FIRST TERM Further Mathematics E-LEARNING NOTE

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SUBJECT: FURTHER MATHEMATICS **CLASS: SS1**

SCHEME OF WORK

WEEK	TOPIC
1	Indices: Basic Laws & Application of indices
2	Indicial and Exponential Equations
3	Logarithms - Laws and application
4	General review of basic concept of set theory
5	Operation of sets and Venn diagrams
6	Review of First Half Terms Lesson & Periodic Test
7	Binary operations and basic laws of binary operations (i) Definition (ii) Solution of simple problems on binary operations (iii) Closure, commutative, associative and distributive laws
8	Binary operations continues: (i) Solution to problems on laws of binary operations (ii) Identity and inverse elements of a given binary operations (iii) Addition and multiplication tables for binary operations
9	Surds: (i) Definition of surds (ii) Rules and manipulation of surds (iii) Rationalization of surds at the denominator and equality of surds.
10	Measures of central tendency: (i) Mean, Median and Mode of grouped and ungrouped data (ii) Estimation of mode from the histogram of a grouped data.
11	Revision
12	Examination

REFERENCE(S)

- **Further Mathematics project 1 by Tuttuh Adegun et al**
- **New General Mathematics for SSS1, SSS 2 and SSS 3 by M. F. Macrae et al**

WEEK ONE
TOPIC: INDICES
CONTENT

- Basic Concept of Laws of Indices
- Application of Laws of Indices

Basic Concept of Laws of Indices

A number of the form a^m where a is a real number, ***a*** is multiplied by itself ***m*** times,
The number **a** is called the **base** and the super script **m** is called the **index** (plural indices) or exponent.

1. $a^m \times a^n = a^{m+n}$ **--------------------Multiplication law**

Example: $p^3 \times p^2 = (p \times p \times p) \times (p \times p) = p^5$
Or $p^3 \times p^2 = p^{3+2} = p^5$

2. $a^m \div a^n = a^{m-n}$ **---------------------Division law**
Example: $p^6 \div p^4 = p^{6-4} = p^2$

3. $(a^m)^n = a^{mn}$ **----------------Power law**
Example: $(p^3)^2 = p^3 \times p^3 = p^{3+3} = p^6$
Or $p^{3 \times 2} = p^6$

4. $a^m \div a^m = a^{m-m} = a^0 = 1$
$a^m \div a^m = a^m/a^m = a^o = 1$
$a^0 = 1$ **..........................Zero Index**

Note : Any number raised to power of zero is 1

Example: $3^o = 1$, $c^o = 1$, $y^o = 1$

5. $(ab)^m = a^m b^m$ **-------------Product power law**
e.g. $(2xy)^2 = 4x^2y^2$

6. $a^{-m} = 1/a^m$ **------------- Negative Index**
Example: $2^{-1} = ½$, and $3^{-2} = 1/3^2 = 1/9$

7. $a^{1/n} = {}^n\sqrt{a}$ **-------------- Root power law**
Example **:** $9^{½} = \sqrt{9} = 3$
$27^{1/3} = {}^3\sqrt{27} = 3$ ie $(3)^3 = 3$

8. $a^{m/n} = (a^{1/n})^m = ({}^n\sqrt{a})^m$ **-----------Fraction Index**
or $a^{m/n} = (a^m)^{1/n} = ({}^n\sqrt{a})^m$

Example: $27^{2/3} = 3\sqrt{27} = 3^2 = 9$.

Evaluation

1. $27^{5/3}$ 2. 1000000000^0 3. $2^{x-1} \times 2^{2x+2}$

Application of Laws of Indices

Examples

Solve the following

(i) $32^{3/5}$ (ii) $343^{2/3}$ (iii) $64^{2/3}$ (iv) 0.001 (v) 14^0

Solution:

i) $32^{3/5} = (32^{1/5})^3 = (\sqrt[5]{32})^3$
$= 2^3 = 8$

ii) $343^{2/3} = (343^{1/3})^2 = (\sqrt[3]{343})^2$
$= (7^3)^{1/3})^2$
$= 7^2 = 49$

iii) $64^{2/3} = (64^{1/3})^2 = (4^3)^{1/3})^2 = 4^2$

iv) $(0.001)^3 = (1/100)^3 = (1/10)^3)^3 = (10^{-3})^3$
$= 10^{-9} = 1/10^9$

v) $14^0 = 1$

General Evaluation

Simplify the following (a) $216^{4/3}$ (b) $25^{1.5}$ (c) $(0.00001)^2$ (d) $32^{2/5}$ (e) $81^{3/4}$ (f) $625^{3/8} \times 25$

Reading Assignment : *Further Mathematics project book 1(New third edition).Chapter 2 pg.4 - 6*

Weekend Assignment

1) Evaluate $3^x = 1/81$ (a) 4 (b) -4 (c) -2 (d) 2
2) Simplify $2r^5 \times 9r^3$ (a) p^2 (b) $2p^4$ (c) P^3 (d) $18r^8$
3) Solve $3^{-y} = 243$ (a) -5 (b) 5 (c) 3 (d) -3
4) Solve $25^{-5n} = 625$ (a) 1/5 (b) 2/5 (c) 1 1/5 (d) – 2/5
5) Simplify $(0.0001)^2$ (a) 10^{-5} (b) 10^{-3} (c) 10^{-8} (d) 10^{-10}

Theory

1. $\frac{16^{3/2} \times 8^{2/3}}{32^{1/5}}$ 2. $\frac{3x^2 \times 4x^3}{6x^7}$

WEEK TWO

TOPIC: INDICIAL AND EXPONENTIAL EQUATIONS

CONTENT

- Exponential Equation of Linear Form
- Exponential Equation of Quadratic Form

Exponential Equation of Linear Form

Under exponential equation, if the base numbers of any equation are equal, then the power will be equal & vice versa.

Examples

Solve the following exponential equations

a) $(1/2)^{x} = 8$ b) $(0.25)^{x+1} = 16$ c) $3^{x} = 1/81$ d) $10^{x} = 1/0.001$ e) $4/2^{x} = 64^{x}$

Solution

a) $(1/2)^{X} = 8$
$(2^{-1})^{x} = 2^{3}$
$2^{-x} = 2^{3}$
$-x = 3$
$x = -3$

b) $(0.25)^{x+1} = 16$
$(25/100)^{x+1} = 4^{2}$
$(1/4)^{x+1} = 4^{2}$
$(4^{-1})^{x+1} = 4^{2}$
$4^{-x-1} = 4^{2}$
$-x-1 = 2$
$-x = 2 + 1$
$-x = 3$
$X = -3$

c) $3^{x} = 1/81$
$3^{x} = 1/3^{4}$
$3^{x} = 3^{-4}$
$x = -4$

d) $10^{x} = 1/0.001$
$10^{x} = 1000$
$10^{x} = 10^{3}$
$10^{x} = 10^{3}$
$x = 3$

e) $4/2^{x} = 64^{x}$
$4 \div 2^{x} = 64^{x}$
$2^{2} \div 2^{x} = 64^{x}$
$2^{2-x} = (2^{6})^{x}$
$2^{2-x} = 2^{6x}$
$2 - x = 6x$
$2 = 6x + x$
$2 = 7x$
Divide both sides by 7
$2/7 = 7x/7$
$x = 2/7$

Evaluation

Solve the following exponential equations

a) $2^{x} = 0.125$ b) $25^{(5x)} = 625$ c) $10^{x} = 1/100000$

Exponential Equation of Quadratic Form

Some exponential equation can be reduced to quadratic form as can be seen below.

Example:

Solve the following equations.

a) $2^{2x} – 6 (2^{x}) + 8 = 0$
b) $5^{2x} + 4 x 5^{x+1} – 125 = 0$
c) $3^{2x} – 9 = 0$

Solution

a) $2^{2x} – 6 (2^{x}) + 8 = 0$
$(2^{x})^{2} – 6 (2^{x}) + 8 = 0$
Let $2^{x} = y$
Then $y^{2} – 6y + 8 = 0$
Then factorize
$y^{2} – 4y – 2y + 8 = 0$
$y (y - 4) -2 (y -4) = 0$
$(y -2) (y - 4) = 0$
$y – 2 = 0$ or $y – 4 = 0$
$y = 2$ or $y= 4$
$y = 2, 4$

When y = 4 then, and When y = 2 then,
$2^{x} = 4$ $2^{x} = 2$
$2^{x} = 2^{2}$ $2^{x} = 2^{1}$
$x = 2$ $x = 1$
$x = 1$ and 2

b) $5^{2x} + 4 x 5^{x+1} – 125 = 0$
$(5^{x})^{2} + 4 x (5^{x} x 5^{1}) – 125 = 0$
Let $5^{x} = p$
$P^{2} + 4 x (p x 5) – 125 = 0$
$P^{2} + 4 (5p) – 125 = 0$
$P^{2} + 20p – 125 = 0$
Then, Factorize $p^{2} + 25p – 5p – 125 = 0$
$p (p + 25) - 5 (p + 25) = 0$
$(p - 5) (p + 25) = 0$
$p – 5 = 0$ $p + 25 = 0$
$p = 5$ or $p = - 25$
Since $5^{x} = p$, $p = 5$
$5^{x} = 5^{1}$
$x = 1$
$5^{x} = -25$
$x = \log_5 -25$ $x = \log_5 -25$

c) $3^{2x} – 9 = 0$
$(3^{x})^{2} - 9 = 0$
Let $3^{x} = a$
$a^{2} – 9 = 0$
$a^{2} = 9$
$a = \pm\sqrt{9}$
$a = \pm 3$
$a = 3$ or $– 3$
Since $3^{x} = a$, when $a = 3$
$3^{x} = 3^{1}$

$x = 1$

Since $3^x = a$, when $a = -3$

$3^x = -3$

$x = \log_3 -3$

Evaluation:

Solve: (a) $3(2^{2x+3}) - 5(2^{x+2}) - 156 = 0$ (b) $9^{2x+1} = (81^{x-2}/3^x)$

General Evaluation

Solve the following exponential equations.

a) $2^{2x+1} – 5(2^x) + 2 = 0$

b) $3^{2x} – 4(3^{x+1}) + 27 = 0$

Reading Assignment: *Further Mathematics Project Book 1(New third edition).Chapter 2 pg. 6- 10*

Weekend Assignment

1. Solve for x : $(0.25)^{X+1} = 16$ (a) -3 (b) 3 (c) 4 (d) -4
2. Solve for x : $3(3)^X = 27$ (a) 3 (b) 4 (c) 2 (d) 5
3. Solve the exponential equation : $2^{2x} + 2^{x+1} – 8 = 0$ (a) 1 (b) 2 (c) 3 (d) 4
4. The second value of x in question 3 is (a) -1 (b) 1 (c) 2 (d) No solution
5. Solve for x : $10^{-X} = 0.000001$ (a) 4 (b) 6 (c) -6 (d) 5

Theory

Solve the following exponential equations

(1) $(3^x)^2 + 2(3^x) – 3 = 0$ (2) $5^{2x+1} - 26(5^x) + 5 = 0$

WEEK THREE

TOPIC: LOGARITHM - SOLVING PROBLEMS BASED ON LAWS OF LOGARITHM

CONTENT

- Logarithm of numbers (Index & Logarithmic Form)
- Laws of Logarithm
- Logarithmic Equation
- Change of Base
- Standard forms
- Logarithm of numbers greater than one
- Multiplication and divisions of numbers greater than one using logarithm
- Using logarithm to solve problems with roots and powers (no > 1)
- Logarithm of numbers less than one.
- Multiplication and division of numbers less than one using logarithm
- Roots and powers of numbers less than one using logarithm

Logarithm of numbers (Index & Logarithmic Form)

The logarithm to base **a** of a number **P**, is the index **x** to which **a** must be raised to be equal to **P**.

Thus if $P = a^x$, then x is the logarithm to the base **a** of **P**. We write this as x = log a P. The relationship $\log_a P = x$ and

$a^x = P$ are equivalent to each other.

$a^x = P$ is called the index form and $\log_a P = x$ is called the logarithm form

Conversion from Index to Logarithmic Form

Write each of the following in index form in their logarithmic form

a) $2^6 = 64$ b) $25^{1/2} = 5$ c) $4^4 = 1/256$

Solution

a) $2^6 = 64$

$\text{Log}_2 64 = 6$

b) $25^{1/2} = 5$

$\text{Log}_{25} 5 = 1/2$

c) $4^{-4} = 1/256$

$\text{Log}_4 1/256 = -4$

Conversion from Logarithmic to Index form

a) $\text{Log}_2 128 = 7$ b) $\log_{10} (0.01) = -2$ c) $\text{Log}_{1.5} 2.25 = 2$

Solution

a) $Log_2 128 = 7$
$2^7 = 128$

b) $Log_{10}(0.01) = -2$
$10^{-2} = 0.01$

c) Log1.5 2.25 = 2
$1.5^2 = 2.25$

Laws of Logarithm

a) let $P = b^x$, then $log_b P = x$
$Q = b^y$, then $log_b Q = y$
$PQ = b^x \text{ X } b^y = b^{x+y}$ (laws of indices)
$Log_b PQ = x + y$
:. $Log_b PQ = log_b P + Log_b Q$

b) $P \div Q = b^x \div by = b^{x+y}$
$Log_b P/Q = x - y$
:. $Log_b P/Q = logbP - logbQ$

c) $P^n = (b^x)^n = b^{xn}$
$Log_b p^n = nb^x$
:. $LogP^n = logbP$

d) $b = b^1$
:. $Log_b b = 1$

e) $1 = b^0$
Logb1 = 0

Example

Solve each of the following:

a) $Log_3 27 + 2log_3 9 - log_3 54$

b) $Log_3 13.5 - log_3 10.5$

c) $Log_2 8 + log_2 3$

d) Given that $log_{10}2 = 0.3010$ $log_{10}3 = 0.4771$ and $log_{10}5 = 0.699$ find the $log_{10}64 + log_{10}27$

Solution

a) $Log_3 27 + 2 \, log_3 9 - log_3 54$
$= log_3 27 + log_3 9^2 - log_3 54$
$= log_3 (27 \text{ x } 9^2/54)$
$= log_3 (27^1 \text{ x } 81/54) = log_3 (81/2)$
$= log_3 3^4/ log32$
$= 4log_3 3 - log_3 2$
$= 4 \text{ x } (1) - log_3 2 = 4 - log_3 2$
$= 4 - log_3 2$

b) $log_3 13.5 - log_3 10.5$
$= log_3 (13.5) - Log310.5 = log_3 (135/105)$
$= log_3 (27/21) = log_3 27 - log_3 21$
$= log_3 3^3 - log_3 (3 \text{ x } 7)$
$= 3log_3 3 - log_3 3 - log_3 7$
$= 2 - Log_3 7$

c) $Log_2 8 + Log_3 3$

$= \log_2 2^3 + \log_3 3$

$= 3\log_2 2 + \log_3 3$

$= 3 + 1 = 4$

d) $\log_{10} 64 + \log_{10} 27$

$\log_{10} 2^6 + \log_{10} 3^3$

$6 \log_{10} 2 + 3 \log_{10} 3$

$6 (0.3010) + 3(0.4771)$

$1.806 + 1.4314 = 3.2373$.

EVALUATION

1. Change the following index form into logarithmic form.
   (a) $6^3 = 216$ (b) $3^3 = 1/27$ (c) $9^2 = 81$
2. Change the following logarithm form into index form.
   (a) $\text{Log}_8 8 = 1$ (b) $\log_{½} ¼ = 2$
3. Simplify the following
   a) $\text{Log}_5 12.5 + \log 52$ b) $½ \log_4 8 + \log_4 32 – \log_4 2$ c) $\log_3 81$
4. Given that log 2 = 0.3010, log3 0.4770, log5 = 0.6990, find the value of log 6.25 + log1.44

Logarithmic Equation

Solve the following equation:

a) $\text{Log}10 (x^2 – 4x + 7) = 2$

b) $\text{Log}_8 (r^2 – 8r + 18) = 1/3$

Solution

a) $\text{Log}_{10} (x^2 – 4x + 7) = 2$

$x^2 – 4x + 7 = 10^2$ (index form)

$x^2 – 4x + 7 = 100$

$x^2 – 4x + 7 – 100 = 0$

$x^2 – 4x – 93 = 0$

Using quadratic formula

$$x = \frac{- b \pm \sqrt{b^2 – 4ac}}{2a}$$

$a = 1, b = - 4, c = - 93$

$$x = \frac{- (- 4) \pm \sqrt{(- 4)^2 – 4 \times 1 \times (- 93)}}{2 \times 1}$$

$$= \frac{+ 4 \pm \sqrt{16 + 372}}{2}$$

$= + 4 \pm \sqrt{388}/2$

$= x = 4 + \sqrt{388}/2$ or $4 - \sqrt{388}/2$

$x = 11.84$ or $x = - 7.85$

2) $\text{Log}_8 (x^2 – 8x + 18) = 1/3$

$x^2 – 8x + 18 = 8^{1/3}$

$x^2 – 8x + 18 = (2)^{3X1/3}$

$x^2 – 8x + 18 = 2$

$x^2 – 8x\ 18 – 2 = 0$

$x^2 – 8x + 16 = 0$

$x^2 – 4x – 4x + 16 = 0$

$x(x - 4) -4 (x - 4) = 0$

$(x - 4) (x - 4) = 0$

$(x - 4)$ twice

$x = + 4$ twice

Change of Base

Let $\log_b P = x$ and this means $P = b^x$

$\text{Log}_c P = \log_c b^x = x \log_c b$

If $x \log_c b = \log_c P$

$$x = \frac{\log_c P}{\log_c b}$$

$$\therefore \quad \log_c P = \frac{\log_c P}{\log_c b}$$

Example :

Shows that $\log_a b \ \text{x} \ \log_b a = 1$

$$\text{Log}_a b = \frac{\log_c b}{\log_c a}$$

$$\text{Log}_b a = \frac{\log_c a}{\log_c b}$$

$$\therefore \quad \log_a b \ \text{x} \log_b a = \log_c b \ \text{x} \log_c a$$

$$\log_c a + \log_c b = 1$$

Evaluation

Solve (i) $\text{Log}_3 (x^2 + 7x + 21) = 2$ (ii) $\text{Log}_{10} (x^2 - 3x + 12) = 1$

(iii) $5^{2x+1} - 26(5^x) + 5 = 0$ find the value of x

Logarithm of numbers greater than one

Numbers such as 1000 can be converted to its power of ten in the form 10^n where n can be term as the number of times the decimal point is shifted to the front of the first significant figure i.e. $10000 = 10^4$

Number	Power of 10
100	10^2
10	10^1
1	10^0
0.01	10^{-3}
0.10	10^{-1}

Note: One tenth; one hundredth, etc are expressed as negative powers of 10 because the decimal point is shifted to the right while that of whole numbers are shifted to the left to be after the first significant figure.

A number in the form $A \times 10^n$, where A is a number between 1 and 10 i.e. $1 \leq A < 10$ and n is an integer is said to be in ***standard form*** e.g. 3.835×10^3 and 8.2×10^{-5} are numbers in standard form.

Examples

Express the following in standard form

1) 7853 2) 382 3) 0.387 4) 0.00104

Solution

1) $7853 = 7.853 \times 10^3$
2) $382 = 3.82 \times 10^2$
3) $0.387 = 3.87 \times 10^{-1}$
4) $0.00104 = 1.04 \times 10^{-3}$

Base ten logarithm of a number is the power to which 10 is raised to give that number e.g.

$628000 = 6.28 \times 10^5$

$628000 = 10^{0.7980} \times 10^5$

$= 10^{0.7980\ +\ 5}$

$= 10^{5.7980}$

Log 628000 = 5.7980

Integer Fraction (mantissa)

If a number is in its standard form, its power is its integer i.e. the integer of its logarithm e.g. log 7853 has integer 3 because 7853 = 7.853 x 10^3

Examples: Use tables (log) to find the complete logarithm of the following numbers.

(a) 80030 (b) 8 (c) 135.80

Solution:

(a) 80030 = 4.9033

(b) 8 = 0.9031

(c) 13580 = 2.1329

Multiplication and Division of number greater than one using logarithm

To multiply and divide numbers using logarithms, first express the number as logarithm and then apply the addition and subtraction laws of indices to the logarithms. Add the logarithm when multiplying and subtract when dividing.

Examples: Evaluate using logarithm.

1. 4627 x 29.3
2. 8198 ÷ 3.905
3. $\frac{48.63 \ \text{x} \ 8.53}{15.39}$

Solutions

1. 4627 x 29.3

	No	Log
	4627	3.6653
	29.3	+ 1.4669
Antilog →	135600	5.1322

∴ 4627 x 29.3 = **135600**

To find the Antilog of the log 5.1322 use the antilogarithm table:

Check 13 under 2 diff 2 (add the value of the difference) the number is 0.1356. To place the decimal point at the appropriate place, add one to the integer of the log i.e. 5 + 1 = 6 then shift the decimal point of the antilog figure to the right (positive) in 6 places.

= 135600

2. 819.8 x 3.905

	No	Log
	819.8	2.9137
	3.905	0.5916
antilog →	**209.9**	2.3221

∴ 819.8 ÷ 3.905 = 209.9

3. $\frac{48.63 \ \text{x} \ 8.53}{15.39}$

No	Log

	48.63	1.6869
	8.53	+0.9309
		2.6178
	÷ 15.39	-1.1872
antilog →	**26.95**	1.4306

$$\therefore \quad \frac{48.63 \div 8.53}{15.39} = 26.96$$

Evaluation:

1. Use table to find the complete logarithm of the following:
   (a) 183 (b) 89500 (c) 10.1300 (d) 7

2 Use logarithm to calculate. $\frac{3612 \times 750.9}{113.2 \times 9.98}$

Using logarithm to solve problems with powers and root (numbers greater than one).

Examples:

Evaluate

(a) 3.53^3 (b) $\sqrt[4]{40000}$ (c) $\sqrt{\frac{94100 \times 38.2}{5.68^3 \times 8.14}}$ correct to 2s.f.

Solution

No.	**Log**
3.53^3	0.5478 x 3
44.00	1.6434

$\therefore 3.53^3 = 44.00$

(b) $\sqrt[4]{4000}$

No.	**Log**
$\sqrt[4]{4000}$	3.6021 ÷ 4
7.952	0.9005

$\therefore \quad \sqrt[4]{4000} = 7.952$

(c) $\sqrt{\frac{94100 \times 38.2}{5.683^3 \times 8.14}}$

Find the single logarithm representing the numerator and the single logarithm representing the denominator, subtract the logarithm then find the anti log.
(Numerator – Denominator).

No	**Log**	
$\sqrt{94100}$	4.9736 ÷ 2 = 2.4868	
38.2	1.5821	
Numerator	**4.0689**	→ 4.0689

5.68^3 $0.7543 \times 3 = 2.2629$
8.14 $\underline{0.9106}$
Denominator $\underline{\mathbf{3.1735}} \rightarrow \underline{\mathbf{3.1735}}$
7.859 $\underline{\mathbf{0.8954}}$

$$\therefore \sqrt{\frac{94100 \times 38.2}{5.68^3 \times 8.14}} = 7.859$$

$$\simeq \mathbf{7.9\ (2.sf)}$$

Evaluation:

Evaluate using logarithm.

$$\frac{95.3 \times \sqrt{318.4}}{1.29^5 \times 2.03}$$

Logarithm of number less than one

To find the logarithm of number less than one, use negative power of 10 e.g.

$0.037 = 3.7 \times 10^{-2}$
$10^{0.5682 \times 10-2}$
$10^{0.5682 + (-2)}$
$10^{-2\ 5682}$

Log 0.037 = 2 . 5682

$\bar{2}$. 5682

Integer decimal fraction (mantissa)

Example:

Find the complete log of the following.

(a) 0.004863 (b) 0.853 (c) 0.293

Solution

Log 0.004863 = $\bar{3}.6369$
Log 0.0853 = $\bar{2}.9309$
Log 0.293 = $\bar{1}.4669$

Evaluation

1. Find the logarithm of the following:
   (a) 0.064 (b) 0.002 (c) 0.802

Using logarithm to evaluate problems of Multiplication, Division, Powers and roots with numbers less than One

Examples:

1. 0.6735 x 0.928 2. 0.005692 ÷ 0.0943 3. 0.6104^3

4. $\sqrt[4]{0.00083}$ 5. $\sqrt[3]{0.06642}$

Solution

1. 0.6735 x 0.928

No.	Log.
0.6735	$\bar{1}.8283$
0.928	$\bar{1}.9675$
0.6248	$\mathbf{\bar{1}.7958}$

∴ 0.6735 x 0.928 = 0.6248

2. $0.005692 \div 0.0943$

No	Log
0.005692	$\bar{3}.7553$
$\div$ 0.0943	$\bar{2}.9745$
0.06037	**2.7808**

3. 0.6104^3

No	Log
0.6104^3	$\bar{1}.7856 \times 3$
0.2274	1.3568
$\therefore\ 0.6104^3$	= **0.2274**

$\therefore\ 0.005692 \div 0.943 =$ **0.6037**

4. $\sqrt[4]{0.00083}$

No.	Log.
$\sqrt[4]{0.00083}$	$\bar{4}.9191 \div 4$
0.1697	1.2298

$\therefore \sqrt[4]{0.06642} =$ **0.1697**

5. $\sqrt[3]{0.6642}$

No.	Log.
$\sqrt[3]{0.6642}$	$\bar{2}.8223 \div 3$
	$\bar{2}.1 + 1 + 0.8223 \div 3$
	$\bar{3} + 1 .8223 \div 3$
	$\bar{1} + 0.6074$
0.405	$\bar{1}.6074$

$\sqrt[3]{0.6642} =$ **0.405**

Note: 3 cannot divide 2 therefore subtract 1 from the negative integer and add 1 to the positive decimal fraction so as to have 3 which is divisible by 3 without remainder.

Evaluation:

Evaluate using logarithm tables:

(1) $\dfrac{\sqrt{12.3} \times 0.0034^3}{132.5}$

(2) $\dfrac{23.97 \times 0.7124}{3.877 \times \sqrt{52.18}}$

General Evaluation

1. Solve the logarithmic equation: $\log_4 (x^2 + 6x + 11) = \frac{1}{2}$
2. $\log_2 (x^2 - 2) = \log_2 (x-1) + 1$

3. Evaluate $\sqrt[5]{(0.1684)^3}$

4. $6.28 \times \sqrt{\frac{304}{981}}$

5. $\frac{16^{3/2} \times 8^{2/3}}{32^{1/5}}$

Reading Assignment: *Further Mathematics Project Book 1(New third edition).Chapter 2 pg.10- 16*

Weekend Assignment

1.) If log81/64 = x, find the value of x (a) 2 (b) 1 (c) -3 (d) -4.

2.) Solve $9^{(1-x)} = (1/27)^{x+1}$ (a) -5 (b) -1 (c) 1 (d) ½

Use table to find the log of the following:

3.) 900 (a) 3.9542 (b) 1.9542 (c) 2.9542 (d) 0.9542

4.) 0.000197 (a) 4.2945 (b) 4.2945 (c) 3.2945 (d) 3.2945

5.) Use antilog table to write down the number whose logarithms is 3.8226.
(a) 0.6646 (b) 0.06646 (c) 0.006646 (d) 66.46

Theory

(1.) Find the value of x for which $\log_{10}(4x^2 + 1) - 2\log_{10} x - \log_{10} 2 = 1$ is valid.

(2.) Evaluate using logarithm. $\sqrt[3]{69.5^2 - 30.5^2}$

WEEK FOUR

TOPIC: REVIEW OF BASIC CONCEPT OF SET

WEEK FIVE

TOPIC: SETS

- Idea of a set, set notations and applications.
- Disjoint sets , Venn diagram

A. IDEA OF SET, NOTATIONS, APPLICATIONS.

Definitions:

A set can be defined as a group or a collection of well defined objects or numbers e.g collection of books, cooking utensils.

A set is denoted by capital letters such as P, Q, and R e.t.c while small letters are used to denote the elements e.g. a, b, c

Elements of a set: These are the elements or members of a given set. The elements are separated by commas and enclosed by a curly bracket {}
e.g M ={ 1, 3 ,5, 7, 11}, 1 is an element of M.

Example: Write down the elements in each of the following sets.

A = {Odd numbers from 1 to 21}
F = {factors of 30}
M = {Multiples of 4 up to 40}

Solution:

A = { 1,3,5,7,9,11,13,15,17,19,21}
F = {1, 3, 5, 6, 10, 15, 30}
M = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40}

Cardinality of a set: This is the number of elements in a set.

Example: Given that μ= {all the days of the week}, B= {all days of the week whose letter begin with s}

1. List all the elements of μ
2. List the members of B
3. What is n (μ) 4. What is n(μ) + n(B)

Solution:

1.μ = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
2. B = {Sunday, Saturday}
3. n (μ) = 7
4. n (μ) + n(B) = 7 + 2= 9

Set notation: A set can be described algebraically using inequality and other symbols. E.g B = {x: -10≤x ≤ 3, x is an integer}

Example: List the members of the following sets

1. A= {x: 5< x< 8} 2. B= {x: 0≤ x≤ 5}

Solution;

1. A = {6, 7} 2. B = {0, 1, 2, 3, 4, 5}

GENERAL/REVISION EVALUATION: If μ= {all positive integers ≤ 30}, M= {all even number ≤ 20}, N = {all integers: 10≤ x≤ 30}

Find 1. n (μ) 2.n (N) 3. n (N) + n(s) 4. n (M) + n(N)

B. Types of sets:

Finite and Infinite set: Finite set is a set in which all its members can be listed.

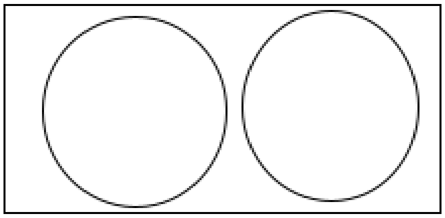
Infinite set: An infinite set is a set in which all its members cannot be listed.

Empty (Null) set: A set without any member. It is usually denoted by { } or Ø.

Subset and Supersets: If we have 2 sets A and B such that all the elements in A is contained in B, then A is a subset of B. Subset is denoted by $\underline{C}$ e.g. A $\underline{C}$ B. If there is at least one element in set B but not in A, then B is a superset of A.

Disjoint set: Two sets are disjoint when there is no common element between them. i.e no intersection.

A n B = Ø



Universal set: This is a set that contains all the members under consideration for any given problem. It is denoted by μ or €.

Complementary Set: This is a set that contains the members in the universal set that are not in set A. It is denoted by A^c or A^1.

Intersection of sets: This is the set which consists all the common elements in a given two or more sets. It is denoted by n.

Union of sets: This is the set of all members that belong to A or to B or to both A and B. It is denoted by u.

Example: If the universal set μ= {x: 1≤ x ≤ 12} and its subsets D, F and G are given as follows. D = {x: 2<x<8}, F={x: 4≤ x≤ 10}, G={x: 1< x ≤ 4}
Find (a) D U F (b) D n F (c) G^1 (d) (D n G)^1

Solution:

μ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
D = {3, 4, 5, 6, 7}
F = {4, 5, 6, 7, 8, 9, 10}
G = {2, 3, 4}

(a) D U F ={ 3, 4, 5, 6, 7,8, 9,10}
(b) DnF = {4, 5, 6, 7}
(c) G^1 = { 1, 5, 6, 7, 8,9,10,11,12]
(d) (D n G)^1
D n G = {3, 4}
(D n G)^1 = {1, 2, 5, 6, 7,8, 9,10,11,12}

Relationship between union and intersection of sets

n(A or B) = n(A) + n(B) –n(A and B)\
or n(AUB) = n(A) + n(B) - n(A n B)

Example:
If n(A)=23, n(B)= 12, n(AUB) = 35, find n(AnB) and comment on set A and B.

Solution

n(AUB) = n(A) + n(B) – n(AnB)
35 = 23 + 12 –n(AnB)
n(AnB) = 35- 35
n (AnB) = 0
Set A and B are disjoint.

Evaluation:

1. A and B are two sets. The number of elements in AUB is 49, the number in A is 22 and the number in B is 34.How many elements are in AnB?

2. The universal set μ ={ set of all integers}, p= {x:x≤ 2}, Q= { x: -7≤ x ≤15}R ={x: -2 ≤x ≤ 19}
Find 1. PnQ 2. P n (Q UR^1)

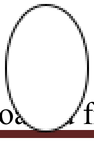
Venn diagrams:

The Venn diagram is a geometric representation of sets using diagrams which shows different relationship between two or more sets. In order words, it is the diagrammatical representation of relationships between two or more sets. The operations of intersection, union and complementation of sets can be demonstrated by using Venn diagrams.

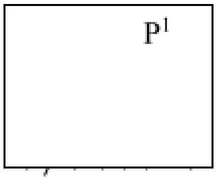
Venn diagram representation

E or U

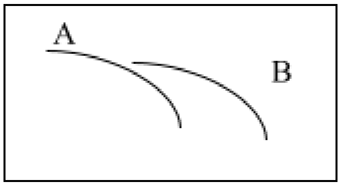
The rectangle represents the universal set i.e E or U



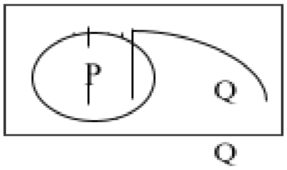
The oval shape represents the subset A.



The shaded portion represents the complement of set P i.e p^1 or P^c

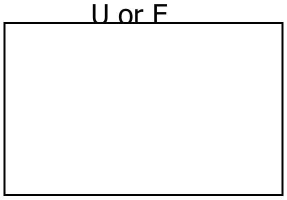


The shaded portion shows the elements common to A and B i.e A∩B or A intersection B.

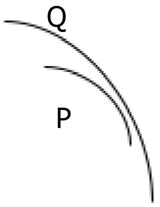


The shades portion shows P intersection $Q^{|}$ i.e $P \cap Q^{|}$

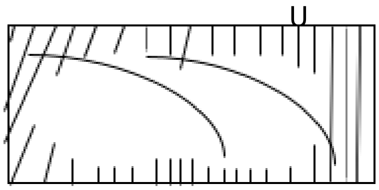
The shaded portion shows A U B i.e A union B



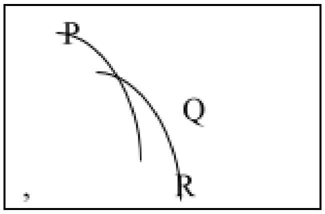
This shows that P and Q have no common element. i.e P and Q are disjoint sets i.e P∩Q= Φ



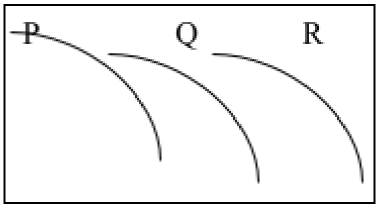
P is a subset of Q i.e P C Q



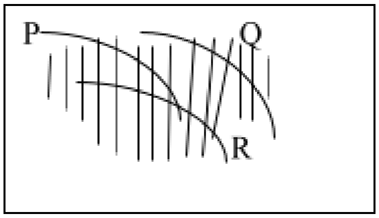
P| ∩ Q| or (P U Q)|. This shows elements that are neither in P nor Q but are represented in the universal set



This shows the element common to set P,Q and R i.e the intersection of three sets P,Q and R i.e $P \cap Q \cap R$



This shows the elements in P only, but not in Q and R i.e $P \cap Q^{l} \cap R^{l}$



This shaded region shows the union of the three sets i.e $P \cup Q \cup R$

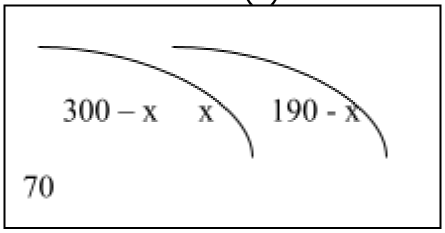
Use of Venn diagrams to solve problems involving two sets

Examples:

1. Out of the 400 final year students in a secondary school, 300 are offering Biology and 190 are offering Chemistry. If only 70 students are offering neither Biology nor Chemistry. How many students are offering (i) both Biology and Chemistry? (ii) At least one of Biology or Chemistry?

<u>Solution</u>

n(E)= 400



Let the number of students who offered both Biology and Chemistry be X i.e $(B \cap C)= X$. from the information given in the question

$n(E)= 400$

$n(B)= 300$

$n(C)= 190$

$n(B \cup C)^{l}= 70$

since the sum of the number of elements in all region is equal to the total number of elements in the universal sets, then:

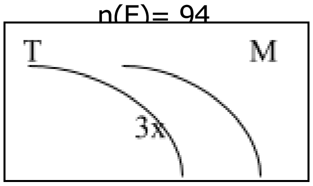
$300 - x + x + 190 – x + 70 = 400$

560 – x= 400
-x= 400 – 560
X= 160
Number of students offer both Biology and Chemistry= 160
(ii)no of students offering at least one of biology and chemistry from the Venn diagram this includes those who offered biology only, chemistry only and those whose offered both i.e
300 – x + 190 – x + x= 490
490 – 160 (from (i) above)= 330

2. In a youth club with 90 members, 60 likes modern music and 50 likes traditional music. The member of them who like both traditional and modern music are three times those who do not like any type of music. How many members like only one type of music

Solution
Let the members who do not like any type of music = X
Then,
n(TnM)= 3X
Also,
n(E)= 94
n(M)=60
n(T)= 50
$n(MUT)^{I}$= X

n(E)= 94

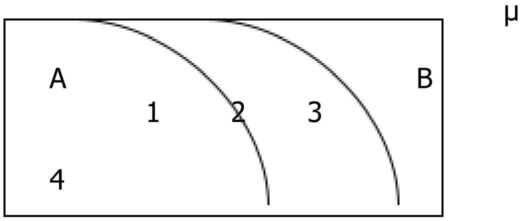


Since the sum of the number of elements in all region is equal to the total number of elements in the universal set, then
60 – 3X + 3X + 50 – 3X = 94
110 – 2X= 94
16= 2X
Divide both sides by 2

$$\frac{16}{2} = \frac{2X}{2}$$

X= 8
Therefore number of member who likes only one type of music are those who like modern music only + those who like traditional music only
60 -3x + 3X + 50 – 3X= 110
110 – 6 x 8 (from above)
= 110 – 48
= 62

Two Venn diagram;



Where 1 = AnB^{I} , 2 = AnB, 3 = $A^{I}nB$, 4 = $(AUB)^{I}$
Therefore, μ = 1 + 2 + 3 + 4

$\mu = n(AnB^I) + n(AnB) + n(A^InB) + n(AUB)^I$

Example 1: In a class of 40 students, every student had to study French or Russian or both subjects. 25 students studied French and 20 studied Russian. Find the number of students who studied both languages.

Solution:
Let μ = {All the students}
F = {French students}, R = {Students studying Russian}
μ = 40, n(F)= 25, n(R) = 20
n(Fn R)= x
$n(FnR^I) = 25-x$
$n (Rn F^I)= 20- x$
μ = 25 –x +x + 20-x
40= 45 –x
x = 45- 40
x=5, n(FnR) = 5 students.

Evaluation
Two questions A and B were given to 50 students as class work 23 of them could answer question A but not B. 15 of them could answer B but not A. If 2x of them could answer none of the two questions and 2 could answer both questions.
a) Represent the information in a Venn diagram. (b) Find the value of x

General evaluation
1. In a senior secondary school, 90 students play hockey or football. The numbers that play football is 5 more than twice the number that play hockey. If 5 students play both games and every students in the school plays at least one of the game. Find:
a) The number of students that play football
b) The number of student that play football but not hockey
c) The number of students that play hockey but not football
2. A, B and C are subset of the universal set U such that
U={0,1,2,3,4………….12}
A={X: 0≤X7} B= {4,6,8,10,12} C= {1<y<8} where Y is a prime number.
a) Draw a venn diagram to illustrate the information
b) Find (i) BUC (ii) A ∩ B∩C

Reading assignment: NGM bk1 pg 89 – 92 and Ex 8d number 11 pgs 91- 92

Weekend Assignment
1. Given that μ= {-10≤ x ≤ 10}, p= { -10 < x< 10}, Q= { -5 < x ≤ 3}. Which of the following is correct? I P^I n Q II P U Q =μ III P^I C Q^I
A. I and II only B. I and III only C II and III only
2. P and Q are subsets of the μ={x is an integer and 1< x < 15}, P= { x is odd} and Q= { x is prime}, find $n(P^I n Q^I)$ A. 3 B. 4 C. 5
Use the information below to answer question 3 and 4, μ= {1, 2, 3… 10}, A= {2, 4, 6, 8, 10} B= {1, 3, 9} and C = {2, 5, 7}
3. A^I n C is A.{5, 7} B. { 1, 3, 4} C. { 6,7,8,9}
4. B^I U C A.{2,4,5,7,8,10} B.{2,4,5,6,7,8,10} C.{ 1, 2,3,4,5, 9}
5. A set contains 7 members; find the number of subsets that can be obtained from it. A. 32 B. 64 C. 128

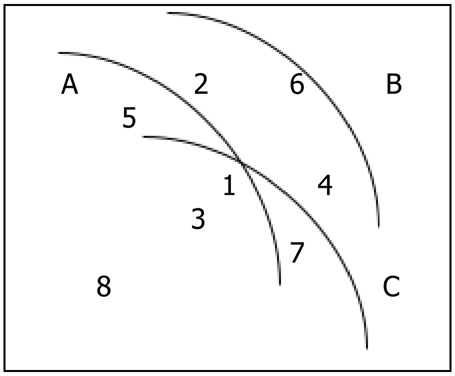
Theory

1. During one year in a school, 5/8 of the students had measles, ½ had chickenpox and 1/8 had neither. What fraction of the school had both measles and chickenpox?
2. In a class of 50 pupils, 24 like oranges, 23 like apple and 7 like the two fruits.

a) How many do not like oranges and apples (b) What percentage of the class like apples only

WEEK FIVE
TOPIC: OPERATION OF SET AND VENN DIAGRAMS
CONTENT: Use of Venn diagrams to solve problems involving three sets

Three Venn diagram:



1= AnBnC
2 = $AnBnC^I$
3 = AnB^InC
5 = AnB^InC^I
6 = A^1nBnC^1
7 = A^1nB^InC

$4 = A^{I}nBnC$ $8 = (AnBnC)^{1}$

Example: A school has 37 vacancies for teachers, out of which 22 are for English, 20 for History and 17 for Fine Art. Of these vacancies 11 are for both English and History, 8 for both History and Fine Art and 7 for English and Fine Art. Using a Venn diagram, find the number of teachers who must be able to teach:

(a.) all the three subjects
(b.) Fine Art only
(c.) English and History but not Fine Art.

Solution:

Let μ = {All vacancies for teachers}
E = {English vacancies}
H = {History vacancies}
F = {Fine Art vacancies}

μ = 37, n(E)= 22, n(H)= 20, n(F)= 17, n{EnH}= 11, n(HnF)= 8, n(EnF)= 7

(1) Let n(EnFnH) = y

$n\ (EnH^{I}nF)= n(E)- (7-y+y+11-y)$
$= 22- (18-y) \quad = \quad 4 + y$

$n(E^{I}nHnF) = n(H) – (11-y+y+8-y)$
$= 20- (19-y) \quad = \quad 1+y$

$n(E^{I}nH^{1}nF)= n(F) – (7-y +y+8-y)$
$= 17 – (15- y) \quad = \quad 2 +y$

μ= 4+y+11-y+1+y+y+8-y+7-y+2+y
37= 33 + y
y = 37- 33
y = 4.
n(EnHnF) = 4 teachers

(2.) Fine Art only, $n(E^{I}nH^{I}nF) = 2+ y$
$= 2+4 \quad = 6$ teachers

(3.) English and History but not Fine Art i.e English and History only
$n(EnHnF^{I}) = 11-y$
$= 11- 4 = 7$ teachers.

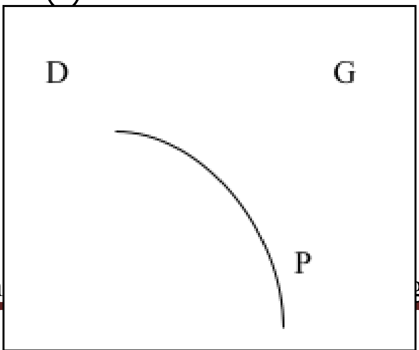
Examples:

1. In a survey of 290 newspaper readers, 181 of them read daily times, 142 read the Guardian, 117 read the Punch and each read at least one of the paper, if 75 read the Daily Times and the Guardian,60 read the Daily Times and Punch and 54 read the Guardian and the punch

a) Draw a venn diagram to illustrate the information
b) How many read:
   (i) all the three papers
   (ii) exactly two of the papers
   (iii) exactly one of the paper

Solution

n (E)= 290



$n(E)= 290$
$n(D)= 181$
$n(G)= 142$
$n(D\cap G)= 75$
$n(D\cap P)= 60$
$n(G\cap P)= 54$
from the venn diagram, readers who read Daily Times only
$=181 - (160 - X + 75 - X +X)$
$=181 - (135 - X)$
$= 46 + X$
Punch readers only
$=117 - (60 - X + 54 - X + X)$
$117 - (114 - X)$
$117 - 114 + X$
$=3 +X$
Guardian readers only
$=142 - (75 - X + 54 - X + X)$
$=142 - (129 - X)$
$=142 - 129 + X$
$=13 + X$
Where:
X is the number of readers who read all the three papers
Since the sum of the number of elements in all regions is equal to the total number of elements in the universal set, then:
$46 + X + 75 - X + 13 + X + 60 - X + X + 54 - X + 3 + X = 90$
$251 + X = 290$
$X = 290 - 251$
$X= 39$

B(i): number of people who read all the three paper= 39
(ii) from the venn diagram, number of people who read exactly two papers
$= 60 - X + 75 - X + 54 - X$
$=189 - 3X \quad = 189 - 3(39)$ from the above
$=189 - 117 \quad = 72$
(iii) also, from the venn diagram, number of people who read exactly only one of the papers
$=46 + X + 13 + X + 3 +X$
$=162 +3X \quad =162 + 3(39)$
$=162 + 117 \quad = 179$
(iv)number of Guardian reader only
$=13 + X$
$=13 + 39 \quad = 52$

2. A group of students were asked whether they like History, Science or Geography. There responds are as follow

Subject liked	**Number of students**
All three subject	7
History and Geography	11
Geography and Science	09

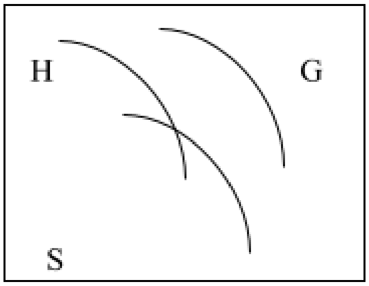
History and Science	10
History only	20
Geography only	18
Science only	16
None of the three subject	03

a) Represent the information in a Venn diagram

b) How many students were in the group? c) How many students like exactly two subjects

Solution

a) n(E)= ?



b) Number of students in the group = sum of the elements in all the regions i.e
Number of students in the group = 20 + 18 + 16 + 4 + 3 + 2 + 7 + 3 = 73

c) Number of students who like exactly two subject = 4 + 3 + 2= 9

Evaluation

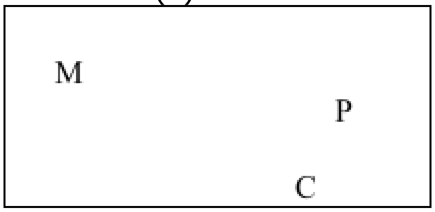
1. In a community of 160 people, 70 have cars ,82 have motorcycles, and 88 have bicycles, 20 have both cars and motorcycles,25 have both cars and bicycles, while 42 have both motorcycles and bicycles each person rode on at least any of the vehicles

a) Draw a venn diagram to illustrate the information

b) Find the number of people that has both cars and bicycles

c) How many people have either one of the three vehicles?

N(U)



The score of 144 candidates who registered for mathematics, physics and chemistry in an examination in a town are represented in the venn diagram above.

a) How many candidate register for both mathematics and physics?

b) How many candidate register for both mathematics and physics only?

General Evaluation

1. n(P) =4 means that these are 4 element in set P. given that n(XUY)= 50, n(X)=20 and n(Y)= 40. Find n(X∩Y)

2. find the sum of the first five terms of GP 2,6,18……..
3. the twelfth term of a linear sequence is 47 and the sum of the first three term is 12. Find the sum of the first 15 terms of the sequence
4. At a meeting of 35 teachers, the analysis of how Fanta, Coke and Pepsi were served as refreshments is as follows. 15 drank Fanta, 6 drank both Fanta and coke, 18 drank Coke, 8 drank both Coke and Pepsi, 20 drank Pepsi, and 2 drank all the three types of drink. How many of the teachers drank I Coke only II Fanta and Pepsi but not Coke.
5. Given n(XUY) = 50, n(X) = 20 and n(Y) = 40, determine n(XnY)

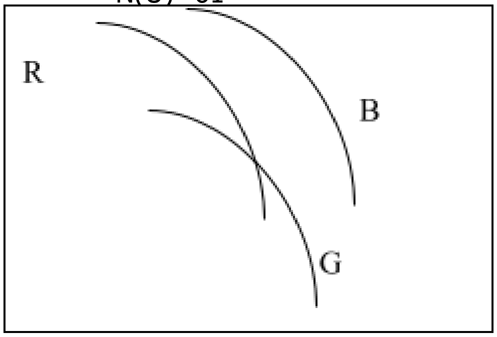
Reading Assignment: Read Sets, Further Mathematics Project II, page 1- 13.

Weekend Assignment

1. In a class of 50 pupils, 24 like oranges, 23 like apples and 7 like the two fruits. How many students do not like oranges and apples? (a)7 (b) 6 (c) 10 (d)15
2. In a survey of 55 pupils in a certain private schools, 34 like biscuits, 26 like sweets and 5 of them like none. How many pupils like both biscuits and sweet? (a) 5(b) 7 (c)9 (d)10
3. In a class of 40 students, 25 speak Hausa, 16 speak Igbo, 21 speak Yoruba and each of the students speak at least of the three languages.
   If 8 speak Hausa and Igbo. 11 speak Hausa and Yoruba.6 speak Igbo and Yoruba. How many students speak the three languages? (a) 3 (b) 4 (c) 5 (d) 6

Use the information to answer question 4 and 5

N(U)=61



The venn diagram above shows the food items purchased by 85 people that visit a store one week. Food items purchased from the store were rice, beans and gari.

4. How many of them purchased gari only? (a)8 (b)10 (c) 14 (d)12
5. How many of them purchased the three food items? (a) 5 (b)7 (c) 9 (d)11

Theory

1. In a school of 300 students, 110 offered French, 110 Hausa language, 180 History, 40 French and Hausa, 50 Hausa and History, 60 French and History while 30 did not offer any of the three subjects.
   a. Draw a Venn diagram to represent the data
   b. Find the number of students who offered I all the three subjects II History alone.

2. In a certain class 22, pupils take one or more of chemistry, economic and government. 12 take economics (e), 8 take government (G) and 7 take chemistry (c). nobody takes economics and chemistry and 4 pupils takes economic and government
   a) Using set notation and the letters to indicate above, write down the two statements in the last sentence
   b) Draw the venn diagram to illustrate the information
   c) How many pupils take; (i.) Both chemistry and government (ii.) Government only

WEEK SIX

First Half Term Revision Questions

1 . Evaluate the following (a) $32^{3/5}$ (b) $25^{1.5}$ (c) $(0.000001)^2$ (d) $343^{2/3}$ (e) 19^0

2 . Solve the following exponential equations (a) $2^x = 0.125$ (b) $3^{-x} = 243$ (c) $25^x = 625$ (d) $10^x = 1/0.001$ (e) $4/2^x = 64^x$

3 . Solve the following exponential equations (a) $2^{2x} - 6(2^x) + 8 = 0$ (b) $2^{2x+1} - 5(2^x) + 2 = 0$ (c) $3^{2x} - 4(3^{x+1}) + 27 = 0$ (d) $3^{2x} - 9 = 0$ (e) $7^{2x} - 2(7^x) + 1 = 0$

4 . Change each of the following index form to their logarithmic form (a) $2^6 = 64$ (b) $3^{-3} = 1/27$ (c) $25^{1/2} = 5$ (d) $3^5 = 243$ (e) $(0.01)^2 = 0.0001$

5 . Change the following logarithmic form into index form (a) $\log_2 128 = 7$ (b) $\log_{1/2}(1/4) = 2$ (c) $\log_7 49 = 2$ (d) $\log_5 1/125 = -3$ (e) $\log_5 1 = 0$

6 . Simplify each of the following (a) $\log_3 27 + 2\log_3 9 - \log_3 54$ (b) $1/2\log_4 8 + \log_4 32 - \log_4 2$ (c) $\log_2\sqrt{8} + \log_3\sqrt{3}$ (d) $\log_x x^9$ (e) $\log_5 12.5 + \log_5 2$

7 . Solve the following logarithmic equations (a) $\log_{10}(x^2 - 4x + 7) = 2$ (b) $\log_8(x^2 - 8x + 18) = 1/3$
(c) $\log_5(x^2 - 9) = 0$ (d) $\log_4(x^2 + 6x + 11) = ½$

8. If $\log_x 27 + \log_y 4 = 5$ and $\log_x 27 - \log_y 4 = 1$.find x and y

9 . Use logarithm table to evaluate the following (a) $\dfrac{(3.68)^2 \times 6.705}{\sqrt{0.3581}}$ (b) $\dfrac{\sqrt{0.897} \times 3.536}{0.00249}$

10. $\dfrac{83.67 \times \sqrt[3]{0.07124}}{352.18}$

WEEK SEVEN

TOPIC: BINARY OPERATIONS: BASIC CONCEPT OF BINARY OPEATIONS

CONTENT

- Concept of binary operations,
- Closure property
- Commutative property
- Associative property and
- Distributive property.

Definition:

Binary operation is any rule of combination of any two elements of a given non empty set. The rule of combination of two elements of a set may give rise to another element which may or not belong to the set under consideration.

It is usually denoted by symbols such as, *, Ө e.t.c.

P**roperties:**
A. Closure property: A non- empty set z is closed under a binary operation * if for all a, b € Z.
Example; A binary operation * is defined on the set S= {0, 1, 2, 3, 4} by
X*Y = x + y –xy. Find (a) 2 * 4 (b) 3* 1 (c) 0* 3. Is the set S closed under the operation *?
<u>Solution</u>

(a) 2 * 4, i.e, x= 2,y=4
2+ 4 – (2x4) = 6-8 = -2.
(b) 3* 1 = 3+1-(3x 1) = 4 – 3= 1
(c) 0*3 = 0 + 3 –(0 x3) = 3
Since -2€ S, therefore the operation * is not closed in S.

B. Commutative Property: If set S, a non empty set is closed under the binary operation *, for all a,b€ S. Then the operation * is commutative if a*b= b*a
Therefore, a binary operation is commutative if the order of combination does not affect the result.
Example; The operation * on the set R of real numbers is defined by:
p*q= $p^3 + q^3$-3pq. Is the operation commutative?

Solution
p*q= $p^3 + q^3$ -3pq
Commutative condition p*q= q*p
To obtain q*p, use the same operation q*p, use the same operation p*q but replace p by q and q by p.
Hence, q*p= $p^3+ q^3$ -3qp
In conclusion p*q= q*p, the operation is commutative.

C. Associative Property: If a non – empty set S is closed under a binary operation *, that is a*b €S. Then a binary operation is associative if (a*b) * c= a*(b*c)
Such that C also belongs to S.
Example: The operation Ө on the set Z of integers is defined by; a Ө b = 2a +3b -1. Determine whether or not the operation is associative in Z.
Solution
Introduce another element C
Associative condition: (aӨb) Өc = a Ө (b Өc)
(aӨb)Өc = (2a+ 3b- 1) Ө C
= 2(2a +3b -1) + 3c -1
= 4a + 6b- 2+ 3c- 1
= 4a +6b+3c- 3.
Also, the RHS, a Ө (b Ө c) = a Ө (2b+3c- 1)
= 2a+ 3(2b +3c- 1) - 1
= 2a + 6b +9c -3 -1
a Ө (b Ө c) = 2a+ 6b+ 9c -4
Since, (a Ө b) Ө c ≠ a Ө (b Ө c), the operation is not associative in Z.

Evaluation
1. **A**n operation* defined on the set R of real numbers is
x* y = 3x+ 2y- 1, x,y €R. Determine (a) 2*3 (b) -4* 5 (c) $\frac{1}{3} * \frac{1}{2}$
is the operation closed.

D. Distributive Property: If a set is closed under two or more binary operations
(* Ө) for all a, b and c € S, such that:
a*(bӨ c) = (a*b)Ө(a*c - Left distributive

(BϴC) *a = (b*a) ϴ(c*a) - Right distributive over the operation ϴ

Example: Given the set R of real numbers under the operations * and ϴ defined by:
a*b = a+ b- 3, aϴb= 5ab for all a, b € R. Does * distribute over ϴ.

Solution Let a, b,c € R
a* (bϴc) = (a*b) ϴ (a*c)
a* (bϴc) = a* (5ab)
= a+ 5ab -3.

(a*b) ϴ (a*c) = (a+ b -3) ϴ (a+ c-3)
= 5(a +b-3)(a +c -3)

From the expansion, it's obvious that, a* (bϴc) ≠ (a*b) ϴ (a*c) therefore * does not distribute over ϴ.

Evaluation:

1. A binary operation * is defined on the set R of real numbers by x*y= x +y + 3xy for all x, yεR. determine whether or not * is:

(a) Commutative?
(b) Associative?

2. The operation ⊕ on the set R of real numbers is defined by a ⊕ b = $\frac{a+b}{2}$ + ab for abϵR.

Show that the operation ⊕ is commutative but not associative on R.

General Evaluation

1. The operation * on the set R of real numbers is defined by: x * y = 3x + 2y – 1, x, yϵR. Determine (i) 2 * 3 (ii) 1/3 * ½ (iii) -4*5
2. The operation * on the set R, of real numbers is defined by; p*q = $p^3 + q^3$ – 3pq; p,q ϵR. Is the operation * commutative in R?
3. The operation * and ⊕ are defined on the set R of natural numbers by a*b = ab and a ⊕ b = a/b for all a,bϵR (a) Does * distribute over ⊕ ? (b) Does ⊕ distribute over *?

Weekend Assignment

1. Two binary operation * and ϴ are defined as m * n = mn – n -1 and m ϴ n = mn + n -2 for al real number m n find the value of 3 ϴ (4 * 5) (a) 60 (b) 57 (c) 54 (d) 42
2. If x * y = x + y –xy, find x, when (x*2) + (x*3) = 63 (a) 24 (b) 22 (c) -12 (d) -21
3. A binary operation * is defined by a * b = a^b. If a * 2 = 2 – a, find the possible values of a (a) 1, -1 (b) 1, 2 (c) 2, -2 (d) 1, -2
4. The binary operation * is defined on the set of integers p and q by p*q = pq + p + q. Find 2*(3*4) (a) 59 (b) 19 (c) 67 (d) 38
5. A binary operation ⊕ on real numbers is defined by x ⊕ y = xy + x + y for any two real numbers and y. The value of (-3/4) ⊕ 6 is (a) 3/4 (b) -9/2 (c) 45/4 (d) -3/4

Theory

1. The operation * is defined on the set R of real numbers by a* b= $\frac{a+b}{5}$ _ 1 for all a, b €R . Is the operation * commutative in R?.
2. The operation * is defined on the set R of real numbers by x*y = x + y + xy/2 for all x,y €R (a) is the operation * commutative? (b) is the operation * associative over the set R?

Reading Assignment: Read Binary Operation, Further Mathematics Project II, page 13 – 22

WEEK EIGHT:

TOPIC: BINARY OPERATIONS: IDENTITY AND INVERSE ELEMENTS

Identity element and Inverse element

CONTENT:

Identity Element:

Given a non- empty set S which is closed under a binary operation * and if there exists an element e € S such that a*e = e*a = a for all a € S, then e is called the IDENTITY or NEUTRAL element. The element is unique.

Example: The operation * on the set R of real numbers is defined by $a*b = \frac{2a-1}{2} + b$ for all a, b € R. Determine the identity element.

Solution:

$a*e= e*a = a$

$a*b= \frac{2a-1}{2} + b$

$a*e = \frac{2a-1}{2} + e = a$

$2a-1+ 2e = 2a$

$2e = 2a-2a +1$

$e = ½.$

Evaluation

Find the identity element of the binary operation $a*b = a +b+ab$

Inverse Element;

If x € S and an element x^{-1} € S such that $x*x^{-1} = x^{-1}*x= e$ where e is the identity element and x^{-1} is the inverse element.

Example: An operation * is defined on the set of real numbers by $x*y = x + y -2xy$. If the identity element is 0, find the inverse of the element.

Solution;

$X *y = x+ y- 2xy$

$x*x^{-1} = x-1*x= e, e = 0$

$x + x^{-1}- 2xx^{-1} = 0$

$x^{-1} -2xx^{-1}= -x$

$x^{-1}(1-2x) = -x$

$x^{-1} = -x/ (1-2x)$

The inverse element $x^{-1} = -x/ (1-2x)$

Evaluation:

The operation Δ on the set Q of rational numbers is defined by: $x\Delta y = 9xy$ for x,y € Q

Find under the operation Δ (I) the identity element (II) the inverse of the element a € Q

General Evaluation

1. An operation on the set of integers defined by $a*b = a^2 + b^2 – 2a$,find 2*3*4
2. Solve the pair of equations simultaneously
   (a) $2x + y = 3, 4x^2 – y^2 + 2x + 3y= 16$
   (b) $2^{2x – 3y} = 4, 3^{3x + 5y} – 18 = 0$

Reading Assignment: Read Binary Operation, Further Mathematics Project II, page 16 – 22

Weekend Assignment

1. Find the identity element e under this operation if the binary operation* is defined by c * d = 2cd+ 4c+ 3d for any real number.
   A. $\frac{-3}{2C+3}$ B. -2C+3 C. $\frac{X-3}{2C}$
2. An operation is defined by $x*y = \text{Log}^{y}_{x}$, evaluate 10* 0.0001
   A. 4 B. -4 C. 3
3. The binary operation * is defined by $x*y= x^y- 2x -15$, solve for x if x*2= 0

A.x= -3 or -5 B. x= -3 or 5 C. x = 3 or 5

4. A binary operation * is defined on the set R of real numbers by
$m*n = m + n^2$ for all m, n € R. If k*3 = 7*4, find the value of k
A. 8 B.28/3 C.14

5 .Find the inverse function a^{-1} in the binary operation Δ such that for all a,b € R
a Δ b = ab/ 5
A. 25/a B.-25/a C. a/5

Theory

1. A binary operation * is defined on the set R of real numbers by
$x*y = x^2 + y^2 + xy$ for all x, y € R. Calculate (a) (2*3)* 4
(b) Solve the equation 6*x = 27
2. Draw a multiplication table for modulo 4.
(b) Using your table or otherwise evaluate (2X3) X (3X2)

WEEK NINE
TOPIC: SURDS
CONTENT

- Rules of surds
- Basic Form of Surds
- Similar Surds

- Conjugate Surds
- Simplification of Surds
- Additional & Subtraction of Surds
- Multiplication and Division of Surds
- Rationalization of Surds
- Equality of Surds

Rules of Surds

Surds are irrational numbers. They are the root of rational numbers whose value cannot be expressed as exact fractions. Examples of surds are: √2, √7, √12, √18, etc.

1. √(a X b) = √a X √ b
2. √(a / b) = √a / √b
3. √(a + b) ≠ √a + √b
4. √(a – b) ≠ √a - √b

Basic Forms of Surds

√a is said to be in its basic form if A does not have a factor that is a perfect square. E.g. √6, √5, √3, √2 etc. √18 is not in its basic form because it can be broken into √ (9x2) = 3√2. Hence 3√2 is now in its basic form.

Similar Surds

Surds are similar if their irrational part contains the same numerals e.g.

1. 3√n and 5√n
2. 6√2 and 7√2

Conjugate Surds

Conjugate surds are two surds whose product result is a rational number.

(i)The conjugate of √3 - √5 is √3 + √5
The conjugate of -2√7 + √3 is 2√7 - √3
In general, the conjugate of √x + √y is √x - √y
The conjugate of √x - √y = √x + √y

Simplification of Surds

Surds can be simplified either in the basic form or as a single surd.

Examples

Simplify the following in its basic form (a) √45 (b) √98

Solution

(a) √45 = √ (9 x 5) = √9 x √5 = 3√5
(b) √98 = √ (49 x 2) = √49 x √2 = 7√2

Examples

Simplify the following as a single surd (a) 2√5 (b) 17√2

Solution

(a) 2√5 = √4 x √5 = √ (4 x 5) = √20
(b) 17√2 = √289 x √2 = √ (289 x 2) = √578

Addition and Subtraction of Surds

Surds in their basic forms which are similar can be added or subtracted.

Examples

Evaluate the following

(a)√32 + 3√8 (b) 7√3 - √75 (c) 3√48 - √75 + 2√12

Solution

(a) $(\sqrt{32} + 3\sqrt{8}$

$= \sqrt{(16 \times 2)} + 3\sqrt{(4 \times 2)}$

$=4\sqrt{2} + 6\sqrt{2}$

$= 10\sqrt{2}$

(b) $7\sqrt{3} - \sqrt{75}$

$= 7\sqrt{3} - \sqrt{(25 \times 3)}$

$=7\sqrt{3} – 5\sqrt{3} \quad =2\sqrt{2}$

(c) $3\sqrt{48} - \sqrt{75} + 2\sqrt{12}$

$= 3\sqrt{(16 \times 3)} - \sqrt{(25 \times 3)} + 2\sqrt{(4 \times 3)}$

$= 12\sqrt{3} - 5\sqrt{3} + 4\sqrt{3}$

$= 11\sqrt{3}$

Evaluation

1. Simplify the following **(a)** $5\sqrt{12} - 3\sqrt{18} + 4\sqrt{72} + 2\sqrt{75}$ (b) $3\sqrt{2} - \sqrt{32} + \sqrt{50} + \sqrt{98}$
2. Simplify the following as a single surd (i) $8\sqrt{3}$ (ii) $13\sqrt{2}$

Multiplication and Division of Surds

Example: Evaluate the following (a) $\sqrt{45} \times \sqrt{28}$ (b) $\sqrt{24} /\sqrt{50}$

Solution

(a) $\sqrt{45} \times \sqrt{28}$

$= \sqrt{(9 \times 5)} \times \sqrt{(4 \times 7)}$

$= 3\sqrt{5} \times 2\sqrt{7}$

$= 3 \times 2 \times \sqrt{(5 \times 7)}$

$= 6\sqrt{35}$

(b) $\sqrt{24} / \sqrt{50}$

$= \sqrt{(24 / 50)}$

$= \sqrt{(12 / 25)}$

$= \sqrt{12} / \sqrt{25}$

$= \sqrt{(4 \times 3)} / 5$

$= 2\sqrt{3} / 5$

Evaluation:

Simplify 1. $\sqrt{6} \times (3 - \sqrt{5})$ 2. $(2\sqrt{3} - \sqrt{7})(2\sqrt{3} + \sqrt{7})$

2. Multiply the following by their conjugate (a) $\sqrt{3} - 2\sqrt{5}$ (b) $3\sqrt{2} + 2\sqrt{3}$

Surds Rationalisation

Rationalisation of surds means multiplying the numerator and denominator by the denominator or by the conjugate of the denominator.

(a) Example: Evaluate the following (a) $6/\sqrt{3}$ (b) $\dfrac{3}{\sqrt{3} + \sqrt{2}}$

Solution

(a) $6/\sqrt{3}$

$= \dfrac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \underline{6\sqrt{3}}$

(b) $\dfrac{3}{\sqrt{3} + \sqrt{2}}$

$= \dfrac{3(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})}$

3

$= 2\sqrt{3}$

$= \frac{3\sqrt{3} - 3\sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2}$

$= \frac{3\sqrt{3} - 3\sqrt{2}}{3 - 1}$

$= \frac{3\sqrt{3} - 3\sqrt{2}}{1}$

$= 3(\sqrt{3} - \sqrt{2})$

Equality of Surds

Given two surds i.e $P + \sqrt{m}$ and $q + \sqrt{n}$ if $P + \sqrt{m} = q + \sqrt{n}$ then
$\sqrt{P} - q = \sqrt{n} - m$ the L.H.S
Of the equation is a rational number while the L.H.S and R.H.S can only be equal if they are both equal to zero (0)

$P – q = 0$
:. $P = q$ and $n - m = 0$ i.e.
$\sqrt{n} = \sqrt{m}$

Examples:

Find the square root of the following?
a) $7 + 2\sqrt{10}$ b) $14 - 4\sqrt{6}$

Solution

(a) Let the square root of $7 + 2\sqrt{10}$ be $\sqrt{m} + \sqrt{n}$
$(\sqrt{m} + \sqrt{n})^2 = 7 + 2\sqrt{10}$
$m + \sqrt{2mn} + n = 7 + 2\sqrt{10}$
$m + n = 7$ (1)
$2\sqrt{mn} = 2\sqrt{10}$
$mn = 10$
Squaring both surds we have
$mn = 10$ _______(ii)
$m + n = 7$ ______ (i)
$m n = 10$ _______ (ii)
From equation (1) $m = 7 – n$
Put m in (ii) we have
$(7 – n) n = 10$
$7n – n^2 = 10$
In sum; $n^2 – 7n + 10 = 0$
$n^2 – 2n – 5n + 10 = 0$
$n (n – 2) – 5 (n – 2) = 0$
$(n - 5) (n – 2) = 0$
$n = 5$ or 2
$m = 7 – 2$, where $n = 2$
$m = 5$,
$m = 7 – 5$, when $n = 5$
$m = 2$
$m = 5$ or 2
The square root of $7 + \sqrt{10}$ are 5 & + 2.

(b) Let the square root of $14 – 4\sqrt{6}$ be $\sqrt{P} - \sqrt{Q}$
The $(\sqrt{P} - \sqrt{Q})^2 = 14 – 4\sqrt{6}$
$P - 2\sqrt{PQ} + Q = 14 – 4\sqrt{6}$
$P + Q = 14$ …………………… (1)

$-2\sqrt{PQ} = -4\sqrt{6}$

$\overline{-2}$ $\quad$ $\overline{-2}$

$\sqrt{PQ} = 2\sqrt{6}$ (squaring both sides)

$PQ = (2\sqrt{6})^2$

$PQ = 4 \times 6$ …………………………….. (11)

$P + Q = 14$ ………………………………. (1)

$PQ = 24$ …………………………………… (11)

From equation……………… (1) $P = 14 - Q$

Sub for p in equation ……………….. (11)

$(14 – Q) Q = 24$

$14Q – Q^2 = 24$

In turn we have:

$Q^2 – 14Q + 24 = 0$

$Q^2 – 12Q – 2Q + 24 = 0$

$Q (Q -12) – 2 (Q – 12) = 0$

$Q = 2$ or 12

If $P = 14 – Q$,when $Q= 12$

$P = 14 – 12$

$P = 2$

If $P = 14 – Q$ when $Q = 2$

$P = 14 - 2$

$= 12$

$(\sqrt{12} - \sqrt{2}) = (2\sqrt{3} - \sqrt{2})$ and

$(\sqrt{2} - \sqrt{12}) = (\sqrt{2} - 2\sqrt{3})$

Evaluation:

1. Express $\frac{3\sqrt{2} - \sqrt{3}}{2\sqrt{3} - \sqrt{2}}$ in the form $\frac{\sqrt{m}}{\sqrt{n}}$ where m and n are whole number.

2. Express $\frac{1}{\sqrt{5} + \sqrt{3}}$ in the form $p\sqrt{5} + q\sqrt{3}$, where p and q are rational numbers.

General Evaluation

1. Simplify $\frac{3x^2 \times 4x^3}{6x^7}$
2. Evaluate $\frac{23.97 \times 0.7124}{3.877 \times 52.18}$
3. Solve $9^{(1 - x)} = (1/27)^{x+1}$
4. $\text{Log}_8 (r^2 – 8r + 18) = 1/3$
5. Simplify: $2\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

Reading Assignment: Further Mathematics Project Book 1(New third edition).Chapter 3 pg.19-27

Weekend Assignment

1. Expand $(3\sqrt{2} - 1) (3\sqrt{2} + 1)$ (a) 16 (b) 20 (c) 17 (d) 24
2. Simplify $\sqrt{200}$ in its basic form (a) $10\sqrt{2}$ (b) $5\sqrt{4}$ (c) $2\sqrt{10}$ (d) $2\sqrt{50}$
3. Simplify $9/\sqrt{3}$ (a) $3\sqrt{2}$ (b) $3\sqrt{3}$ (c) 1/3 (d) $2\sqrt{2}$
4. Express $3\sqrt{5}$ as a single surd (a) $\sqrt{40}$ (b) $\sqrt{55}$ (c) $\sqrt{45}$ (d) $\sqrt{35}$implify
5. Simplify $\sqrt{`}\,128 - 4\sqrt{8}$ (a) 0 (b) 1 (c) 2 (d) 3

Theory

1.Express $\underline{3\sqrt{2} - \sqrt{3}}$ in the form $\underline{\sqrt{m}}$ where m and n are whole number.

$2\sqrt{3} - \sqrt{2}$ $\sqrt{n}$

2.Express $\frac{1}{\sqrt{5} + \sqrt{3}}$ in the form ***p√5 + q√3***, where p and q are rational numbers.

WEEK TEN

TOPIC: STATISTICS: MEASURES OF CENTRAL TENDENCY

MEASUREMENT OF CENTRAL TENDENCY

- Mean, Median and Mode of ungrouped
- Mean, Median and Mode of grouped data

Measures of central tendency: This is a measure of how the data are centrally placed. The three commonest measures of position, depending on the information required are the arithmetic mean, median and the mode.

MEAN: It is most widely used measure and sometimes called the arithmetical averages. The mean of the number x_1, x_2, x_3, x_4 …………………x_n is given by:

$X = \frac{\sum x}{n}$ where ∑x is the sum of all items. n = number of items

When the data involves frequency; mean = ∑fx/∑f

Examples:

1. Calculate the mean of the numbers 15, 17, 19, 21, 23, 25, 27, 29.

Solution:

Mean (x) = $\frac{15 + 17 + 19 + 21 + 23 + 25 + 27 + 29}{8} = \frac{176}{8} = 22$

2. The table shows the number of suitcases possessed by a group of travelers.

No of suitcases	0	1	2	3	4	5
Travelers	2	7	7	2	3	9

Calculate the mean to the nearest whole number.

Solution:

Mean (x) = ∑ fx/∑ f = 84/30 = 2.8 = 3

X	F	FX
0	2	0
1	7	7
2	7	14
3	2	6
4	3	12
5	9	45
Total	**30**	**84**

EVALUATION

1. Calculate the mean of the numbers 37.5, 25.5, 30.5, 41.5, 52.5, 28.5.
2. Calculate the mean score of the scores represented in the table below.

Scores	10	12	14	16	18
No of Students	5	2	3	4	4

Mode:

The mode of a distribution is the value of the variable which occurs most often in the distribution. It is also possible for a distribution to have more than one mode, if there were more than one item having the highest frequency.

Example:

1. Find the mode of the data 5, 4, 8, 9, 6, 8, 9, 3, 8. The mode is 8 (it appears 3 times more than others)

Median:

This is the middle value of a set of data, when arranged in ascending or descending order.

Example:

Find the median of these numbers: (1). 35, 28, 42, 28, 56, 70, 35 (2) 18, 20, 25, 30, 22, 25, 28, 15

Solution:

1. Re – arranging the numbers: 70, 56, 42, [35] 35, 28, 28. The median is 35
2. 15, 18, 20, [22, 25], 25, 28, 30. Median = $\frac{22 + 25}{2} = \frac{47}{2} = 23.5$

General Example:

The table below is the distribution of the test scored in a class:

Scores	1	2	3	4	5	6	7	8	9	10
Frequency	1	1	5	3	X	0	6	2	3	4

If the mean score of the class is 6, find the (i) value of x (ii) median score (iii) modal score.

Solution*:*

X	F	FX
1	1	1
2	1	2
3	5	15
4	3	12
5	X	5x
6	0	0
7	6	42
8	2	16
9	3	27
10	4	40
Total	25 + x	155 + 5x

(i) Mean = $\Sigma fx/\Sigma x$

$$6 = \frac{155 + 5x}{25 + x}$$

Cross multiplying: $6(25 + x) = 155 + 5x$

$$150 + 6x = 155 + 5x$$

$$6x - 5x = 155 - 150$$

$$x = 5.$$

(ii)Median score: the median score lies between the 15th and 16th scores, hence: median = (5 + 7)/2 = 6.

(iii)Mode: 7

Evaluation:

Calculate the mode and median of the scores below; 2, 2, 1, 1, 0, 3, 3, 4, 4, 4, 5, 1, 2, 2.

MEAN, MEDIAN AND MODE OF GROUPED DATA

Mean: The arithmetic mean of grouped frequency distribution can be obtained using:

Class Mark Method:

$X = \Sigma fx/\Sigma f \; \Sigma fx/\Sigma f$ where x is the midpoint of the class interval.

Assumed Mean Method: It is also called working mean method. $X = A + (\Sigma fd/\Sigma f)$

Where, $d = x - A$, x = class mark and A = assumed mean.

Example*:* The numbers of matches in 100 boxes are counted and the results are shown in the table below:

Number of matches	25 - 28	29 - 32	33 - 36	37 - 40
Number of boxes	18	34	37	11

Calculate the mean (i) using class mark (ii) assumed mean method given that the assumed mean is 30.5.

Solution*:*

Class interval	**F**	**X**	**FX**	**d = x - A**	**Fd**
25 - 28	18	26.5	477	- 4	- 72
29 - 32	34	30.5	1037	0	0
33 - 36	37	34.5	1276.5	4	148
37 - 40	11	38.5	423.5	8	88
Total	100		3214		164

(i) Class Mark Method: X = $\Sigma fx/\Sigma f \, \Sigma fx/\Sigma f$ = 3214/100 = 32. 14 = 32 matches per box (nearest whole no)

(ii) Assumed Mean Method: X = A + (∑ fd/∑f)
= 30. 5 + (164/100) =30.5 + 1.64
= 32.14 = 32 matches per box (nearest whole number)

Evaluation*:*

Calculate the mean shoe sizes of the number of shoes represented in the table below using (i) class mark (ii) assumed mean method given that the assumed mean is 42.

Shoe sizes	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54
No of Men	10	12	8	15	5

Mode

The mode of a grouped frequency distribution can be determined ***geometrically*** and by ***interpolation method.***

Mode from Histogram: The highest bar is the modal class and the mode can be determined by drawing a straight line from the right top corner of the bar to the right top corner of the adjacent bar on the left. Draw another line from the left top corner to the bar of the modal class to the left top corner of the adjacent bar on the right.

Example:

The table gives the distribution of ages of students in an institution.

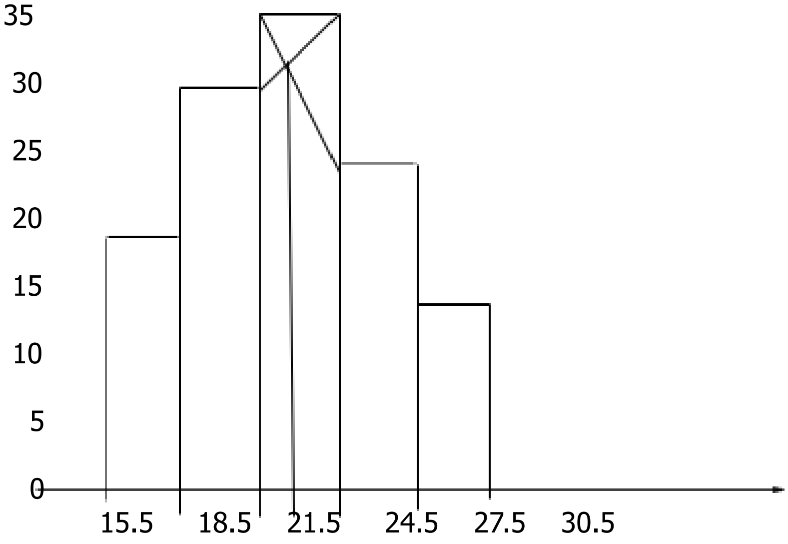
Ages(year)	16 - 18	19 - 21	22 - 24	25 - 27	28 - 30
No of Students	18	30	35	24	13

Draw a histogram and use your histogram to estimate the mode to the nearest whole number.

Solution:

Class Interval (Ages)	F	Class Boundary
16 - 18	18	15.5 - 18. 5
19 - 21	30	18.5 - 21.5
22 - 24	35	21.5 - 24.5
25 - 27	24	24.5 - 27.5
28 - 30	13	27.5 - 30.5

Histogram



Masses(kg)	Frequency	Cumulative Frequency	Upper Class Boundary
10 – 14	3	3	< 14.5
15 – 19	7	10	<19.5
20 – 24	9	19	<24.5
25 – 29	5	24	< 29.5
30 – 34	11	35	< 34.5
35 – 39	6	41	< 39.5
40 – 44	9	50	< 44.5

Modal class = 22 - 24

Mode = 21.5 + 0.9 = 22.4, approximately 22 yrs.

MODE FROM INTERPOLATION: The mode can be obtained using the formula.

$$\text{Mode} = L_m + \left(\frac{\quad}{+ \Delta_2}\right) C$$

Where L_m = lower class boundary of the modal class.

Δ_1 = difference between the frequency of the modal class and the class before it.

Δ_2 = difference between the frequency of the modal class and the class after it.

C = class width of the modal class.

Example: Using the table given in the example above:

Modal class = 22 – 24, $\Delta_1 = 35 – 30 = 5$, $\Delta_2 = 35 – 24 = 11$, C = 3, $L_m = 21.5$

$$\text{Mode} = 21.5 + \left(\frac{\quad}{+ 11}\right) \times 3$$

$$= 21.5 + (15/16) = 21.5 + 0.9375$$

$$= 22.44, \text{ approximately 22 yrs.}$$

MEDIAN FROM INTERPOLATION FORMULA

$$\text{Median} = L_1 + \left(\quad\right) C$$

Where, L_1 = lower class boundary of the median class.

Cfm = cumulative frequency of the class before the median class.

Fm = frequency of the median class.

C = class width of the median class and N = Total frequency

The median class: 30 – 34, L_1 = 29.5, cfm = 24, fm = 11, C = 5

$$\text{Median} = 29.5 + \left(\qquad\right)\frac{4}{1} \times 5$$

$$= 29.5 + 5 \quad = 30\text{kg}$$

Evaluation: Calculate the modal shoe sizes and median of the number of shoes represented in the table below using interpolation method.

Shoe sizes	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54
No of Men	10	12	8	15	5

General evaluation:

1. The table below gives the distribution of masses (kg) of 40 people

Masses(kg)	1 – 5	6 - 10	11 -15	16 - 20	21 - 25	26 - 30	31 - 35	36 - 40
Frequency	9	20	32	42	35	22	15	5

- i. State the modal class of the distribution and find the mode.
- ii. Calculate the mean of the distribution.

2. The following table shows the distribution of marks obtained by a class.

Marks	0	1	2	3	4	5	6	7	8	9
No of students	1	1	3	4	4	12	7	3	3	2

Using this table, find the (1) median mark (2) modal mark (3) mean of the distribution.

Reading Assignment: Further Mathematics Project Book 1(New third edition), pg 328, Exercise18, No 15 -20

Weekend Assignment

Marks	3	4	5	6	7	8
Frequency	5	x – 1	x	9	4	1

If the mean is 5, calculate the (a) value of x (b) mode (c) median of the distribution.

2. The table gives the frequency distribution of a random sample of 250 steel bolts according to their head diameter, measured to the nearest 0.01mm.

Diameter (mm)	23.06 –23.10	23.11 – 23.15	23.16 –23.20	23.21 –23.25	23.26-23.30	23.31 –23.35	23.36-23.40	23.41-23.45	23.46-23.50
No of bolts	10	20	28	36	52	38	32	21	13

- i. State the median class and calculate the median using interpolation method.
- ii. Draw the histogram and use it to estimate the mode.
- iii. Calculate the mean value using a working mean of 23.28mm.
- iv. The table gives the frequency distribution of marks obtained by a group of students in a test.