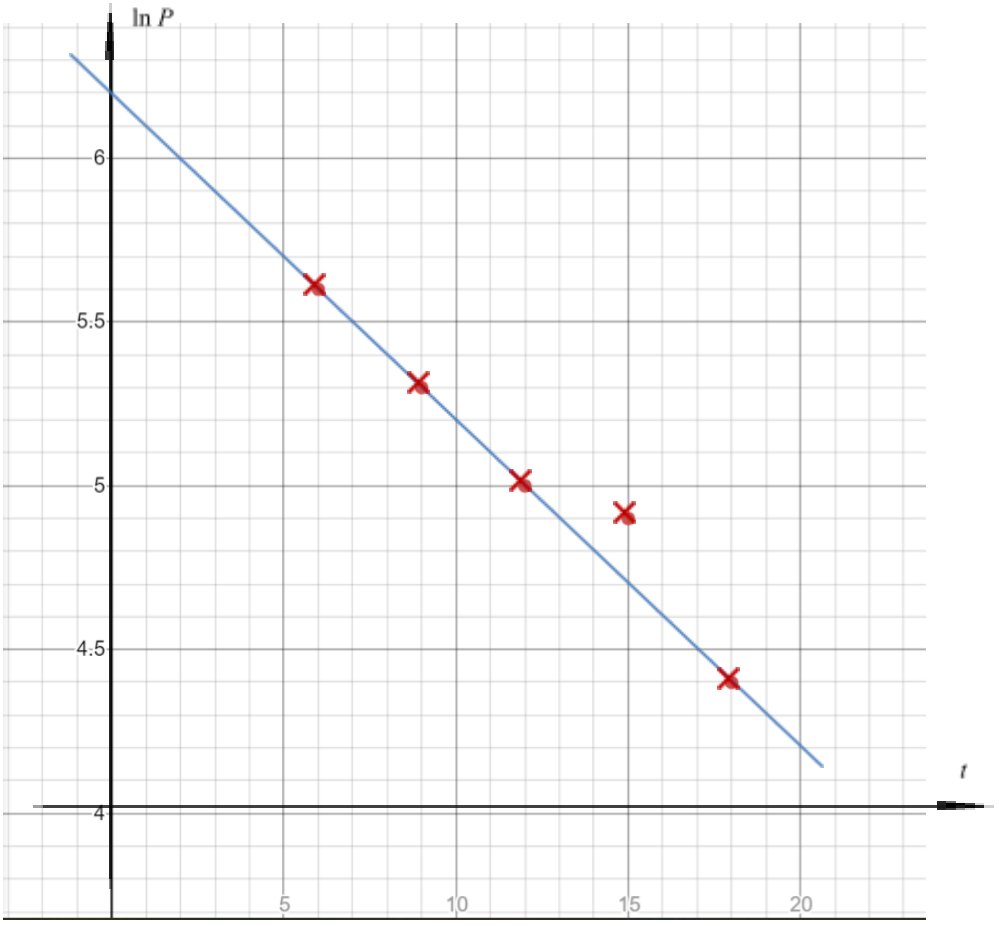
10 (a)	$\int \tan^2(3\theta-2)\,\mathrm{d}\theta$ $=\int\left[\sec^2(3\theta-2)-1\right]\mathrm{d}\theta$ $=\frac{1}{3}\tan(3\theta-2)-\theta+c$	
10 (b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=4\cos 4x-4\sin x\cos^3 x$ When $x=\frac{\pi}{4}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\cos 4\left(\frac{\pi}{4}\right)-4\sin\left(\frac{\pi}{4}\right)\cos^3\left(\frac{\pi}{4}\right)$ $=4(-1)-4\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)^3$ $=-5$	
11	Height $=\frac{\text{Volume}}{\text{Base Area}}$ $=\frac{4\sqrt{2}-3\sqrt{3}}{\left(\sqrt{3}-\sqrt{2}\right)^2}$ $=\frac{4\sqrt{2}-3\sqrt{3}}{3-2\sqrt{6}+2}$ $=\frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}}\times\frac{5+2\sqrt{6}}{5+2\sqrt{6}}$ $=\frac{20\sqrt{2}+8\sqrt{12}-15\sqrt{3}-6\sqrt{18}}{25-4(6)}$ $=20\sqrt{2}+16\sqrt{3}-15\sqrt{3}-18\sqrt{2}$ $=2\sqrt{2}+\sqrt{3}$	

12 (a)	$\frac{dy}{dx} = 1 + ae^{-2x}$ $\therefore$ Gradient of tangent at $(0, -1) = \frac{-1}{\left(-\frac{1}{2}\right)}$ $1 + ae^{-2(0)} = 2$ $1 + a = 2$ $a = 1$	
	$\frac{dy}{dx} = \frac{e^{2x} + a}{e^x}$ Gradient of tangent at $(0, -1) = \frac{-1}{\left(-\frac{1}{2}\right)} = 2$ $\left.\frac{dy}{dx}\right\|_{x=0} = 2$ $\frac{e^0 + a}{e^0} = 2$ $1 + a = 2$ $a = 1$	
12 (b)	$\frac{dy}{dx} = \frac{e^{2x} + a}{e^x} = 1 + e^{-2x}$ Since $e^{-2x} > 0$, $\frac{dy}{dx} > 0$ for all values of x. $\frac{dy}{dx} \neq 0$ $\therefore$, the curve as no stationary point.	
	For stationary point, $\frac{e^{2x} + 1}{e^x} = 0$ $e^{2x} + 1 = 0$ $e^{2x} = -1$ (no solution) Since $\frac{dy}{dx} = 0$ has no solution $\therefore$ y has no stationary point.	

12 (c)	$y = \int \frac{e^{2x}+1}{e^x}\,\mathrm{d}x$ $= \int 1 + e^{-2x}\,\mathrm{d}x$ $= x - \frac{1}{2}e^{-2x} + c$ When $x = 0, y = -1$, $-1 = 0 - \frac{1}{2}e^{-2(0)} + c$ $c = -\frac{1}{2}$ $\therefore y = x - \frac{1}{2e^{2x}} - \frac{1}{2}$	

13 (i)

t	6	9	12	15	18
ln P	5.6	5.3	5.0	4.9	4.4



13 (ii)

$P = P_0 e^{-\omega t}$

Taking ln on both sides,

$\ln P = \ln (P_0 e^{-\omega t})$

$\ln P = -\omega t + \ln P_0$

$\ln P_0 = 6.2$

$P_0 = 492.7$

$P_0 = 500$ (nearest hundred)

$$-\omega = \frac{6.2 - 4.4}{0 - 18}$$

$-\omega = -0.1$

$\omega = 0.1$

13 (iii)	The incorrect value of P is <u>134</u>. $\ln P = 4.7$ $P = 109.9$ $P = 110$ (3sf)	
13 (iv)	$\ln P = \ln (100)$ $= 4.6$ From the graph, $t = 16$ years Year = 2016	