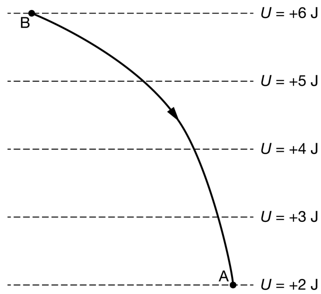
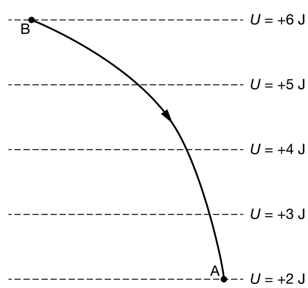
Physics C Solutions 3.3

Potential Energy

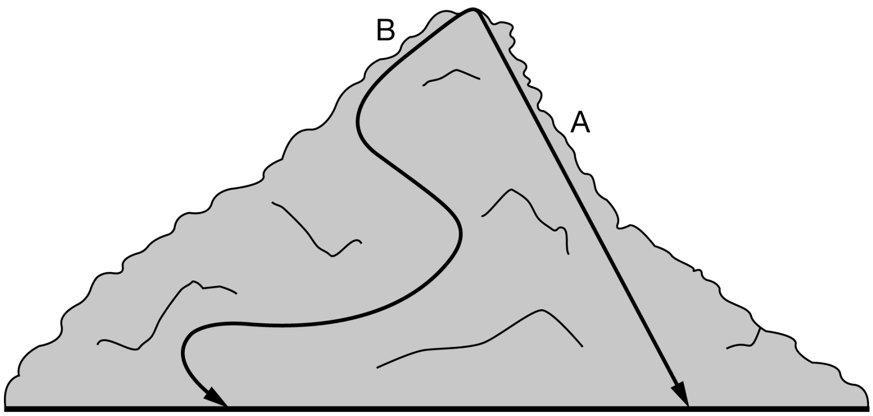


1. The figure shows an object moving from point B to point A and the gravitational potential energy of the object-Earth system at different points along the trajectory. There are no other forces exerted on the object.
Which of the following best describes the direction of the force exerted on the object due to the gravitational field as the object moves along the path from point B to point A ?
A) The force is always directed tangent to the path.
B) The force is always directed perpendicular to the path.
C) The force is always directed downward.
D) The force is always directed upward.
E) The force is always directed opposite the motion of the particle.
Gravitational potential energy U_g is directly related to height h. Both of them are decreasing at an increasing rate, which means the object is accelerating downward, and therefore, there is a downward force present.

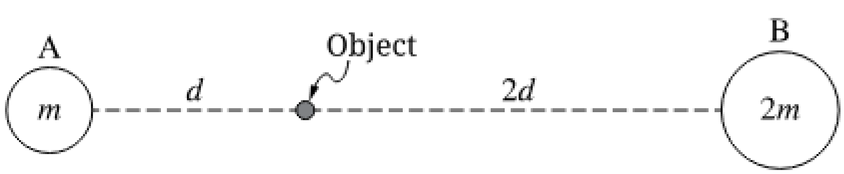


2. The figure shows an object moving from point B to point A and the gravitational potential energy of the object-Earth system at different points along the trajectory. There are no other forces exerted on the object.
The magnitude of the force associated with the gravitational field is constant and has a value F. A particle is launched from point B with an initial velocity and reaches point A having gained U_0 joules of kinetic energy. A resistive force field is now set up such that it is directed opposite the gravitational field with a force of constant magnitude $\frac{1}{2}F$. A particle is again launched from point B. In terms of U_0, how much kinetic energy will the particle gain as it moves from point B to point A ?
The resistive force with magnitude $\frac{1}{2}F$ is combined with the gravitational force with magnitude F, in opposite directions. After they partially cancel out, the total force on the object is $\frac{1}{2}F$.

Work (change in energy) is equal to force multiplied by distance. Because the distance was the same, but total force was $\frac{1}{2}F$, the kinetic energy gained is also half its previous value: $\frac{1}{2}\boldsymbol{U_0}$



3. Two hikers start from the top of the same hill but take different paths to the bottom. Hiker A weighs more than hiker B. The hikers take the paths shown in the figure. Hiker B takes a longer time to descend than hiker A. Which of the following is a correct statement about the change in gravitational potential energy ΔU_A for the Earth-hiker A system and the change in gravitational potential energy ΔU_B for the Earth-hiker B system?
A) $\Delta U_A = \Delta U_B$, because the height descended for both is the same.
B) $\Delta U_A < \Delta U_B$, because the distance traveled along the path for hiker B is greater.
C) $\Delta U_A < \Delta U_B$, because the time required for hiker B to descend the hill is longer.
D) $\Delta U_A > \Delta U_B$, because the time required for hiker A to descend the hill is shorter.
E) $\Delta U_A > \Delta U_B$, because the gravitational force exerted on hiker A is greater.
While the height and gravitational acceleration are the same, and time is irrelevant to potential energy, mass is a factor in gravitational potential energy: $U_g = mgh$.



Note: Figure not drawn to scale.

4. An object of mass M in space is placed between two planets as shown. Planet A has a mass of m and the distance from the center of Planet A to the object is d. Planet B has a mass of $2m$ and the distance from the center of Planet B to the object is $2d$. The object is released from rest and fires a rocket in order

to stay in place. With what force must the rocket continuously fire, in what direction is the rocket force, and how much work is done by the rocket in the first five seconds?

The rocket force must make up the difference between the gravitational forces pulling the object toward planets A and B.

$$\boldsymbol{F_R} = \mathbf{G}\frac{mM}{d^2} - \mathbf{G}\frac{2mM}{4d^2} = \mathbf{G}\frac{mM}{2d^2}$$

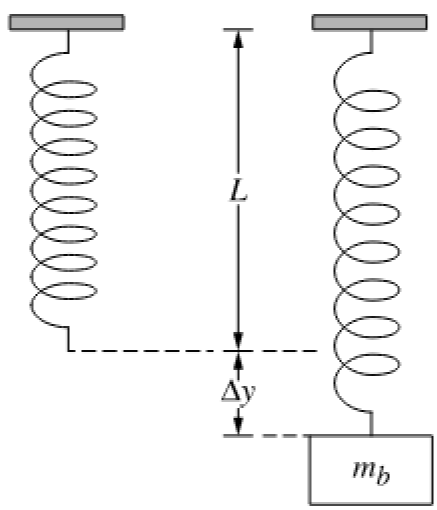
Because the rocket stays in place, there is no change to kinetic or potential energy, so $\boldsymbol{W = 0}$

5. The force exerted by a non-linear spring is given by $F(x) = -kx^{6/5}$. Which of the following expressions correctly models the potential energy stored in the spring when it is compressed a distance of D ?

Work is equal to the change in energy, and is also equal to the integral of force:

$$U_s = W = \int_0^D -kx^{6/5}\,dx = -\frac{5}{11}kx^{11/5}\Big]_0^D = -\frac{5}{11}kD^{11/5}$$

Spring potential energy cannot be negative, so $\boldsymbol{U_s = \frac{5}{11}kD^{11/5}}$



6. One end of a spring of length L and spring constant $k = 400\ \mathrm{N/m^2}$, is attached to the ceiling as shown. The other end of the spring is attached to a small box of mass $m_b = 0.5$ kg, and the block is released from rest. The potential energy U_s of the box-spring system as a function of the stretch of the spring, Δy, can be described as $U_s = \frac{1}{2}k(\Delta y)^3$. The box is allowed to oscillate until the box-spring system comes to rest. Find the magnitude of the stretch of the spring, Δy, when the box-spring system has come completely to rest.

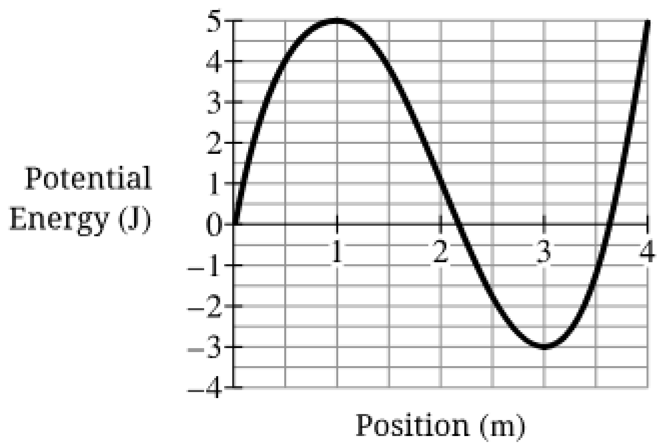
At rest, the gravitational force and the spring force must be equal in magnitude, and spring force is the derivative of potential energy:

$$F_g = F_s = -\frac{dU_s}{dy}$$

$$mg = -\frac{3}{2}k(\Delta y)^2$$

$$(0.5)(-9.8) = -\frac{3}{2}(400)(\Delta y)^2$$

***Δy* = 0.081 m**



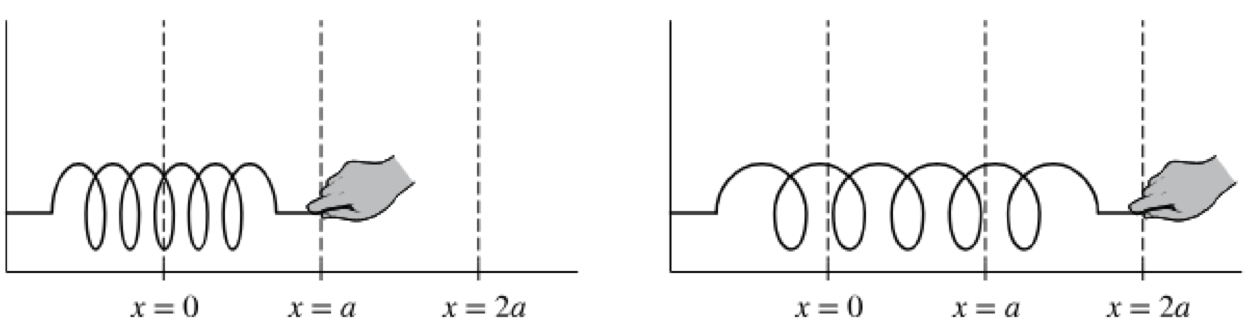
7. An object of mass 3 kg moves along the *x*-axis while a net non-linear conservative force is exerted on it. The potential energy of the system consisting of the object as well as other objects is given by the graph. The speed of the object at $x = 1$ m is 4 m/s. Estimate the maximum speed of the object.

At $x = 1$, $U = 5$ J and $K = \frac{3 \cdot 4^2}{2} = 24$ J, for a total $E = 29$ J, which remains constant.

At $x = 3$, $U = -3$ J, and $E = 29$, so $K = 32$ J.

$32 = \frac{3 \cdot v^2}{2}$

$v = 4.619$ m/s



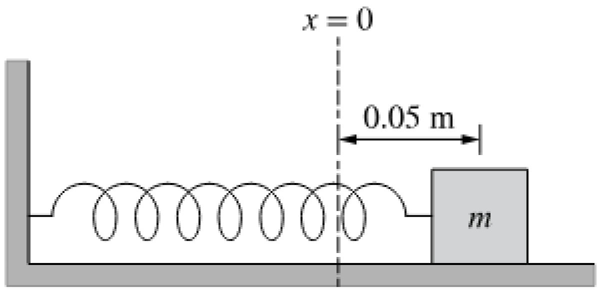
8. One end of an unstretched spring is attached to a wall. When the spring is at its equilibrium length the right end is at position $x = 0$. A student pulls the right end of the spring until the right end of the spring is at position $x = 2a$, as shown in the figures. The magnitude of the force exerted by the spring on the student as a function of position *x* of the right end of the spring is $F(x) = 8x^3 - 3x^2$. Give an expression in

terms of a which indicates the change in potential energy of the spring during the time that the free end of the spring moves from $x = a$ to $x = 2a$.

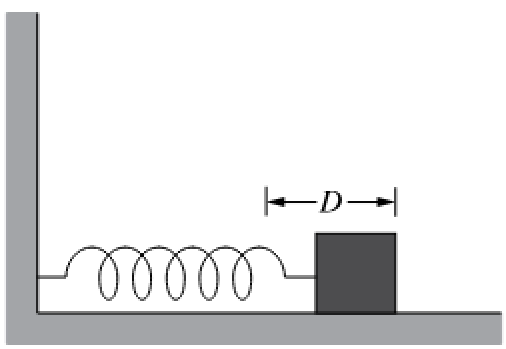
$$\Delta U_s = W = \int_{x_1}^{x_2} F(x)\, dx$$

$$\int_a^{2a} 8x^3 - 3x^2\, dx = 2x^4 - x^3\Big]_a^{2a} = [32a^4 - 8a^3] - [2a^4 - a^3]$$

$30a^4 - 7a^3$



9. One end of an ideal spring is attached to a wall and the other end to a block as shown. The block is initially at equilibrium at $x = 0$ m. A physics student determines that stretching the spring by moving the block from $x = 0$ m to $x = 0.05$ m stores 5.0 J of energy in the block-spring system. How much additional energy is stored in the block-spring system after the block is moved from $x = 0.05$ m to $x = 0.15$ m ?
Potential energy stored in an ideal spring is proportionate to the square of its stretch. By tripling the stretch, the energy is increased ninefold to 45 J. The additional energy is 45 - 5 = **40 J**



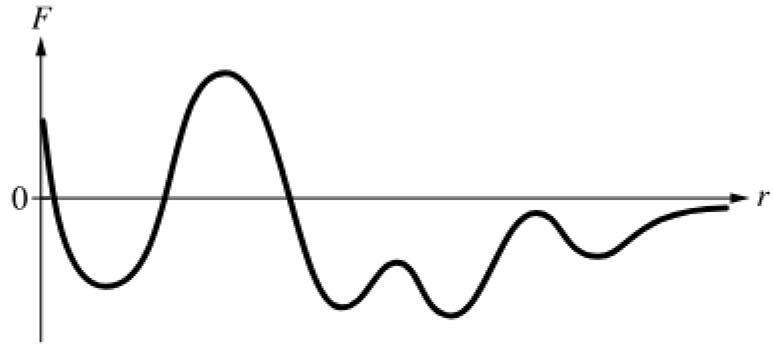
10. A non-linear spring that is neither compressed nor stretched, is connected at one end to a wall and at the other end to a block as shown. The magnitude of the force F required to compress the spring can be described by the equation $F(x) = e^{x/k}$, where k is the spring constant, and x is the distance compressed or stretched from equilibrium. The block is pushed along the surface toward the wall compressing the spring

a distance D. What is the change in potential energy of the block-spring system because of the compression of the spring?

$$\Delta U_s = W = \int_{x_1}^{x_2} F(x)\,dx$$

$$\int_0^D e^{x/k}\,dx = ke^{x/k}\Big]_0^D = ke^{D/k} - ke^0$$

$$\boldsymbol{= ke^{D/k} - k = k(e^{D/k} - 1)}$$



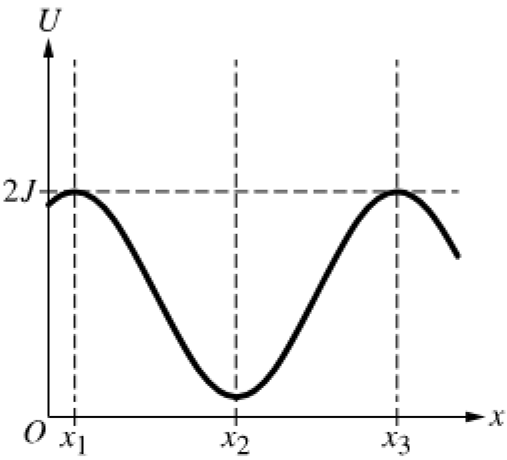
11. The net force on an object as a function of the position of the object along the x-axis is shown. How many points of stable equilibrium exist for this object along the x-axis?

Points of stable equilibrium occur when U reaches a minimum.

Any function will reach a minimum when its derivative changes from negative to positive.

$F = -\frac{dU}{dt}$, so U has a minimum when $F(x)$ changes from positive to negative, which happens twice.

2 times

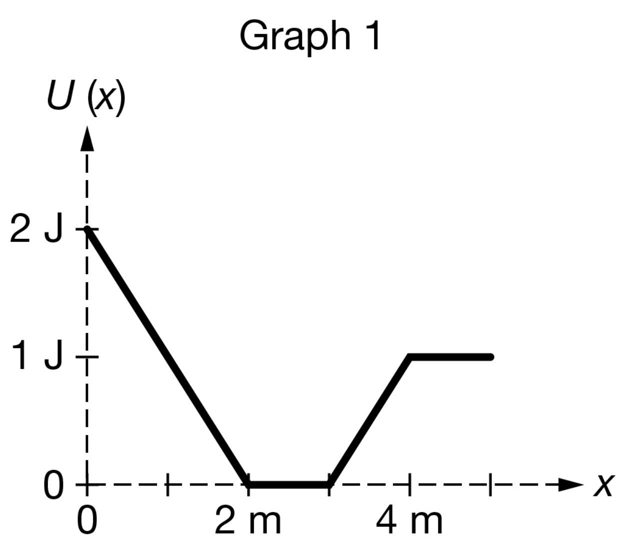


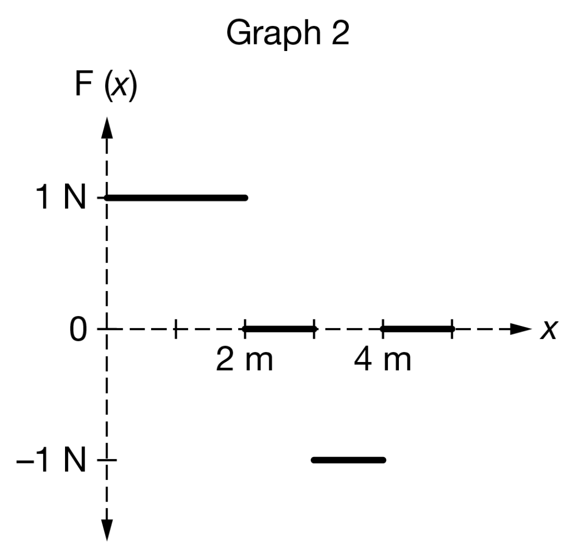
12. A system consists of several objects that interact with each other by conservative forces. One of the objects is free to move only in the x-direction, while the other objects are fixed in space. The potential energy of the system as a function of the position of the moveable object is shown. At what point(s) on this graph is the speed of the moveable object at its least value?

The total energy of the system is conserved, so K and v^2 will both reach a relative minimum when U has a relative maximum.

$x = x_1$ and x_3

13. An object of mass M moves in one dimension along the x-axis. A conservative force $F(x)$ is exerted on the object. The potential energy $U(x)$ associated with this force as a function of position x is shown in graph 1. A student used the potential energy graph to construct the graph of $F(x)$ as a function of x shown in graph 2. Are these graphs consistent with one another, and if not, what is the error?





A) For $0 < x < 2$ m and 3 m $< x < 4$ m, the graph of $F(x)$ should be curved, not constant.
B) For $0 < x < 2$ m, $F(x)$ should be negative, and for 3 m $< x < 4$ m, $F(x)$ should be positive.
C) For $0 < x < 2$ m, $F(x)$ should be negative because the slope of $U(x)$ is negative.
D) For 3 m $< x < 4$ m, $F(x)$ should be positive because $U(x)$ is positive.
E) The two graphs are consistent with each other.
Force is the rate of change of energy. Because this is a conservative force, when the force is positive, the change in potential energy is negative, and vice versa. The *y*-value of the force graph matches the slope (with sign flipped) of the *U* graph at all times.

14. A certain nonlinear spring has a force function given by $F = -ax^2 - b$, where x is the displacement of the spring from equilibrium, $a = 3.0\ \frac{N}{m^2}$ and $b = 4.0$ N. The change in elastic potential energy of the spring as it is stretched from $x = 0$ m to $x = 2.0$ m is
Change in energy is the negative integral of force, with respect to distance. All we have to do is integrate the force expression, with $x = 0$ and $x = 2$ as the bounds:

$$\Delta U = -\int_0^2 -3x - 4\, dx = -(-16)\text{ J} = \mathbf{16\ J}$$

The sign is flipped because the applied force is in the direction opposite to spring force.

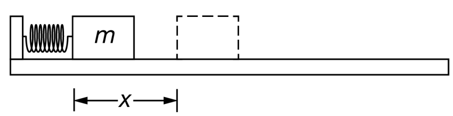


Figure1

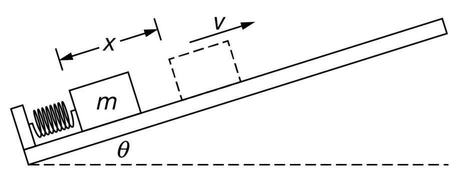
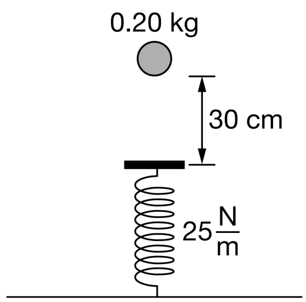


Figure 2

15. Students perform a set of experiments by placing a block of mass m against a spring, compressing the spring a distance x along a horizontal surface of negligible friction, releasing the block, and measuring the velocity v of the block as it leaves the spring, as shown in Figure 1. The experiments indicate that as x increases, so does v in a linear relationship. The surface is now lifted so that the surface is at an angle θ above the horizontal. Which of the following indicates how the relationship between x and v changes?
A) As *x* increases, *v* increases, but the relationship is no longer linear and the values of *v* will be less for the same values of *x*.
B) As x increases, v increases, but the relationship is no longer linear and the values of v will be more for the same values of x.

C) As x increases, v increases, and the relationship is still linear, but the values of v will be less for the same values of x.
D) As x increases, v increases, and the relationship is still linear, but the values of v will be more for the same values of x.
E) The relationship is still linear, but as x increases, v decreases.
With the surface tilted, the block will have to work against the force of gravity, which performs negative work and slows down the exit velocity.
A



16. A 0.20 kg ball is released from rest at a height of 30 cm above an ideal vertical spring. The spring has a spring constant of 25 $\frac{N}{m}$, and all motion is restricted to the vertical direction. The maximum compression of the spring is:
This can be solved with conservation of energy. When the ball hits the spring, its potential energy has changed by $\Delta U_g = mgh = (0.2)(10)(0.3) = 0.6$ J
However, as the spring compresses, the ball continues to lose potential energy, as its height changes by the same distance Δx the spring compresses. The change in energy is therefore:

$$\Delta U_g = 0.6 + mg\Delta x = 0.6 + 2(\Delta x)$$

Set this change equal to spring potential energy and solve for Δx:

$$0.6 + 2\Delta x = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}(25)(\Delta x)^2$$

$$0.048 + 0.16\Delta x = (\Delta x)^2$$

This is a quadratic function, which means it has two solutions. One of them is negative, which in the context of this problem, means the spring extended upward. We know that's not right. Using a calculator, the other solution is ***Δx* = 0.313 m**

1. C
2. $\frac{1}{2}U_0$
3. E
4. $F = G\frac{mM}{2d^2}$ and $W = 0$
5. $U_s = \frac{5}{11}kD^{11/5}$
6. $\Delta y = 0.081$ m
7. $v = 4.619$ m/s
8. $30a^4 - 7a^3$
9. 40 J
10. $ke^{D/k} - k = k(e^{D/k} - 1)$
11. 2 times
12. $x = x_1$ and x_3
13. E
14. 16 J
15. A
16. $\Delta x = 0.313$ m