Linear Algebra MAT313 Spring 2023

Professor Sormani

Lesson 3 Augmented Matrices and the Algorithm

In this lesson we will learn a specific algorithm that we will use all semester for solving many different kinds of problems. It is necessary to use this specific algorithm and not another one taught in other courses because I will be designing homework and quiz problems that can be solved quickly using this algorithm. If you have difficulty learning this algorithm, I will help with feedback and you can resubmit your work.

Warning: do not start this lesson until you have completed and corrected Lesson 2 following the solutions and submitted all the notes, classwork, and homework for that lesson.

You will cut and paste the photos of your notes and completed classwork and a selfie taken holding up the first page of your work in a googledoc entitled:

MAT313S23-lesson1-lastname-firstname

and share editing of that document with me sormanic@gmail.com. You can use your Lehman id and hand instead of your face in your selfie. You will also include your homework and any corrections to your homework in this doc.

If you have a question, type QUESTION in your googledoc next to the point in your notes that has a question and email me with the subject MAT313 QUESTION. I will answer your question by inserting a photo into your googledoc or making an extra video.

This Lesson has two parts and each has its own playlist:

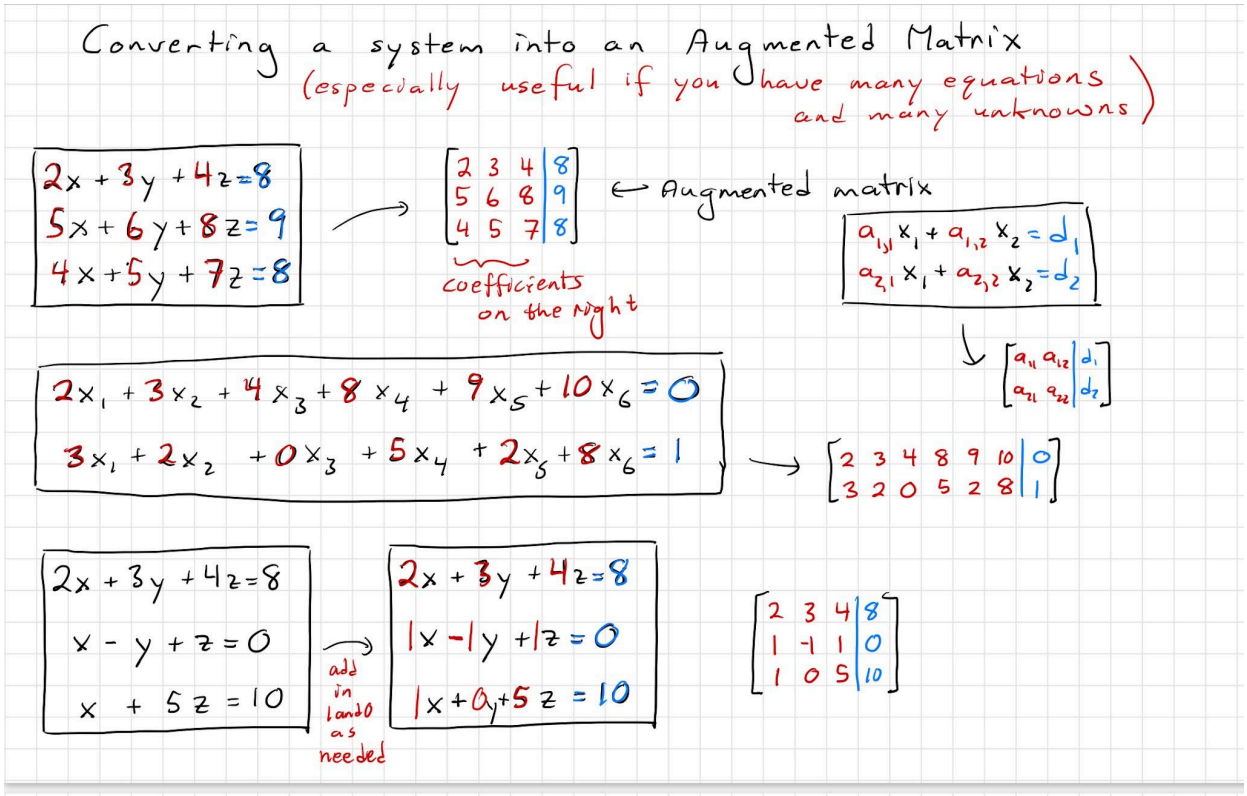
Part I Augmented Matrices

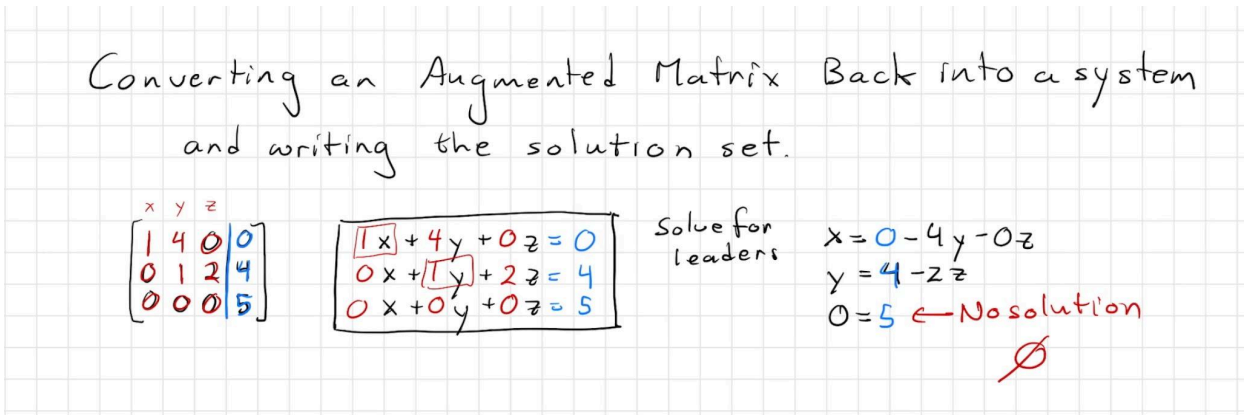
Part II Algorithm

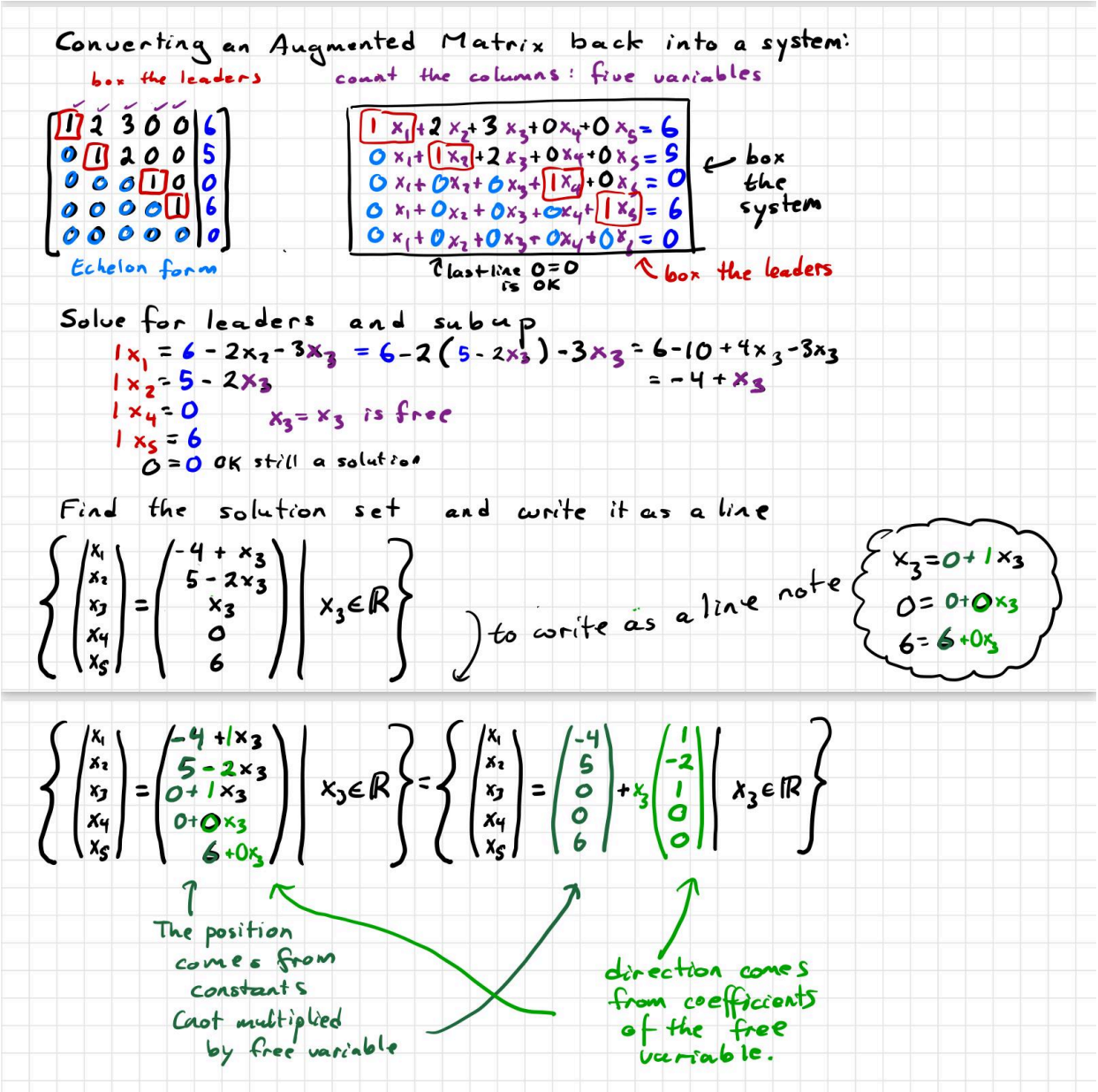
There are 9 homework problems.

Part I Augmented Matrices

Solving a linear system using augmented matrices, reducing to Echelon form, converting back into a system, solving for leaders, subbing up, and writing a solution set. Watch the [Playlist 313F23-L3-P1-1to3]







Part II Algorithm

Watch [Playlist 313S23-L3-P2-1to10]

Our Algorithm to Reduce an Augmented Matrix to Echelon Form
(there are many algorithms but we will use this one in this course)

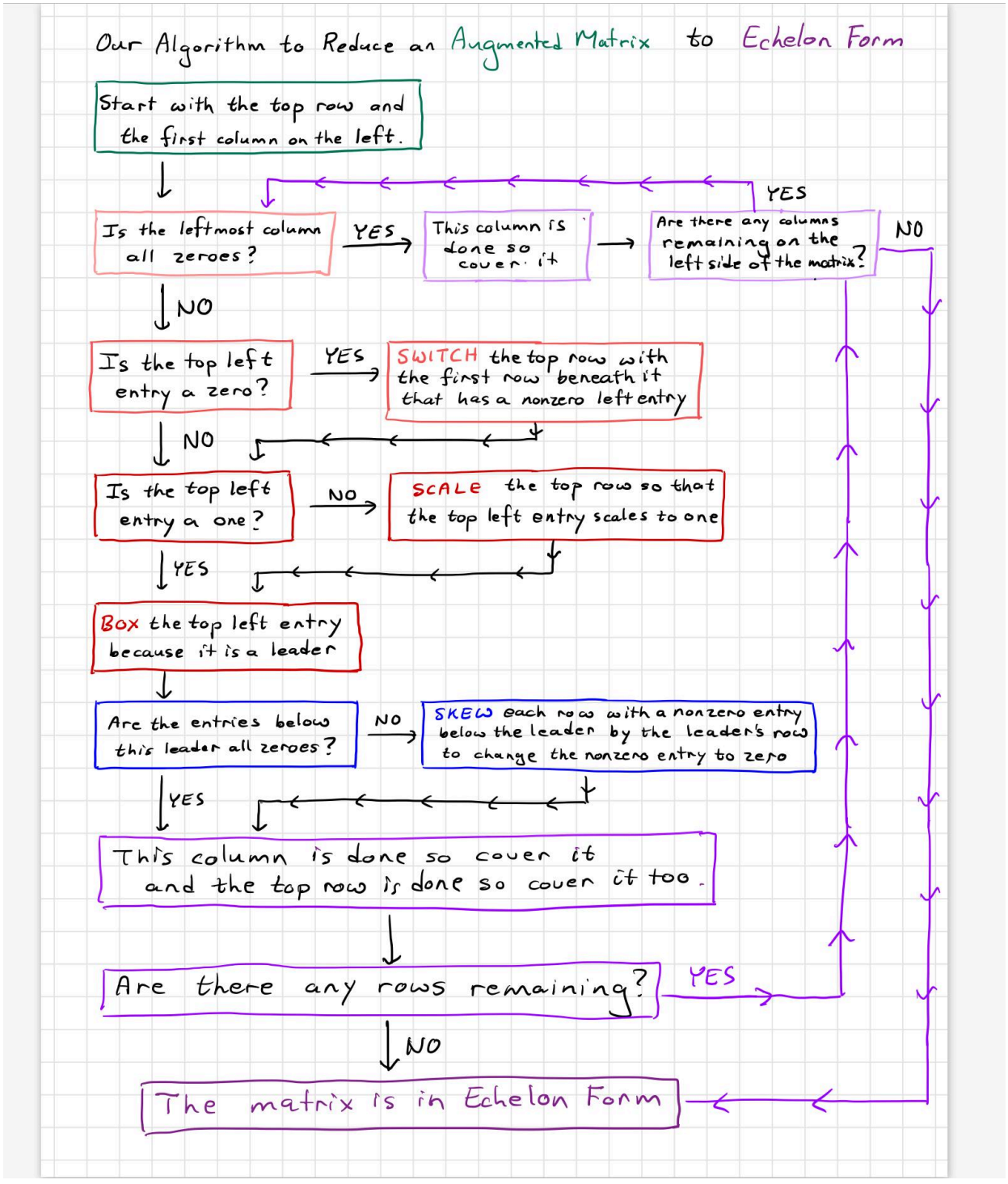
Definition: a matrix is in Echelon Form if
it has zeroes to the left and below all leaders

and all the leaders are ones and
all rows without leaders are at the bottom.

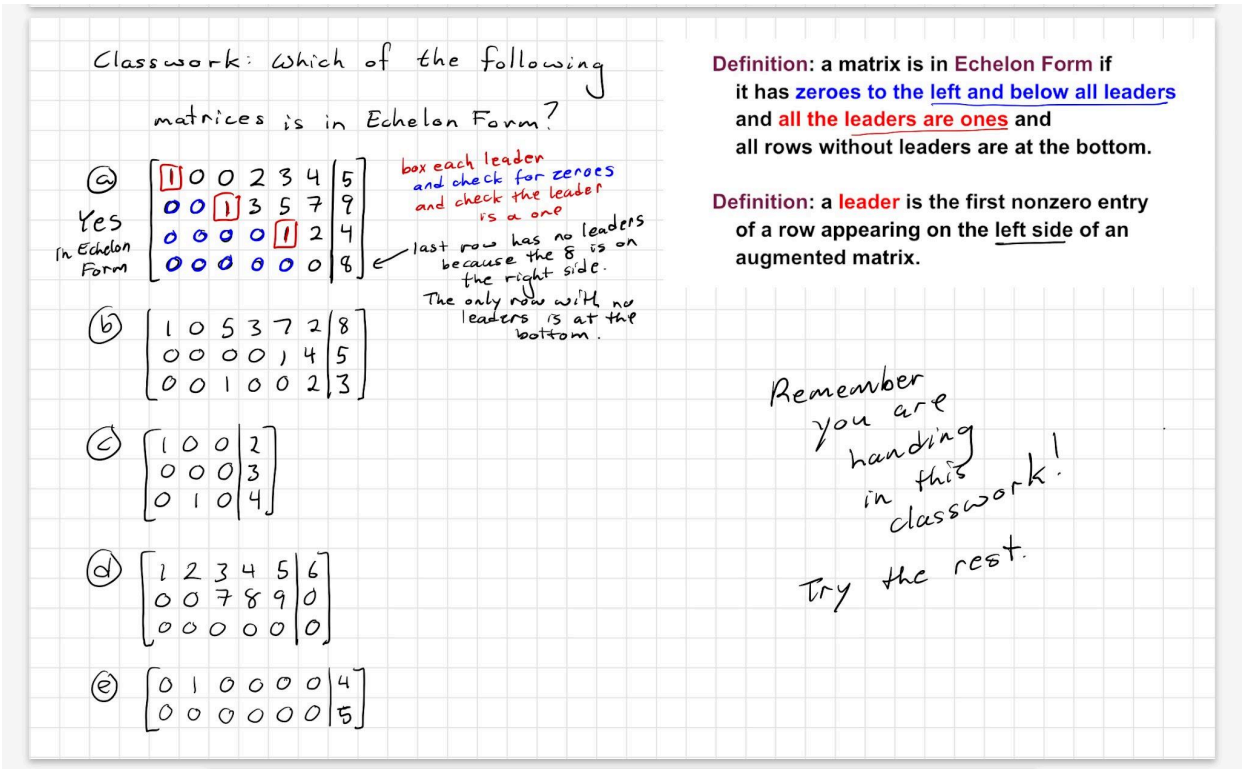
Definition: a **leader** is the first nonzero entry
of a row appearing on the left side of an
augmented matrix.

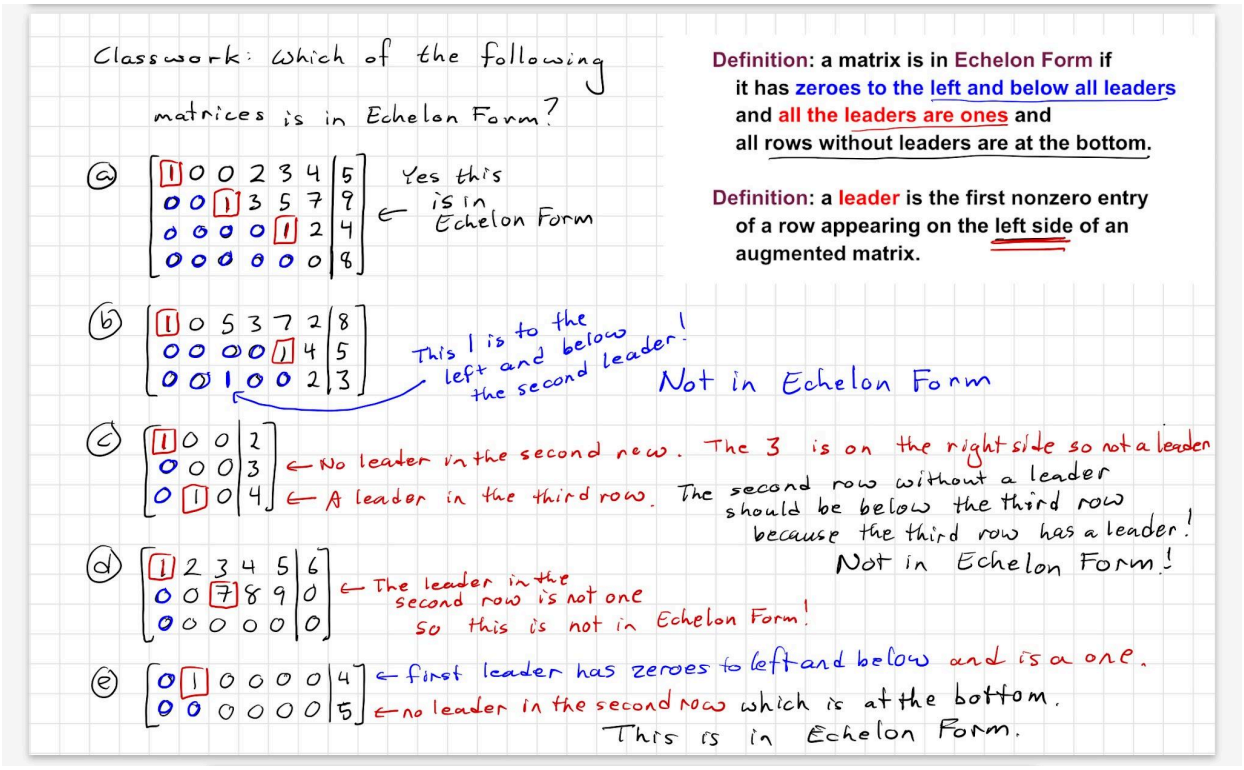
Method: Start on the upper left.
Step 1: Find a leader and make it one using row actions.
Step 2: Get zeroes below the leader using row actions.
Go to the next row and column and repeat Step 1 and Step 2.
When we run out of rows, the matrix is in Echelon Form:
it has zeroes to the left and below all leaders
and all the leaders are ones and
all rows without leaders are at the bottom.

A quick glance at our algorithm:



Echelon Form Classwork: (use graph paper and a multicolor pen)





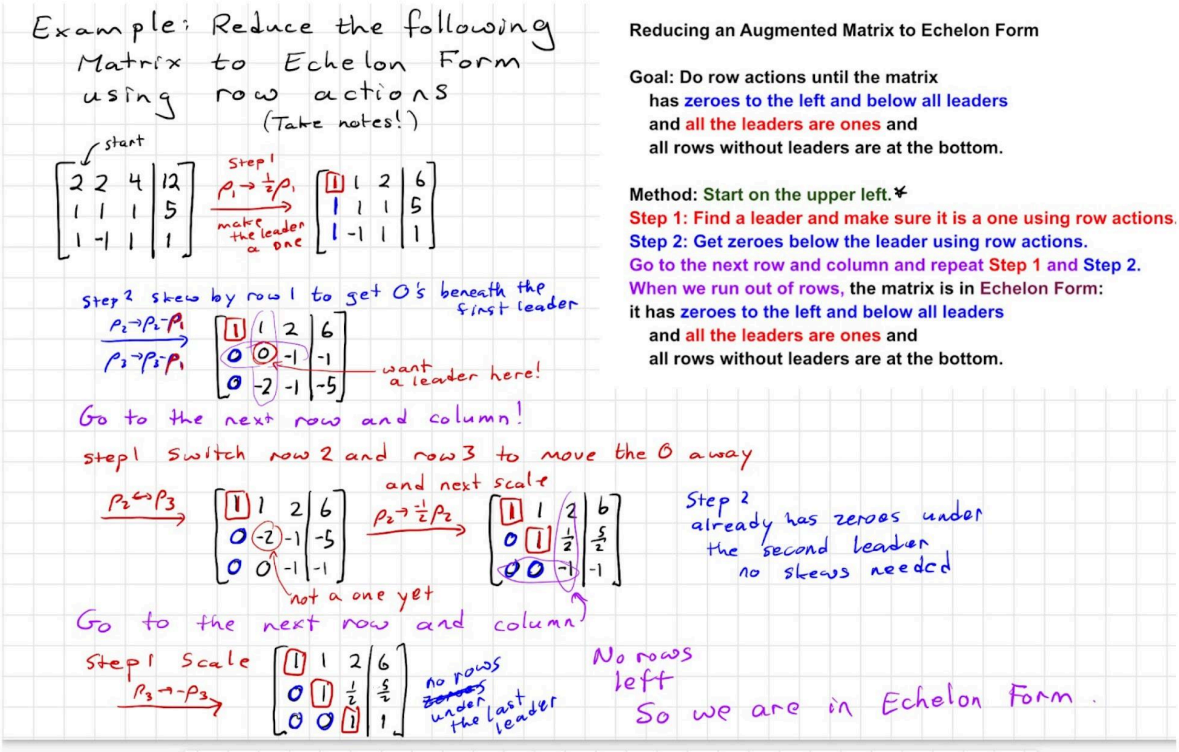
Reducing an Augmented Matrix to Echelon Form

Goal: Do row actions until the matrix

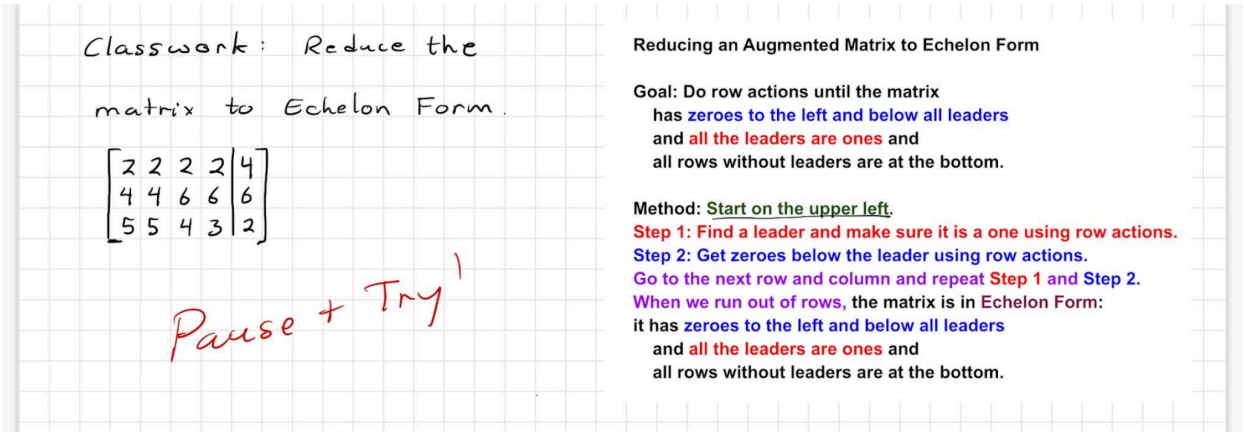
has zeroes to the left and below all leaders
and all the leaders are ones and
all rows without leaders are at the bottom.

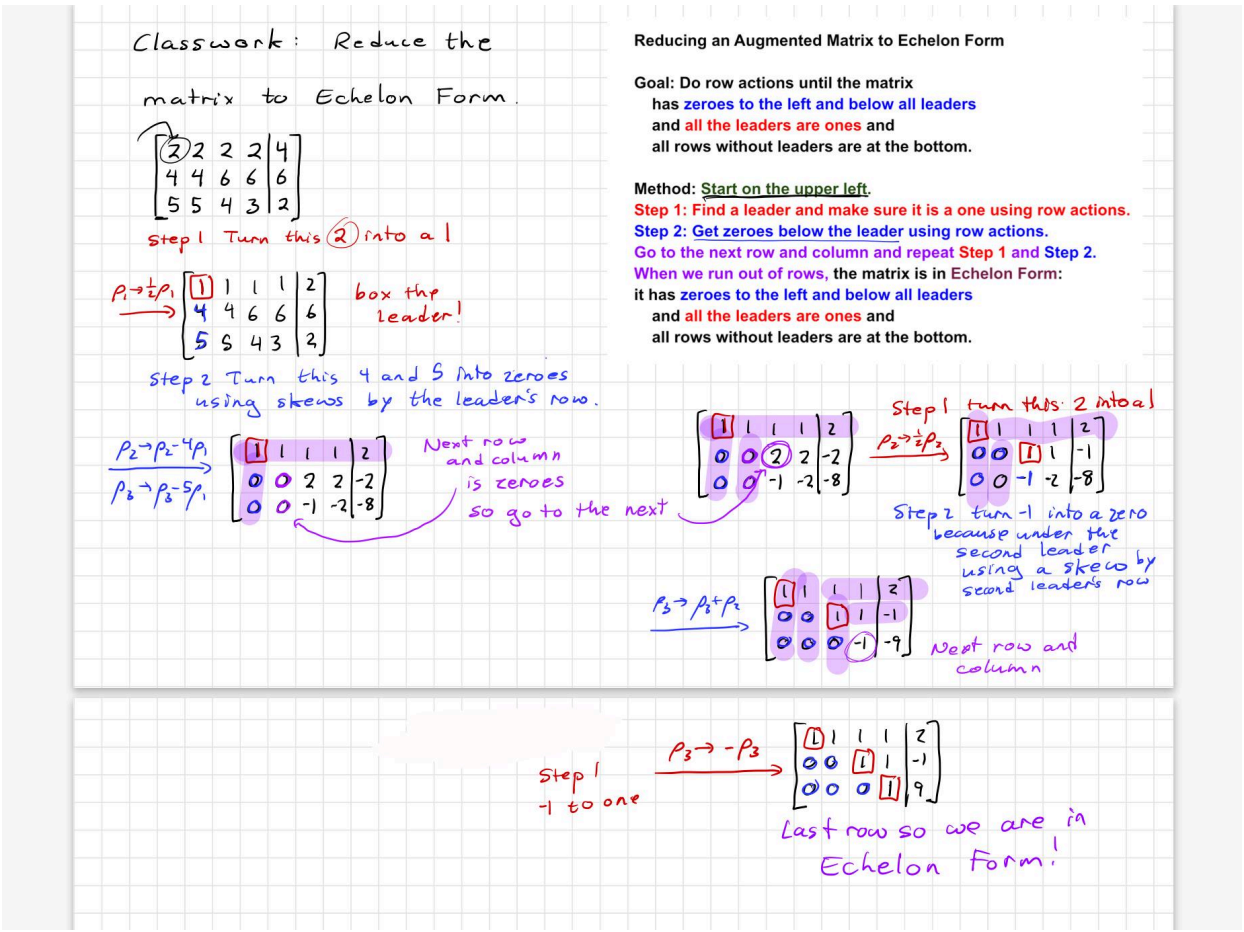
Method: Start on the upper left.
Step 1: Find a leader and make sure it is a one using row actions.
Step 2: Get zeroes below the leader using row actions.
Go to the next row and column and repeat Step 1 and Step 2.
When we run out of rows, the matrix is in Echelon Form:
it has zeroes to the left and below all leaders
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Example:

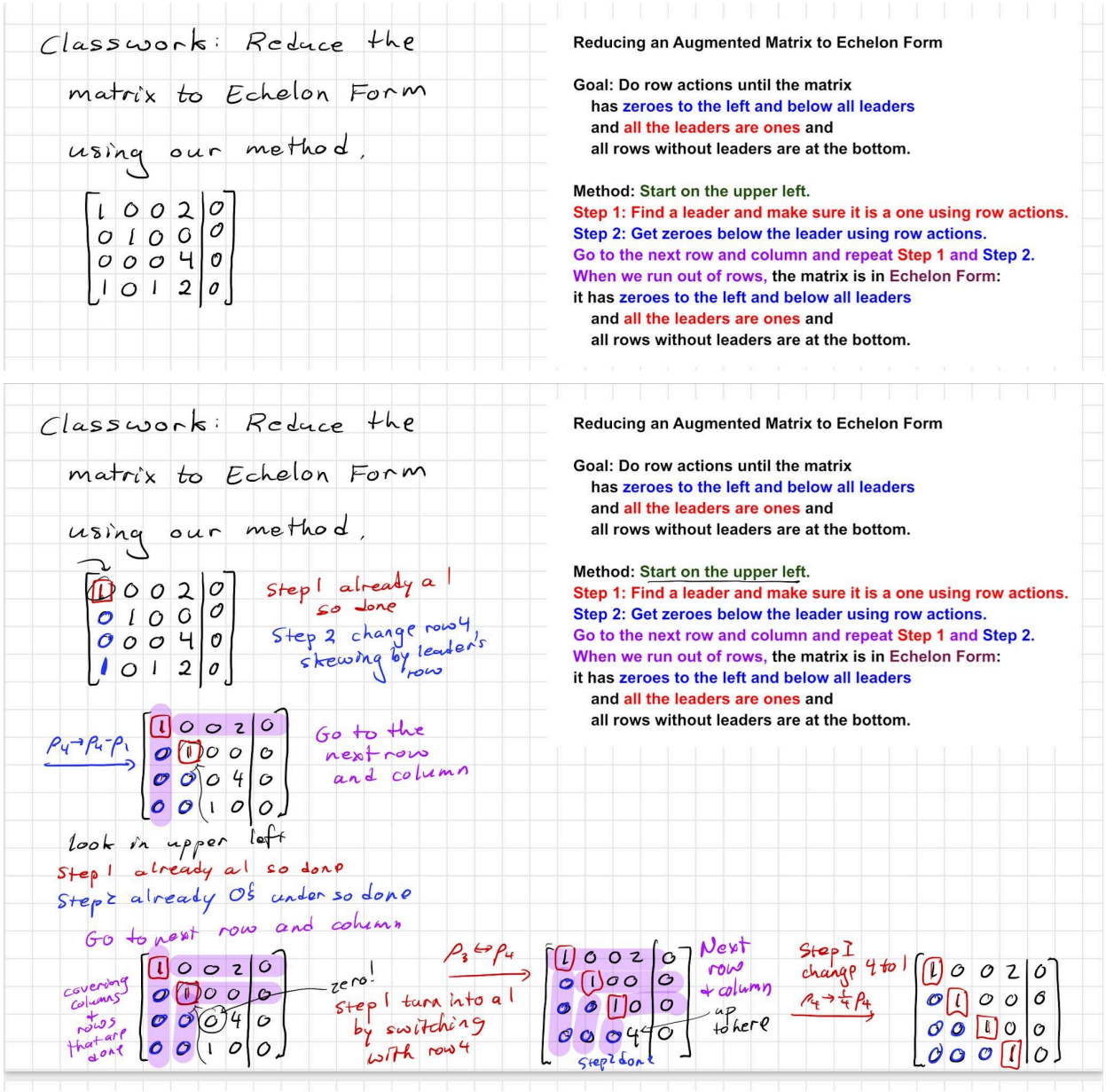


Classwork:


Classwork: Reduce the matrix to Echelon Form.
Pause + Try!
Reducing an Augmented Matrix to Echelon Form
Goal: Do row actions until the matrix
has zeroes to the left and below all leaders
and all the leaders are ones and
all rows without leaders are at the bottom.
Method: Start on the upper left.
Step 1: Find a leader and make sure it is a one using row actions.
Step 2: Get zeroes below the leader using row actions.
Go to the next row and column and repeat Step 1 and Step 2.
When we run out of rows, the matrix is in Echelon Form:
it has zeroes to the left and below all leaders
and all the leaders are ones and
all rows without leaders are at the bottom.


Classwork: Reduce the matrix to Echelon Form.
Step 1 Turn this 2 into a 1
box the leader!
Step 2 Turn this 4 and 5 into zeroes using skews by the leader's row.
Next row and column is zeroes so go to the next
Step 1 turn this 2 into a 1
Step 2 turn -1 into a zero because under the second leader using a skew by second leader's row
Next row and column
Step 1 -1 to one
Last row so we are in Echelon Form!
Reducing an Augmented Matrix to Echelon Form
Goal: Do row actions until the matrix
has zeroes to the left and below all leaders
and all the leaders are ones and
all rows without leaders are at the bottom.
Method: Start on the upper left.
Step 1: Find a leader and make sure it is a one using row actions.
Step 2: Get zeroes below the leader using row actions.
Go to the next row and column and repeat Step 1 and Step 2.
When we run out of rows, the matrix is in Echelon Form:
it has zeroes to the left and below all leaders
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Classwork

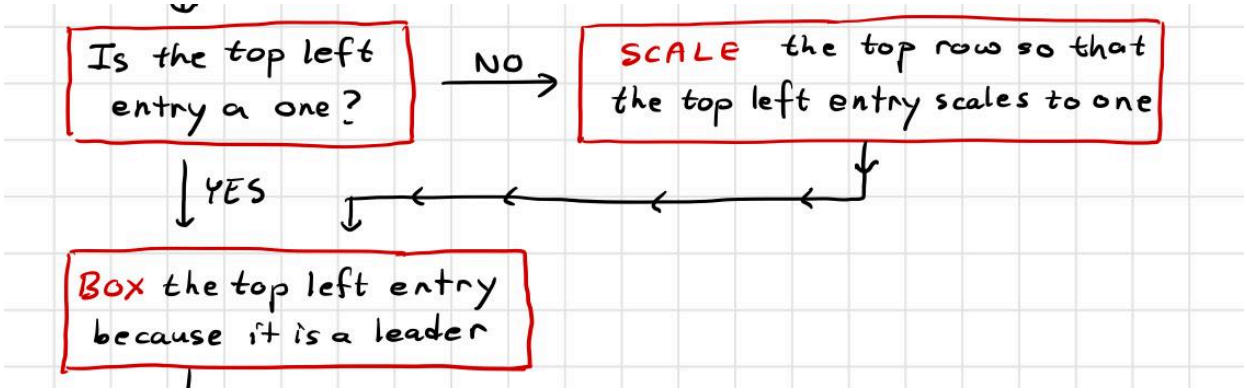


Creating Our Algorithm:

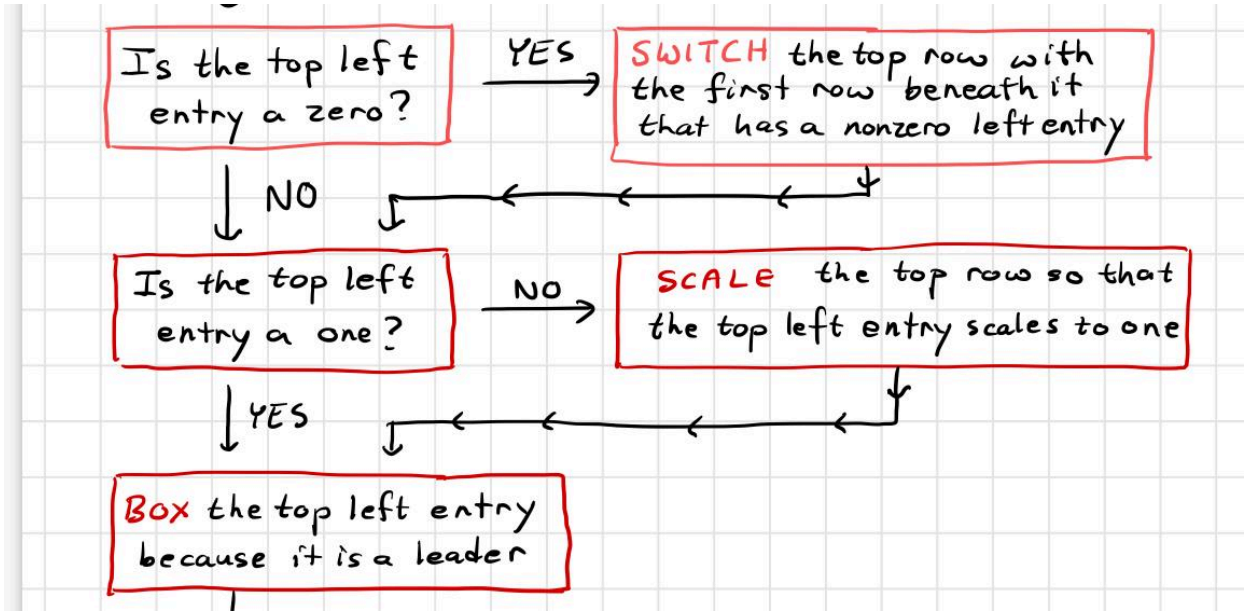
Method: Start on the upper left.
Step 1: Find a leader and make sure it is a one using row actions.
Step 2: Get zeroes below the leader using row actions.
Go to the next row and column and repeat Step 1 and Step 2.
When we run out of rows, the matrix is in Echelon Form:
it has zeroes to the left and below all leaders
and all the leaders are ones and
all rows without leaders are at the bottom.

Method: **Start on the upper left.**

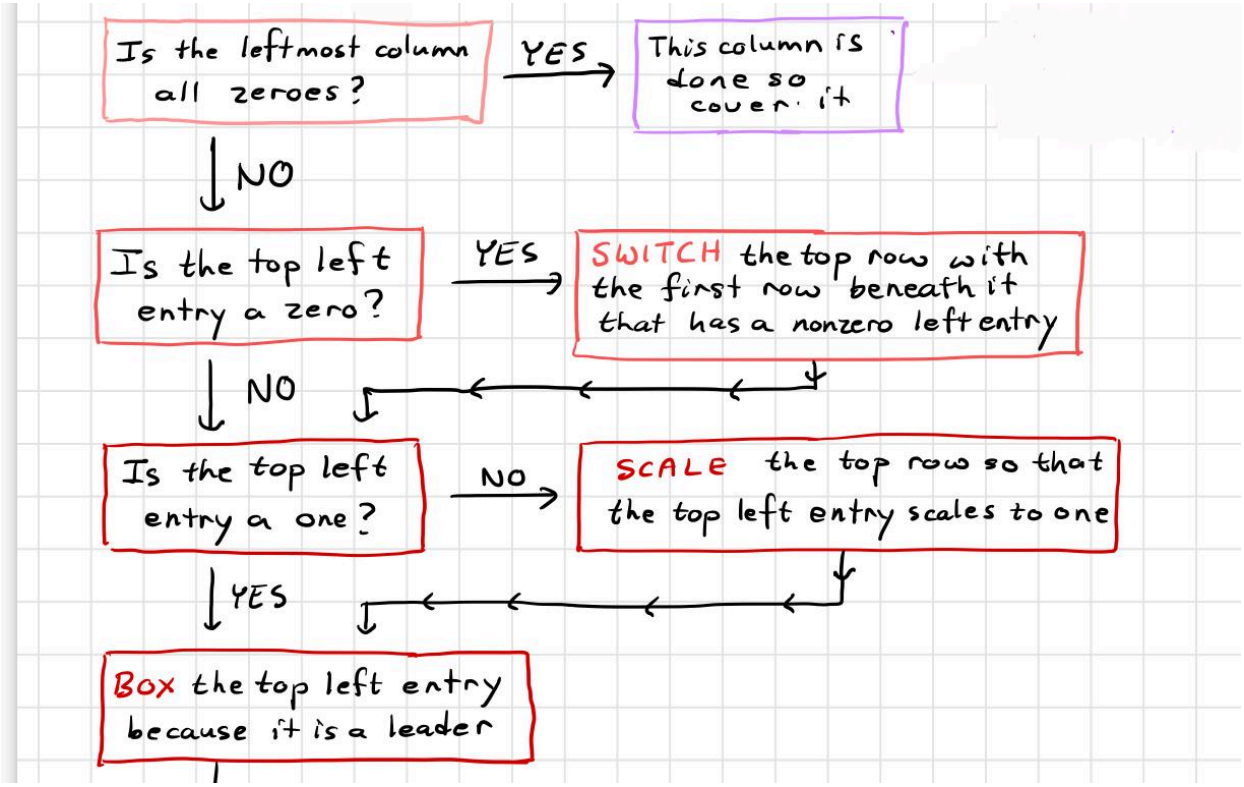
Step 1: Find a leader and make sure it is a one using row actions.



However, what if the top left entry is zero? Then we cannot scale it! Let's eliminate that first by adding this question on top:



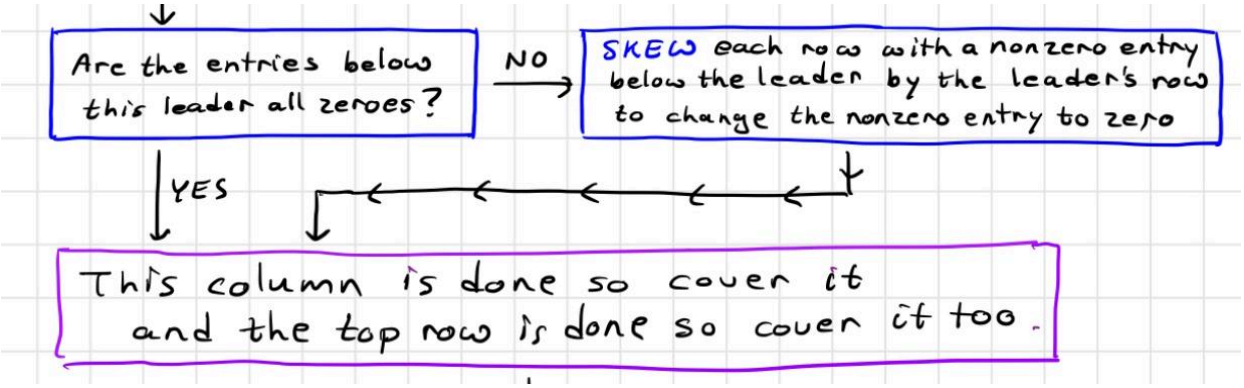
However, what if everything under it is zero too? Then we cannot switch it! Let's eliminate that first by adding this question on top:



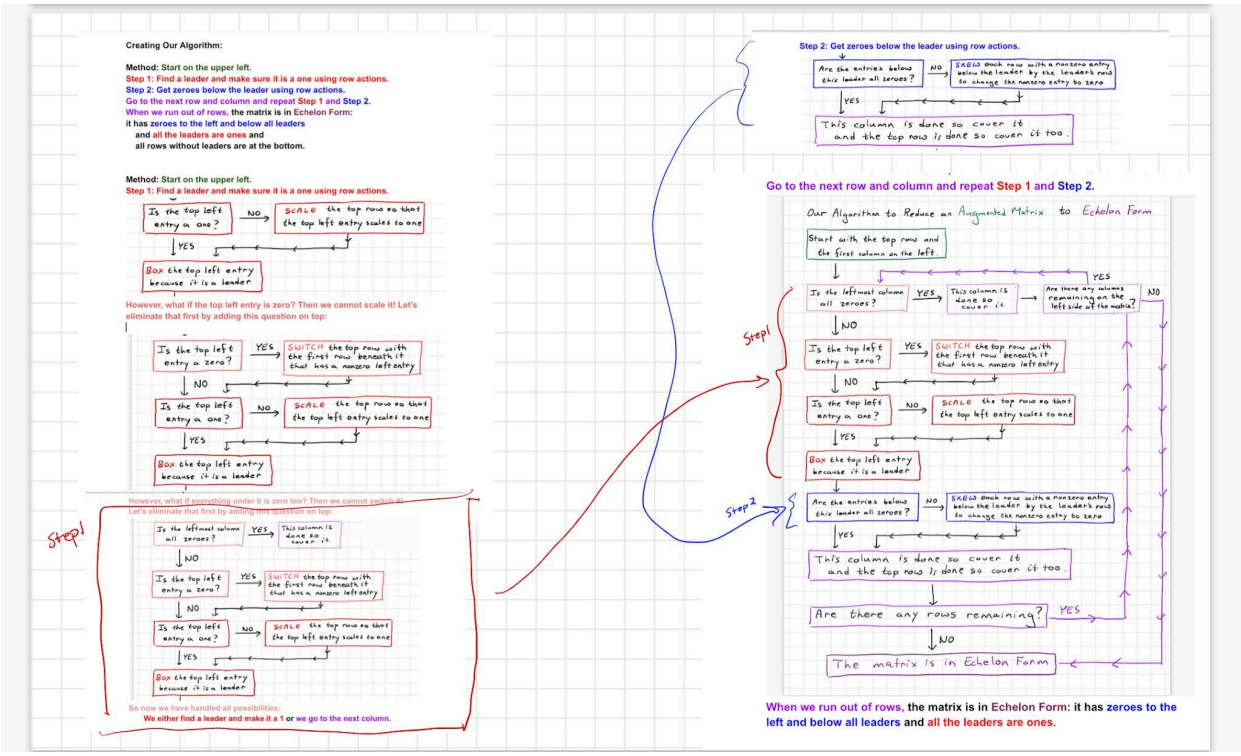
So now we have handled all possibilities:

We either find a leader and make it a 1 or we go to the next column.

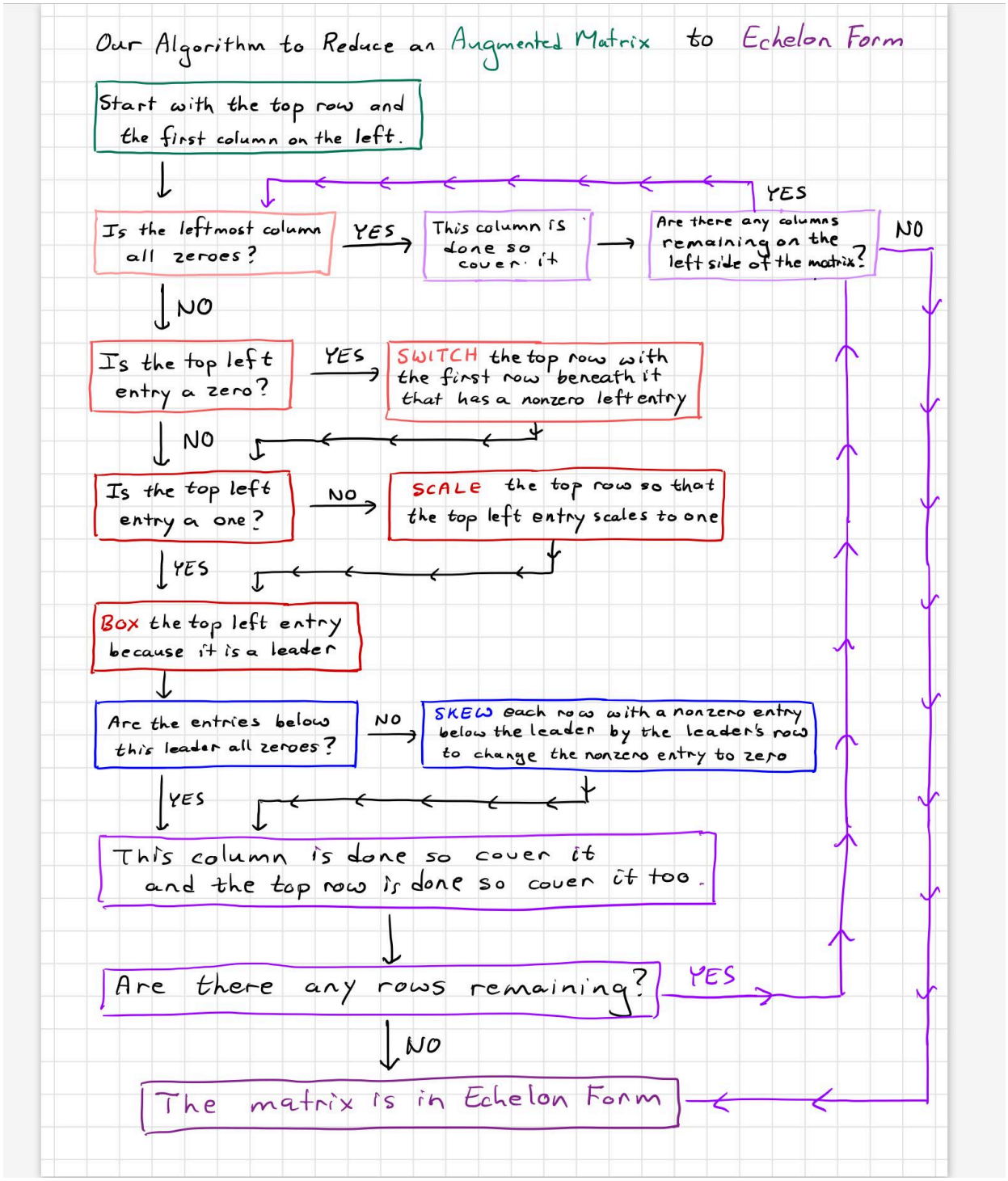
Step 2: Get zeroes below the leader using row actions.



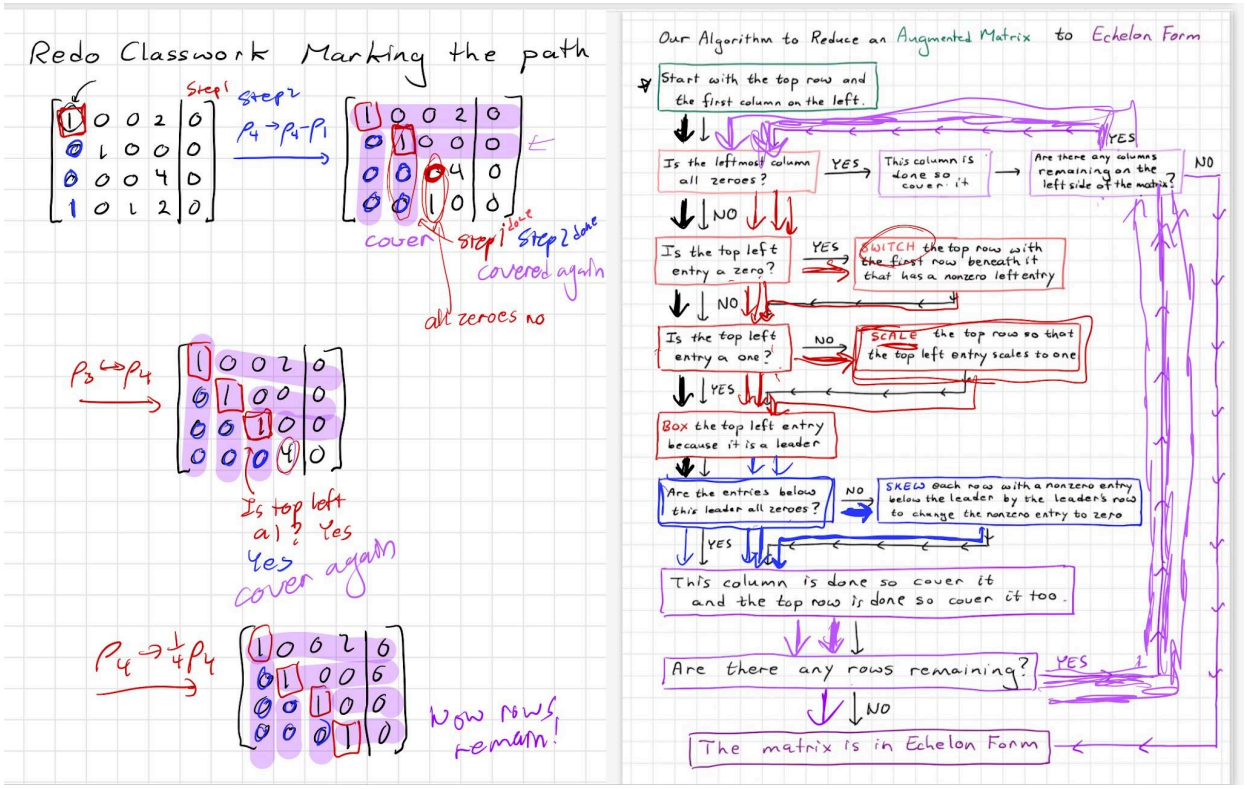
Go to the next row and column and repeat Step 1 and Step 2.



Our Algorithm:



When we run out of rows, the matrix is in Echelon Form: it has zeroes to the left and below all leaders and all the leaders are ones.


Redo Classwork Marking the path
Step 1
Step 2
$R_4 \to R_4 - R_1$
cover
Step 1 done
Step 2 done
covered again
all zeroes no
$R_3 \leftrightarrow R_4$
Is top left a 1? Yes
Yes
cover again
$R_4 \to \frac{1}{4}R_4$
Now rows remain!
Our Algorithm to Reduce an Augmented Matrix to Echelon Form
Start with the top row and the first column on the left.
Is the leftmost column all zeroes?
YES
This column is done so cover it
Are there any columns remaining on the left side of the matrix?
NO
NO
Is the top left entry a zero?
YES
SWITCH the top row with the first row beneath it that has a nonzero left entry
NO
Is the top left entry a one?
NO
SCALE the top row so that the top left entry scales to one
YES
Box the top left entry because it is a leader
Are the entries below this leader all zeroes?
NO
SKEW each row with a nonzero entry below the leader by the leader's row to change the nonzero entry to zero
YES
This column is done so cover it and the top row is done so cover it too.
Are there any rows remaining?
YES
NO
The matrix is in Echelon Form

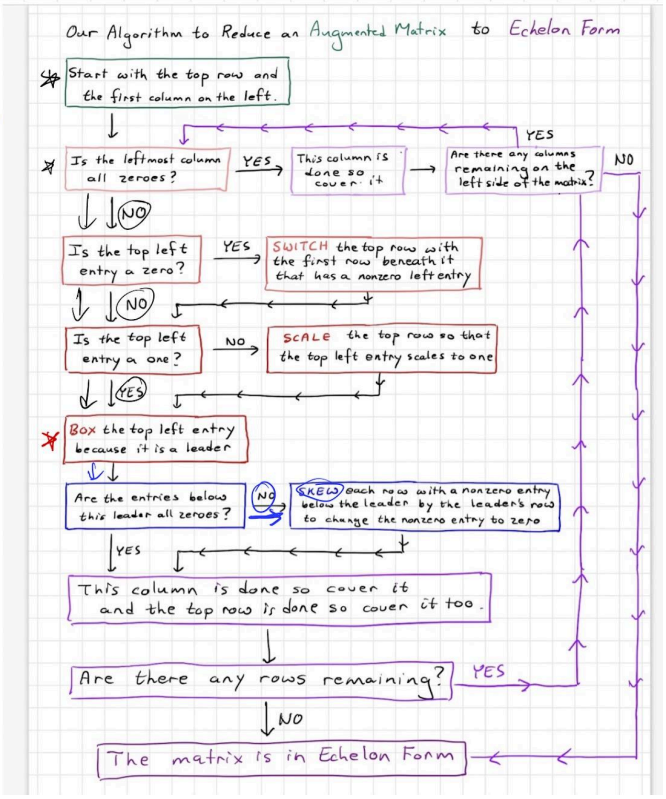
Classwork: Do the next row action for the following matrices

(a) $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 3 \\ 0 & 4 & 1 & 1 & 5 \\ 2 & 3 & 4 & 5 & 6 \end{array}\right]$ SKEW BY LEADER'S ROW $R_3 \to R_3 - 2R_1$ $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 3 \\ 0 & 4 & 1 & 1 & 5 \\ 0 & 3 & 0 & 5 & 0 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 3 \\ 0 & 4 & 1 & 1 & 5 \\ 0 & 0 & 6 & 7 & 8 \end{array}\right]$

(c) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 5 & 6 & 7 \\ 0 & 3 & 2 & 0 & 1 \end{array}\right]$

(d) $\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$



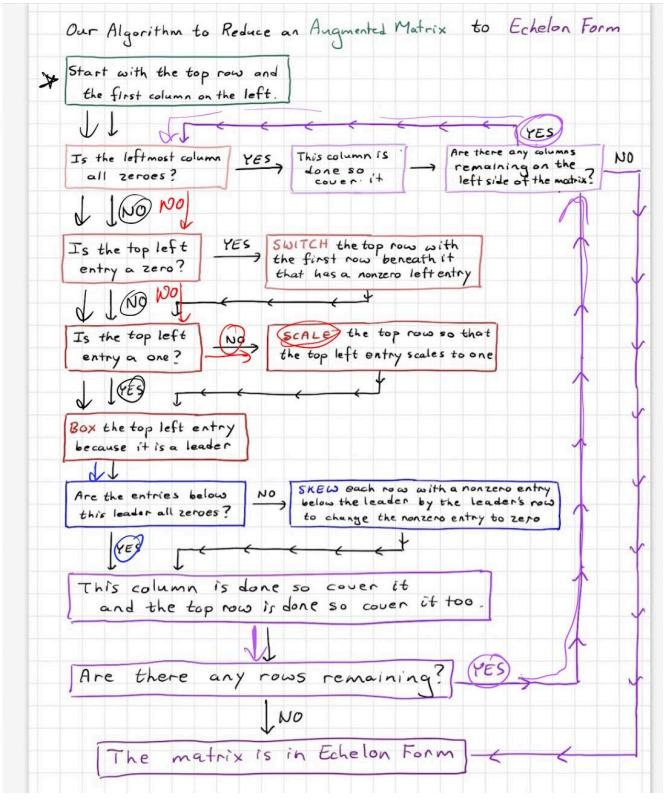
Classwork: Do the next row action for the following matrices

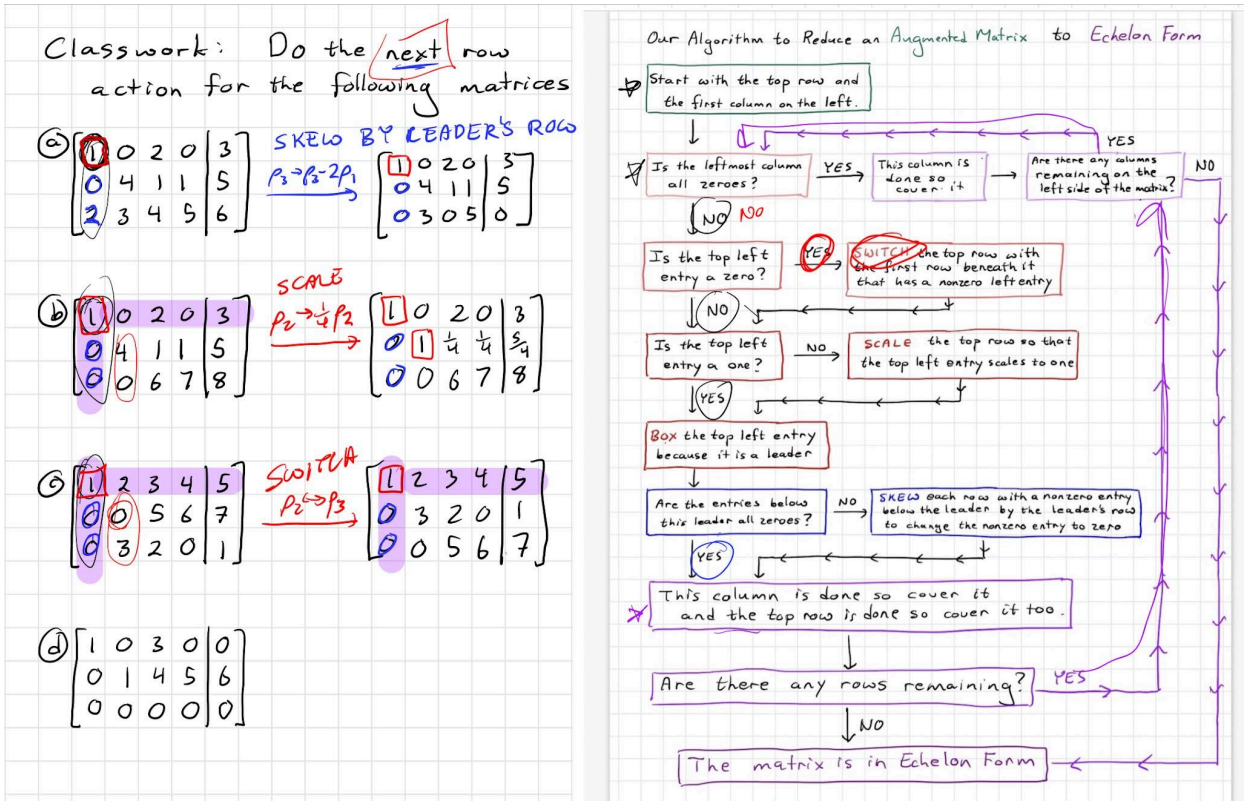
(a) $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 3 \\ 0 & 4 & 1 & 1 & 5 \\ 2 & 3 & 4 & 5 & 6 \end{array}\right]$ SKEW BY LEADER'S ROW $R_3 \to R_3 - 2R_1$ $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 3 \\ 0 & 4 & 1 & 1 & 5 \\ 0 & 3 & 0 & 5 & 0 \end{array}\right]$

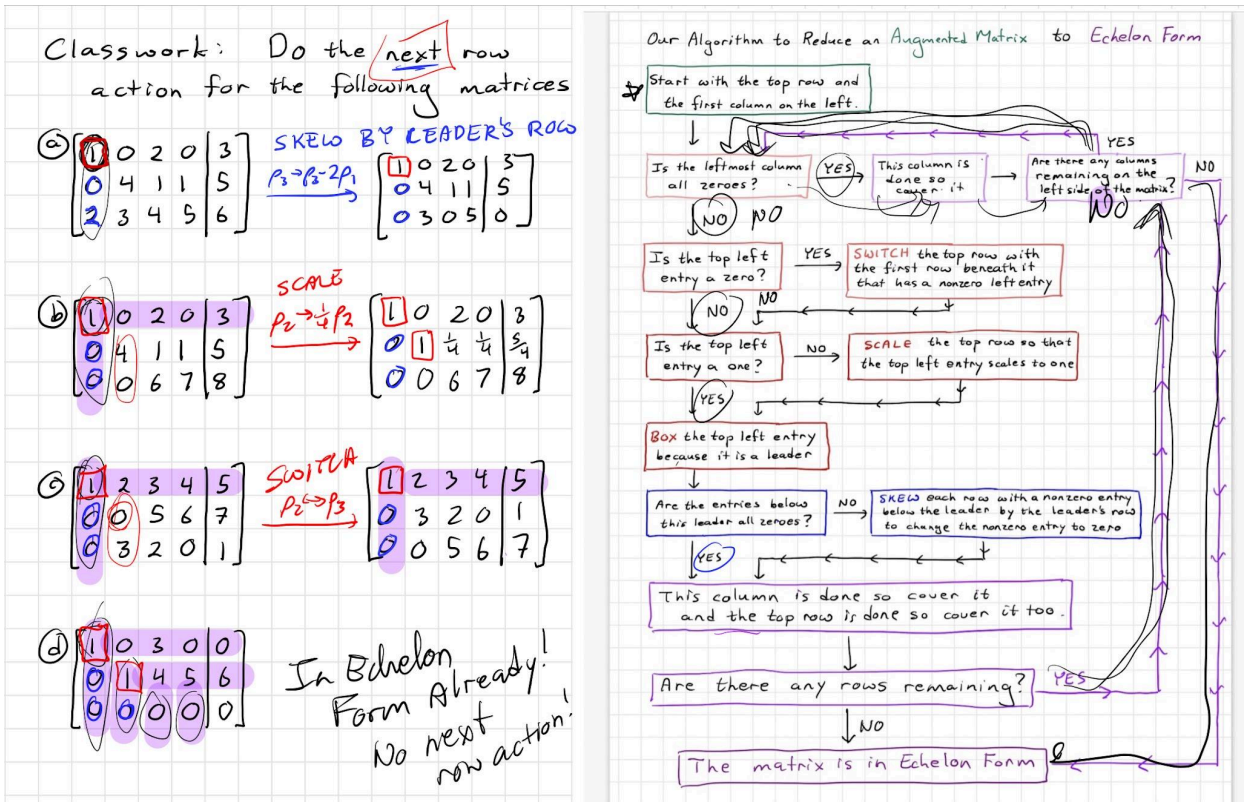
(b) $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 3 \\ 0 & 4 & 1 & 1 & 5 \\ 0 & 0 & 6 & 7 & 8 \end{array}\right]$ SCALE $R_2 \to \frac{1}{4}R_2$ $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 3 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{5}{4} \\ 0 & 0 & 6 & 7 & 8 \end{array}\right]$

(c) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 5 & 6 & 7 \\ 0 & 3 & 2 & 0 & 1 \end{array}\right]$

(d) $\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

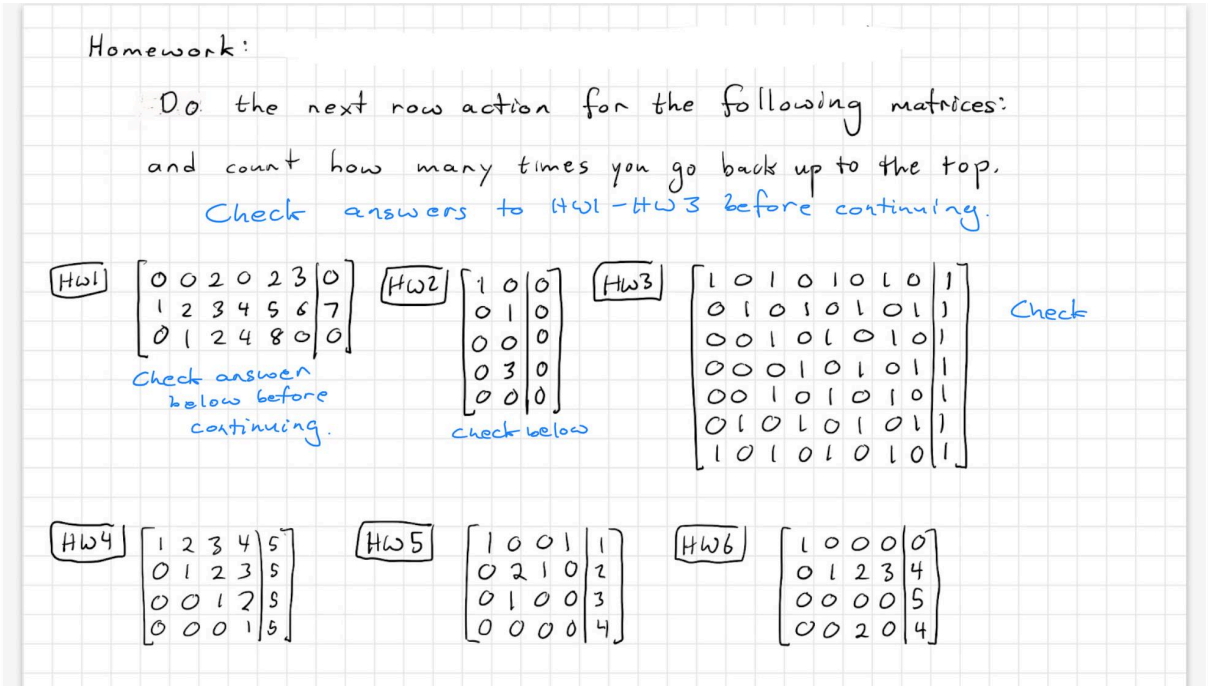






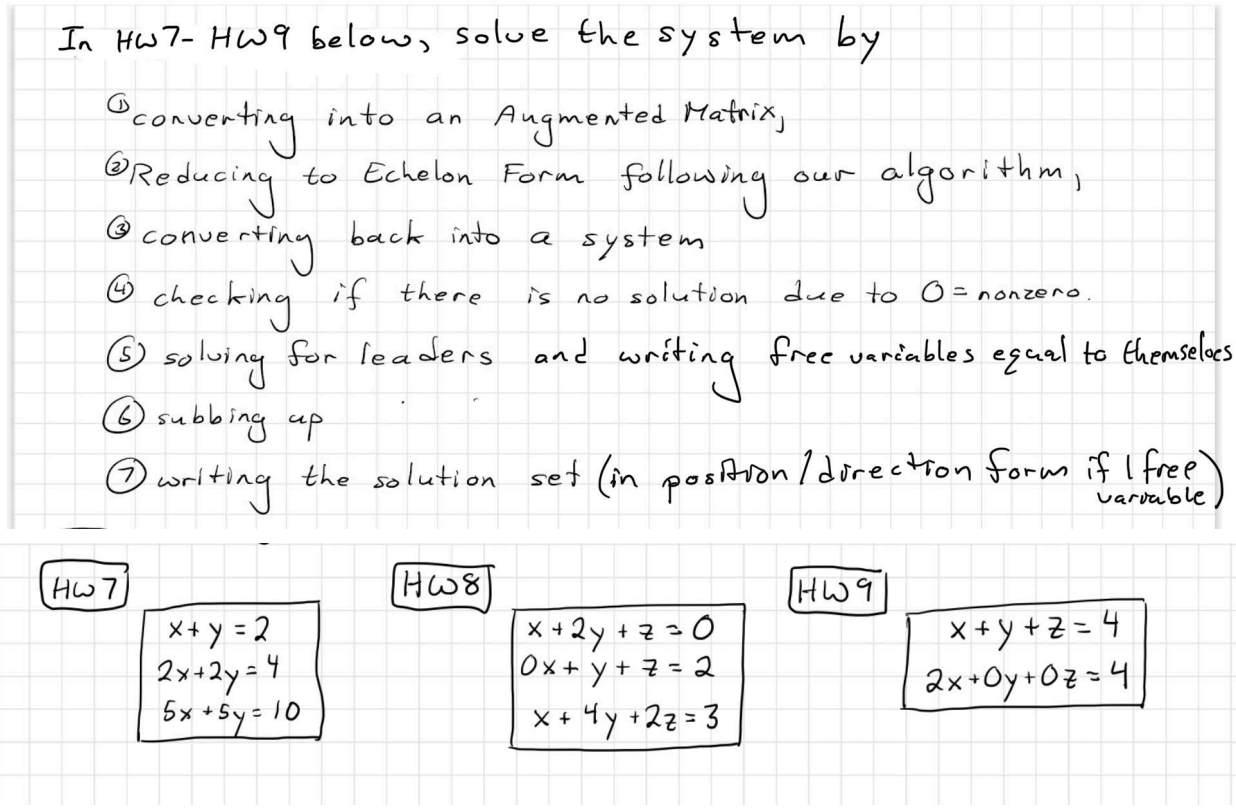
The following homework will prepare you for Quiz 1.

Homework: **Check answers to HW1-HW3 at the end of this doc.**



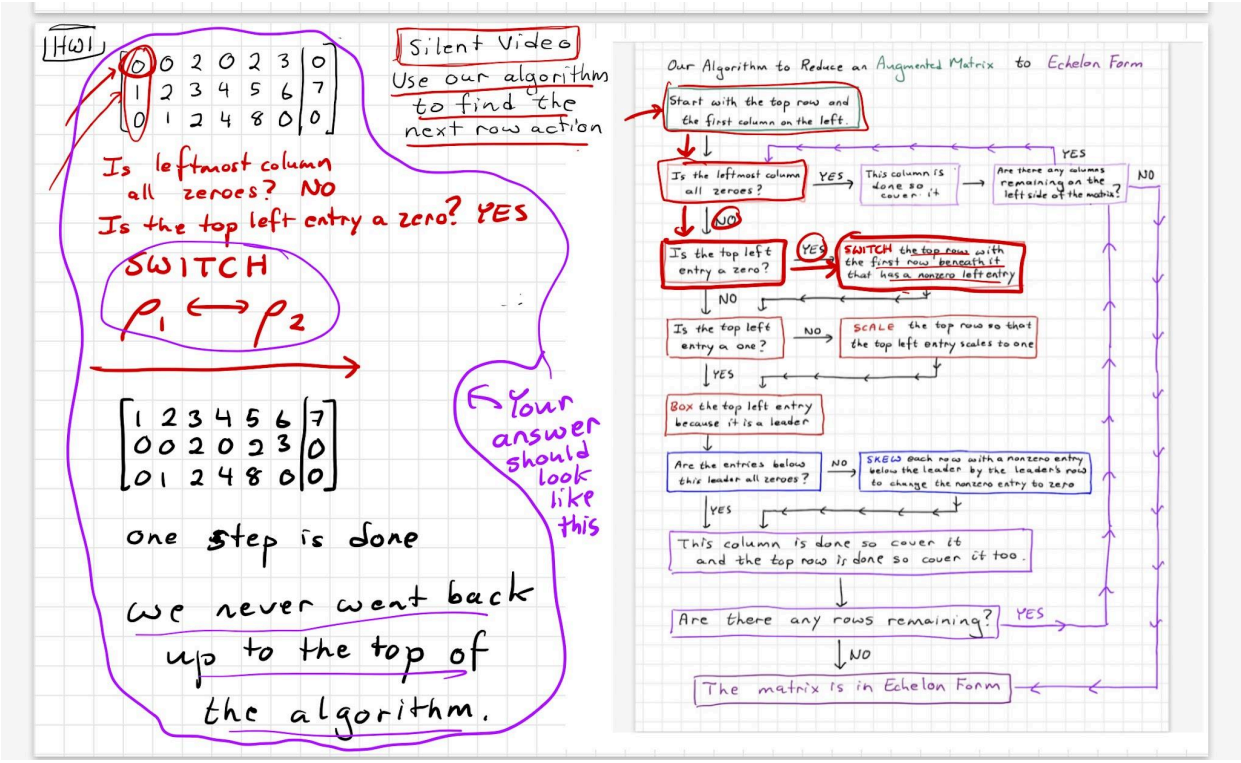
HW1-HW6 above only require one row action each to be completed and students will explain how many times they went to the top of the algorithm for that one action.

HW7-HW9 below require the complete solution including position/direction form when there is one free variable.



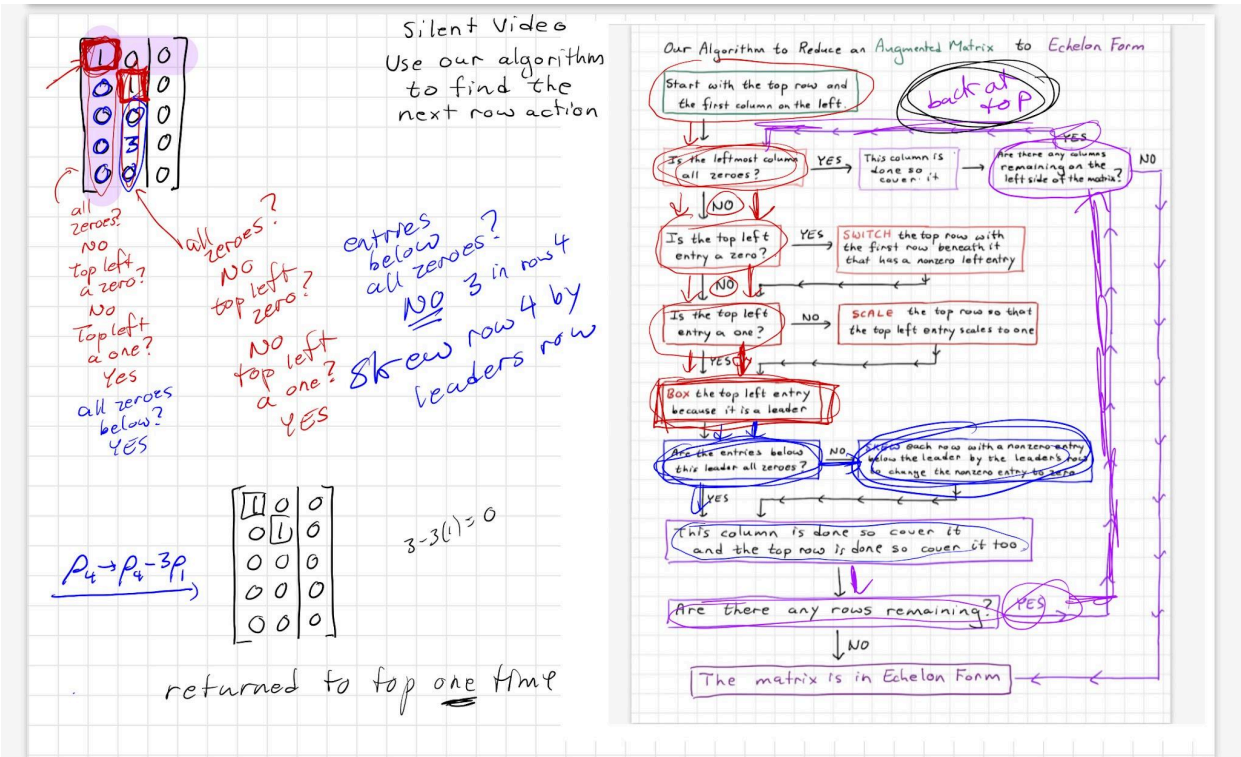
Choose one of the four times on the course webpage for your quiz 1.

Check your answer to HW1 here:



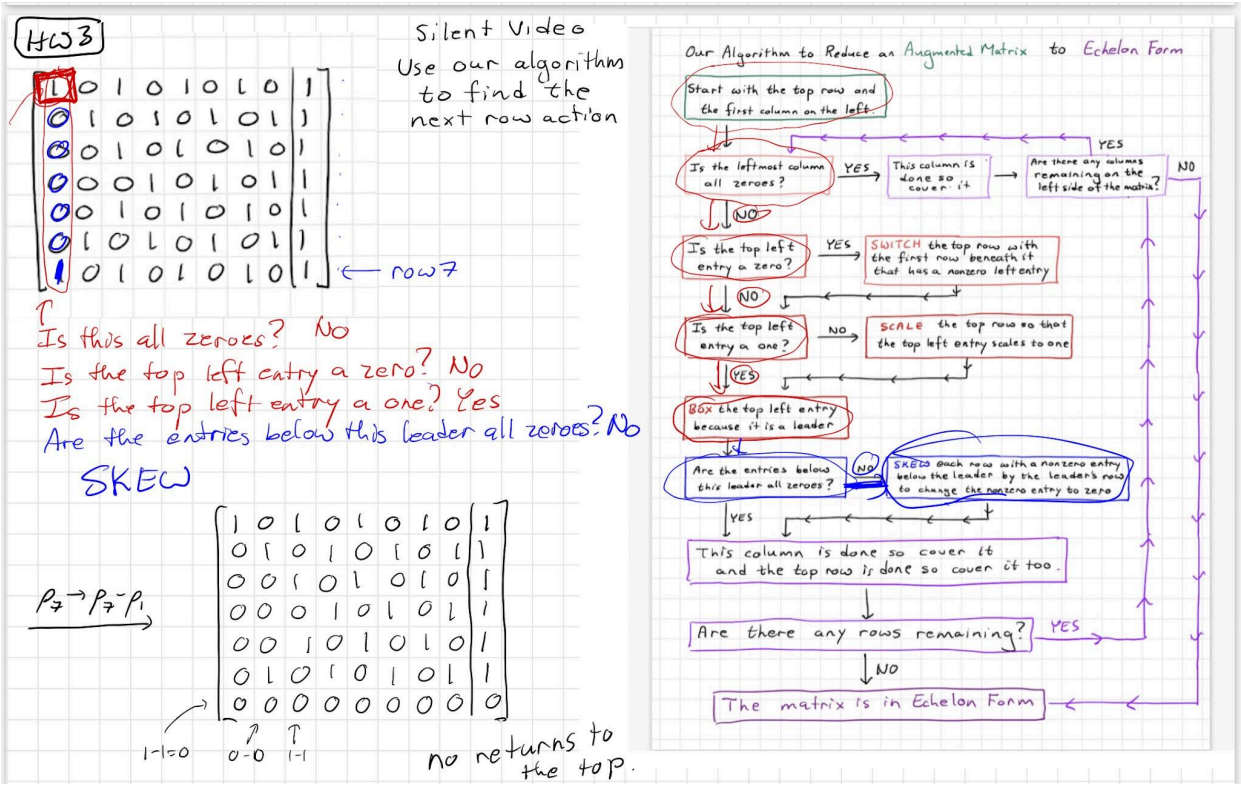
If your answer to HW1 is wrong, watch this [Video of HW1](#).

Check your answer to HW2 here:



If your answer to HW2 is wrong, watch the [Silent Video of HW2](#).

Check your answer to HW3 here:



If your answer to HW3 is wrong, watch the [Silent Video of HW3].

STUDENT QUESTIONS:

QUESTION: If I follow the flow chart, and I reach the box that says SCALE, how do I know which number to scale by?

SCALE explanations: (to change a nonzero leader to 1)

Suppose your leader is a 5 and you are changing it to 1,
then you scale the leader's row by (1/5)
because (1/5)5=1 will appear in the leader's place.
Note any zeroes in that row will go to (⅕)0=0
and all other numbers in the row will be multiplied by ⅕ as well.

If the leader is -2 and you are changing it to 1,
then you scale the leader's row by (-1/2)
because (-1/2)(-2)=1 will appear in the leader's place.
Note any zeroes in that row will go to (-1/2)0=0
and all other numbers in the row will be multiplied by (-½) as well.

If the leader is 1/8 and you are changing it to 1,
then you scale the leader's row by 8
because 8(1/8)=1 will appear in the leader's place.
Note any zeroes in that row will go to (8)0=0
and all other numbers in the row will be multiplied by (8) as well.

In general if the leader is R and you are changing it to 1,
then you scale the leader's row by (1/R) as long as R is not 0
because (1/R)R=1 will appear in the leader's place.
Note any zeroes in that row will go to (1/R)0=0
and all other numbers in the row will be multiplied by (1/R) as well.

QUESTION: If I follow the flow chart, and I reach the box that says SKEW, how do I know which skew to choose?

SKEW explanations: (to change a nonzero entry below a leader to 0)

Suppose you have an entry 5 in row4 below a leader in row1,
then you skew by the leaders' row,
row4 to row4-5row1,
because 5-5(1)=0 will appear under the leader.

Suppose you have an entry -8 in row5 below a leader in row2,
then you skew by the leaders' row,
row5 to row5+8row2,
because -8+8(1)=0 will appear under the leader.

Suppose you have an entry 3/7 in row4 below a leader in row2,
then you skew by the leaders' row,
row4 to row4-(3/7)row2,
because 3/7-(3/7)1=0 will appear under the leader.

In general if you have an entry k in rowi below a leader in rowj,
then you skew by the leaders' row,
rowi to rowi -k rowj,
because k-k(1) =0 will appear under the leader.

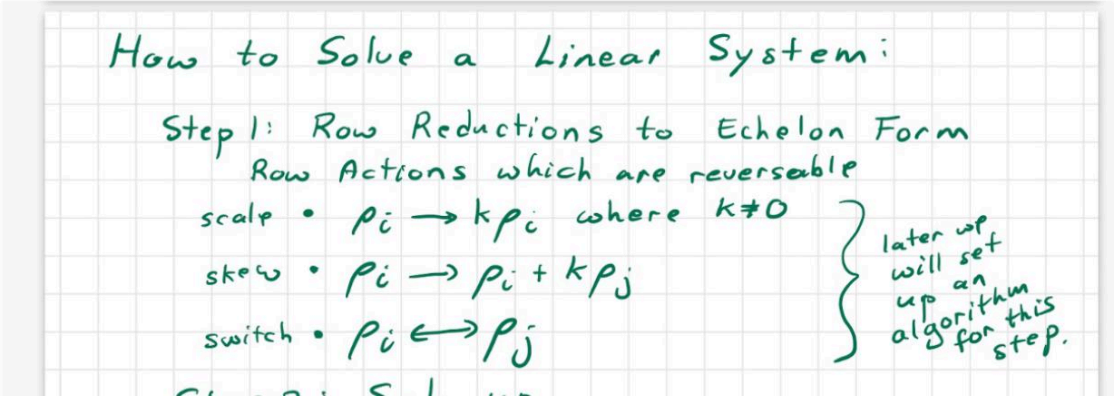
QUESTION: Where did we learn SWITCH, SCALE, and SKEW?

We first learned SWITCH, SCALE, and SKEW in Lesson 2 Part II:

Part II: How to Solve Any Linear System

Watch [Playlist 313F21-2-8to12]

"How to solve a Linear System" is in video 313F21-2-8 and the Example 1 rewritten using this technique is in video 313F21-2-8 and 313F21-2-9:



We've continued to practice using the exact same actions in our classwork today and will continue to use them all semester.