Name____ Date_______________

SUBJECT: MATHEMATICS **CLASS: SS2**

SCHEME OF WORK

WEEK	TOPIC
1	Revision of Logarithm of Numbers Greater than One and Logarithm of Numbers Less than one; Reciprocal and Accuracy of Results Using Straight Calculation.
2	Approximations; Calculations Using Standard Form; Significant Figures; and Percentage Error.
3	Sequence and Series: Concept of Sequence and Series; Terms of Arithmetic Progressions and Sum ; Solving problem on A.P
4	Geometric Progressions: The nth Term and Sum of the First n-terms. Problem Solving on G.P and Geometric Mean.
5	Construction of Quadratic Equation from Sum and Product of Roots. Word Problem Leading to Quadratic Equation.
6	Review of the Half Term Work and Periodic Test.
7	Simultaneous Equations: Solving Simultaneous Equations Using Elimination and Substitution Method; Word Problem Leading to Simultaneous Equations.
8	Simultaneous Equations: Solving Equations Involving One Linear and One Quadratic; Using Graphical Method to Solve Quadratic Equations.
9	Straight Line Graphs: Gradient of a Straight Line; Gradient of a Curve; Drawing of Tangents to a Curve.
10	Revision.

REFERENCE BOOKS

- New General Mathematics SSS2 by M.F. Macrae etal.
- Essential Mathematics SSS2 by A.J.S. Oluwasanmi.

Name____ Date_______________

WEEK ONE

TOPIC: REVISION OF LOGARITHM OF NUMBERS GREATER THAN ONE AND LOGARITHM OF NUMBERS LESS THAN ONE.

CONTENT

- Standard forms
- Logarithm of numbers greater than one
- Multiplication and divisions of numbers greater than one using logarithm
- Using logarithm to solve problems with roots and powers (no > 1)
- Logarithm of numbers less than one.
- Multiplication and division of numbers less than one using logarithm
- Roots and powers of numbers less than one using logarithm

STANDARD FORMS

A way of expressing numbers in the form A x 10^x where 1< A < 10 and x is an integer, is said to be a standard form. Numbers are grouped into two. Large and small numbers. Numbers greater than or equal to 1 are called large numbers. In this case the x, which is the power of 10 is positive. On the other hand, numbers less than 1 are called small numbers. Here, the integer is negative.

Numbers such as 1000 can be converted to its power of ten in the form 10^x where x can be termed as the number of times the decimal point is shifted to the front of the first significant figure i.e. 10000 = 10^4

Number	Power of 10
100	10^2
10	10^1
1	10^0
0.01	10^{-3}
0.10	10^{-1}

Note: One tenth; one hundredth, etc are expressed as negative powers of 10 because the decimal point is shifted to the right while that of whole numbers are shifted to the left to be after the first significant figure.

Examples

1. Express in standard form (i) 0.08356 (ii) 832.8 in standard form

Solution

i 0. 08356 = 8.356 x 10^{-2}

ii 832.8 = 8.328 x 10^2

2. Express the following in standard form

(a) 39.32 = 3.932 x 10^1

(b) 4.83 = 4.83 x 10^0

(c) 0.005321 = 5.321 x 10^{-3}

WORKING IN STANDARD FORM

Example

Evaluate the following leaving your answer in standard form

(i) 4.72 x 10^3 + 3.648 x 10^3

(ii)6.142 x 10^5 + 7.32 x 10^4

(iii) 7.113 x 10^{-5}- 8.13 x 10^{-6}

solution

i. $4.72 \times 10^3 + 3.648 \times 10^3$

$= [\ 4.72 + 3.648\ ] \times 10^3$

$= 8.368 \times 10^3$

ii. $= 6.142 \times 10^5 + 7.32 \times 10^4$

$= 6.142 \times 10^5 + 0.732 \times 10^5$

$= [6.142 + 0.732\ ] \times 10^5$

$= 6.874 \times 10^5$

iii. $= 7.113 \times 10^{-5} - 8.13 \times 10^{-6}$

$= 7.113 \times 10^{-5} - 0.813 \times 10^{-5}$

$= [\ 7.113 - 0.813\ ] \times 10^{-5}$

$= 6.3 \times 10^{-5}$

Example: Simplify : √[P/Q], leaving your answer in standard form given that $P = 3.6 \times 10^{-3}$ and $Q = 4 \times 10^{-8}$.

Solution

$= \sqrt{[P/Q]}$

$= \dfrac{3.6 \times 10^{-3}}{4 \times 10^{-8}}$

$= \sqrt{\dfrac{36 \times 10^{-4}}{4 \times 10^{-8}}}$

$= \sqrt{9 \times 10^{-4-(-8)}}$

$= 3 \times (10^4)^{½}$

$= 3 \times 10^2$

EVALUATION

1. Evaluate $2.5 \times 10^{-3} + 3.2 \times 10^{-2}$
2. Without using table, evaluate the following leaving your answer in standard form,
   i. 4ab given that $a = 3.5 \times 10^{-3}$ and $b = 2.3 \times 10^6$ ii. $\dfrac{0.08 \times 0.000025}{0.0005}$

LOGARITHM OF NUMBERS GREATER THAN ONE

Base ten logarithm of a number is the power to which 10 is raised to give that number e.g.

$628000 = 6.28 \times 10^5$

$628000 = 10^{0.7980} \times 10^5$

$= 10^{0.7980+5}$

$= 10^{5.7980}$

Log 628000 = 5.7980

IntegerFraction (mantissa)

If a number is in its standard form, its power is its integer i.e. the integer of its logarithm e.g. log 7853 has integer 3 because $7853 = 7.853 \times 10^3$

Examples:

Use tables (log) to find the complete logarithm of the following numbers.

(a) 80030 (b) 8 (c) 135.80

Solution

(a) 80030 = 4.9033

(b) 8 = 0.9031

(c) 13580 = 2.1329

Evaluation

Use table to find the complete logarithm of the following:

(a) 183 (b) 89500 (c) 10.1300 (d) 7

Multiplication and Division of numbers greater than one using logarithm

To multiply and divide numbers using logarithms, first express the number as logarithm and then apply the addition and subtraction laws of indices to the logarithms. Add the logarithm when multiplying and subtract when dividing.

Examples

Evaluate using logarithm.

1. 4627 x 29.3
2. 8198 ÷ 3.905
3. $\frac{48.63 \times 8.53}{15.39}$

Solutions

1. 4627 x 29.3

	No	Log
	4627	3.6653
	X 29.3	+ 1.4669
antilog →	135600	5.1322

∴ 4627 x 29.3 = **135600**

To find the Antilog of the log 5.1322 use the antilogarithm table:

Check 13 under 2 diff 2 (add the value of the difference) the number is 0.1356. To place the decimal point at the appropriate place, add one to the integer of the log i.e. 5 + 1 = 6 then shift the decimal point of the antilog figure to the right (positive) in 6 places.

= 135600

2. 819.8 x 3.905

	No	Log
	819.8	2.9137
	3.905	0.5916
antilog →	209.9	2.3221

∴ 819.8 ÷ 3.905 = **209. 9**

3. $\frac{48.63 \text{ X } 8.8.53}{15.39}$

	No	Log
	48.63	1.6869
	8.53	+ 0.9309
		2.6178
	÷ 15.39	- 1.1872
antilog →	26.95	1.4306

∴ $\frac{48.63 \div 8.53}{15.39}$ = 26.96

Evaluation: Use logarithm to calculate. $\frac{3612 \times 750.9}{113.2 \times 9.98}$

USING LOGARITHM TO SOLVE PROBLEMS WITH POWERS AND ROOT (NO. GREATER THAN ONE)

Examples:

Evaluate:

(a) 3.53^3 (b) $\sqrt[4]{40000}$ (c) $\sqrt{\frac{94100 \times 38.2}{5.683^3 \times 8.14}}$ to 2 s.f

Solution

(a) 3.53^3

No.	Log
3.53^3	0.5478 x 3
44.00	1.6434

∴ 3.53^3 = **44.00**

(b) $\sqrt[4]{4000}$

No.	Log
$\sqrt[4]{4000}$	3.6021 ÷ 4
7.952	0.9005

∴ $\sqrt[4]{4000}$ **=** 7.952

(c) $\dfrac{\sqrt{94100} \times 38.2}{5.683^3 \times 8.14}$

Find the single logarithm representing the numerator and the single logarithm representing the denominator, subtract the logarithm then find the antilog.

No	Log	
$\sqrt{94100}$	4.9736 ÷ 2 = 2.4868	
38.2	1.5821	
Numerator	**4.0689** →	4.0689
5.68^3	0.7543 x 3 = 2.2629	
8.14	0.9106	
Denominator	**3.1735** →	**3.1735**
7.859		**0.8954**

∴ $\dfrac{\sqrt{94100} \times 38.2}{5.68^3 \times 8.14}$ = 7.859 ≃ **7.9 (2.sf)**

LOGARITHM OF NUMBERS LESS THAN ONE

To find the logarithm of number less than one, use negative power of 10 e. g.

$0.037 = 3.7 \times 10^{-2}$

$= 10^{0.5682} \times 10^{-2}$

$= 10^{0.5682 + (-2)}$

$= 10^{-2\ 5682}$

Log 0.037 = $\bar{2}$. 5682

$\bar{2}$. 5682

Integer decimal fraction (mantissa)

Example: Find the complete log of the following.

(a) 0.004863 (b) 0. 853 (c) 0.293

Solution

Log 0.004863 = $\bar{3}$.6369

Log 0.0853 = $\bar{2}$.9309

Log 0.293 = $\bar{1}$.4669

Evaluation

1. Find the logarithm of the following:
   (a) 0.064 (b) 0.002 (c) 0.802
2. Evaluate using logarithm.
   $\dfrac{95.3 \times \sqrt{318.4}}{1.29^5 \times 2.03}$

USING LOGARITHM TO EVALUATE PROBLEMS OF MULTIPLICATION, DIVISION, POWERS AND ROOTS WITH NUMBERS LESS THAN ONE

OPERATION WITH BAR NOTATION

Note the following when carrying out operations on logarithm of numbers which are negative.

i.The mantissa (fractional part) is positive, so it has to be added in the usual manner.

ii. The characteristic (integral part) is either positive or negative and should therefore be added or operated as directed numbers.

iii. For operations like multiplication and division, separate the integer from the characteristic before performing the operation.

Examples:

Simplify the following, leaving the answers in bar notation, where necessary

i. $\bar{5}$.7675 + 2.4536
ii. 6.8053 – 4.1124
iii. 2.4423 x 3
iv. 2.2337 ÷ 7

Solution

i. $\bar{5}$.7675 + 2.4536 ii. 6.8053 – 4.1124

$\bar{5}$.7675 6.8053
+ 2. 4536 - 4. 1124
6. 22112.6929

iii. 2.4423 x 3
= 3(2 + 0.4423)
= 6 + 1.3269
= 5.3269

iv. 2. 2337 ÷ 7
= 7 + 5.2337÷ 7
= 1+ 0.7477
= 1 + 0.7477
= 1.7477

Examples: Evaluate the following using the logarithm tables;

1. 0.6735 x 0.928
2. 0.005692 ÷ 0.0943
3. 0.6104^3
4. $\sqrt[4]{0.00083}$
5. $\sqrt[3]{0.06642}$

Solution

1. 0.6735 x 0.928

No.	**Log.**
0.6735	$\bar{1}$.8283
0.928	$\bar{1}$.9675
0.6248	**1.7958**

∴ 0.6735 x 0.928 = 0.6248

2. 0.005692 ÷ 0.0943

No	**Log**
0.005692	3.7553
÷ 0.0943	2.9745
0.06037	**2.7808**

3. 0.6104^3

No	**Log**
0.6104^3	1.7856 x 3
0.2274	1.3568

∴ 0.6104^3 = 0.2274

∴ 0.005692 ÷ 0.943 = 0.6037

4. 4 0.00083

No.	**Log.**
4 0.00083	4.9191 ÷ 4
0.1697	1.2298

∴ 4 0.06642 =**0.1697**

5. $\sqrt[3]{0.06642}$

No.	**Log.**
$\sqrt[3]{0.06642}$	2.8223 ÷ 3
	3) 2 + 0.8223
	3) 3 + 1.8223
	1 + 0.6074
0.405	1.6074

$\sqrt[3]{0.6642}$ = **0.405**

Note: 3 cannot divide 2 therefore subtract 1 from the negative integer and add 1 to the positive decimal fraction so as to have 3 which is divisible by 3 without remainder.

Evaluation: Use the logarithms table to evaluate

$\sqrt[5]{(0.1684)^3}$

GENERAL EVALUATION / REVISION QUESTION

Use tables to evaluate the following, giving your answers correct to 3 s.f.

1. $(0.897)^3$ 2. $(0.896 \times 0.791)^3$ 3. $\sqrt{(800.9 \times 87.25)^2}$

4. $\frac{8750000 \times 8900}{300.5}$ 5. $\frac{80.4^2 \times 78000}{\sqrt{100.5 \times 35.7}}$

WEEKEND ASSIGNMENT

Use table to find the log of the following:

1. 900 (a) 3.9542 (b) 1.9542 (c) 2.9542 (d) 0.9542
2. 12.34 (a) 3.0899 (b) 1.089 (c) 2.0913 (d) 1.0913
3. 0.000197 (a) 4.2945 (b) 4.2945 (c) 3.2945 (d) 3.2945
4. 0.8 (a) 1.9031 (b) 1.9031 (c) 0.9031 (d) 2.9031
5. Use antilog table to write down the number whose logarithms is 3.8226.
   (a) 0.6646 (b) 0.06646 (c) 0.006646 (d) 66.46

THEORY

Evaluate using logarithm.

1. $\frac{23.97 \times \sqrt{0.7124}}{3.877 \times 52.18}$

2. $\sqrt[3]{\frac{76.58}{0.009523}}$

Reading Assignment

Essential Mathematics for SSS2, pages 1-10, Exercise 1.8

Name____ Date________________

Name____ Date_______________

WEEK TWO

TOPIC: PERCENTAGE ERROR

CONTENT

- Definition of percentage error
- Calculation of percentage error
- Percentage error (range of values via approximations)
- Calculations on percentage error in relation to approximation

Definition of Percentage Error

No measurement, however, carefully made is exact (accurate) i.e if the length of a classroom is measured as 2.8m to 2 s.f the actual length may be between 2.75 and 2.85, the error of this measurement is 2.75 – 2.8 or 2.85 – 2.8 = $\pm$ 0.05.

2.75 2.85 2.8 2.9

$$\text{Percentage error} = \frac{\text{error}}{\text{Actual measurement}} \times \frac{100}{1}$$

error = $\pm$ 0.05

actual measurement 2.8

$$\% \text{ error} = \frac{0.05}{2.8} \times \frac{100}{1}$$

$$= 1.785\% = 1.79\%$$

Example 2

Suppose the length of the same room is measured to the nearest cm ,280cm i.e. (280cm) calculate the percentage error.

Measurement = 280cm.

The range of measurement will be between 279.5cm or 280cm

Error = 280 – 279.5 = 0.5cm

$$\% \text{ error} = \frac{\text{error}}{\text{Measurement}} \times \frac{100}{1}$$

$$\% \text{ error} = \frac{0.5}{280} \times \frac{100}{1} = 0.178\%$$

$$= 0.18\% \text{ (2sf)}$$

Example 3

The length of a field is measured as 500m; find the percentage error of the length if the room is measured to

i. nearest metre ii. nearest 10m iii. one significant figure.

Solutions

i. To the nearest metre

Measurement = 500m

Actual measurement = between 499.5 - 500.5

Error = $\pm$ 0.5m

$$\% \text{ error} = \frac{\text{error}}{\text{measurement}} \times 100$$

$$\frac{0.5}{500} \times \frac{100}{1} = 0.10\%$$

$$= \mathbf{0.10\%}$$

ii. Nearest 10m

Measurement = 500m,

range= 495m – 505m

error = $\pm$ 5m

$$\text{error} = \frac{5}{500} \times \frac{100}{1} = 1\%$$

iii. To 1 s.f.

measurement = 500m

range = 450 – 550

error = $\pm$ 50

$$\% \text{ error} = \frac{50}{500} \times \frac{100}{1} = 10\%$$

Evaluation

1. The length of each side of a square is 3.6 cm to 2s.f. (a) Write down the smallest and the largest of each side. (b) Calculate the smallest and the largest values for the perimeter. (c) Find the possible values of the area.

Percentage Error (range of values via approximations)

1. Range of values measured to the nearest whole number i.e. nearest tens, hundreds etc. e.g. Find the range of values of N6000 to:

i.	nearest naira	=	N5999.50	-	6000.50
ii.	nearest N10	=	N5995	-	6005
iii.	nearest N100	=	N5950	-	6050
iv.	nearest N1000	=	N5500	-	6,500

2. Range of values measured to a given significant figure. E.g. find the range of value of 6000 to

1 sf	=	5500 - 6500
2 sf	=	5950 - 6050
3 sf	=	5995 - 6005
5 sf	=	5999.95- 6000.05

3. Range of values measured to a given decimal places e.g. 39.8 to a 1d.p = 39.75 – 39.85.
   Note: if it is 1 d.p, the range of values will be in 2 d.p, if 2 d.p, the range will be in 3 d.p etc. (i.e the range = d.p + 1). The same rule is also applicable to range of values to given significant figure.

Evaluation

Orally: From New General Mathematics SS 2 by J. B. Channon and Co 3rd edition exercise 46 no. 1a – f.

Calculations on percentage error:

Example:

Calculate the percentage error if

1. The capacity of a bucket is 7.5 litres to 1 d.p.
2. The mass of a student is 62kg to 2 s.f.

Solutions

1. Measurement = 7.5litres (1d.p)
   Range of values = 7.45 - 7.55
   Error = 7.5 – 7.45 = 0.05
   $\% \text{ error} = \frac{\text{error}}{\text{measurement}} \times \frac{100}{1}$
   $\frac{0.05}{7.51} \times 100$
   $= 0.67\%$
2. Measurement = 62kg (2 s.f)
   Range of values = 61.5kg to 62.5kg
   error = 6.2 - 61.5 = 0.5kg
   $\% \text{ error} = \frac{\text{error}}{\text{measurement}} \times \frac{100}{1}$
   $\frac{0.5}{62} \times \frac{100}{1} = \mathbf{0.81\%}$

EVALUATION

1. Calculate correct to 2 s.f. the percentage error in approximately 0.375 to 0.4.

GENERAL EVALUATION / REVISION QUESTION

1. A metal rod was measured as 9.20 m. If the real length is 9.43 m, calculate the percentage error to 3 s.f

2.A student measures the radius of a circle as 1.46 cm instead of 1.38 cm. Calculate the percentage error.
3.The weight of sugar was recorded as 8.0 g instead of 8.2 g. What is the percentage error?
4.A student mistakenly approximated 0.03671 to 2 d.p instead of 2 s.f. What is the percentage error correct to 2 s.f
5.A man's weight was measured as 81.5 kg instead of 80 kg. Find the percentage error in the measurement.

WEEKEND ASSIGNMENT
What is the error in the following measurement
1. The distance between two towns is 60km to the nearest km. (a) 5km (b) 0.5km (c) 8.3km (d) 0.83km
2. The area of a classroom is 400m2 to 2 s.f. (a) 50m^2 (b) 1.25m^2(c) 2.5m^2 (d) 5m^2
3. A sales girl gave a girl a balance of N1.15 to a customer instead of N1.25, calculate the % error.
4. A student measured the length of a room and obtained the measurement of 3.99m, if the percentage error of his measurement was 5% and his own measurement was smaller than the length, what is the length of the room?(a) 3.78m (b) 3.80m (c) 4.18m (d) 4.20m
5. A man is 1.5m tall to the nearest cm, calculate his percentage error.
(a) 0.05cm (b) 0.33% (c) 0.033% (d) 0.05cm

THEORY
1. A classroom is 10m by 10m; a student measured a side as 9.5m and the other side as 10m and uses his measurement to calculate the area of the classroom. Find the percentage error in a. the length of one of the sidesb. the area of the room
2. Instead of recording the number 1.23cm for the radius of a tube, a student recorded 1.32cm, find the percentage error correct to 1 d.p.

Reading Assignment
Essential Mathematics for SSS2, pages 13-22, Exercise 2.4

Name____ Date_______________

WEEK THREE

TOPIC: ARITHMETIC PROGRESSION (A. P)

CONTENT

- Sequence
- Definition of Arithmetic Progression
- Denotations in Arithmetic progression
- Deriving formulae for the term of A. P.
- Sum of an arithmetic series

Find the next two terms in each of the following sets of number and in each case state the rule which gives the term.

(a) 1, 5, 9, 13, 17, 21, 25(any term +4 = next term)
(b) 2, 6, 18, 54, 162, 486, 1458 (any term x 3 = next term)
(c) 1, 9, 25, 49, 81, 121, 169, (sequence of consecutive odd no)
(d) 10, 9, 7, 4, 0, -5, -11, **-**18, -26**,** (starting from 10, subtract 1, 2, 3 from immediate no).

In each of the examples below, there is a rule which will give more terms in the list. A list like this is called a SEQUENCE in many cases; it can simply matter if a general term can be found for a sequence e.g.

1, 5, 9, 13, 17 can be expressed as

1, 5, 9, 13, 17 ……………. 4n – 3 where n = no of terms

Check: 5^{th} term = 4(5) -3

20 – 3 = **17**

10^{th} term = 4(10) – 3

40 – 3 = **37**

<u>Example 2</u>

Find the 6^{th} and 9^{th} terms of the sequence whose nth term is

(a) (2n + 1)
(b) 3 – 5n.

<u>Solution</u>

(a) 2n + 1

6^{th} term = 2(6) + 1 = 12 + 1 = 13

9^{th} term = 2 (9) + 1 = 18 + 1 = 19

(b) 3 – 5n

6^{th} term = 3 – 5 (6) = 3 – 30 = **-27**

9^{th} term = 3 – 5 (9) = 3 – 45 = -**42**

Evaluation

For each of the following sequence, find the next two terms and the rules which give the term.

1. 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ____, ____
2 100, 96, 92, 88, _____, ____
3. 2, 4, 6, 8, 10, ____, _____
4. 1, 4, 9, 16, 25, ____, _____
(i) Arrange the numbers in ascending order (ii) Find the next two terms in the sequence
5. 19, 13, 16, 22, 10
6. $-2^1/_2$, $5^1/_2$, $3^1/_2$, $1^1/_2$, $-^1/_2$
7. Find the 15^{th} term of the sequence whose nth term is $\frac{3n - 5}{4}$

DEFINITION OF ARITHMETIC PROGRESSION

A sequence in which the terms either increase or decrease in equal steps is called an Arithmetic Progression.

The sequence 9, 12, 15, 18, 21, ____, _____, _____ has a first term of 9 and a common difference of +3 between the terms.

Denotations in A. P.

a = 1^{st} term
d = common difference
n = no of terms
U_n = nth term

S_n = Sum of the first n terms

Formula for nth term of Arithmetic Progression

e.g. in the sequence 9, 12, 15, 18, 21.

a = 9

d = 12 – 9 or 18 – 15 = 3.

1st term = U_1 = 9 = a

2nd term = U_2 = 9 + 3 = a + d

3rd term = U_3 = 9 + 3 + 3 = a + 2d

10th term = U_{10} = 9 + 9(3) = a + 9d

nth term = U_n = 9+(n-1)3 = a + (n-1)d

∴nth term = U_n = a + (n-1)d

Example:

1.Given the A.P, 9, 12, 15, 18 …… find the 50th term.

a = 9 d = 3 n = 50 $U_n = U_{50}$

U_n = a + (n-1) d

U_{50} = 9 + (50-1) 3

= 9 + (49) 3

= 9 + 147

= **156**

2.The 43rd term of an AP is 26, find the 1st term of the progression given that its common difference is ½ and also find the 50th term.

U_{43} = 26 d = ½ a = ? n = 43

U_n = a + (n-1) d

26 = a + (43-1) ½

26 = a + 42($^1/_2$)

26 = a + 21

26 – 21 = a

5 = a

a = **5**

(b) a = 5 d = ½ n = 50 U_{50} =?

U_n = a + (n-1) d

U_{50} = 5 + (50-1)$^1/_2$

= 5 + 49($^1/_2$)

U_{50} = 5 + 24$^1/_2$

U_{50} = 29$^1/_2$

Evaluation

1. Find the 37th term of the sequence 20, 10, 0, -10…
2. 1, 5… 69 are the 1st, 2nd, and last term of the sequence; find the common difference between them and the number of terms in the sequence.

SUM OF AN ARITHMETIC SERIES

When the terms of a sequence are added, the resulting expression is called series e.g. in the sequence 1, 3, 5, 7, 9, 11.

Series = 1 + 3 + 5 + 7 + 9 + 11

When the terms of a sequence are unending, the series is called infinite series, it is often impossible to find the sum of the terms in an infinite series.

e.g. 1 + 3 + 5 + 7 + 9 + 11 + ………………… Infinite

Sequence with last term or nth term is termed finite series.

e.g.

Find the sum of

1, 3, 5, 7, 9, 11, 13, 15

If sum = 2, n = 8

Then

S = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15

Or S = 15 + 13 + 11 + 9 + 7 + 5 + 3 + 1

Add eqn1 and eqn 2

2s = 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16

$= \frac{48}{2} = \frac{8(16)}{2} = S =$ **64**

Deriving the formula for sum of A. P. The following represent a general arithmetic series when the terms are added.

S = a + (a+d) + a + 2d + ... + (L-2d) + (L-d) + L – eqn

S = L + (L-d) + L – 2d + ... a + 2d + (a+d) + a – eqn

2s = (a + L) + (a + L) + (a + L) + ... (a + L) + (a + L) + (a + L)

$2s = \frac{n(a + L)}{2}$

$S = \frac{n(a+L)}{2}$

L => Un = a + (n-1)d

Substitute L into eq**

$S = \frac{n(a + a+(n-1)d}{2}$

$\mathbf{S = \frac{n(2a + (n-1)d}{22} = \underline{n (2a+ (n-1)d}}$

∴ $\mathbf{S = \frac{n}{2}}[a + L]$ where L is the last term i.e U_n

or

$\mathbf{S = \frac{n}{2}}[2a +(n-1)d]$ when d is given or obtained

Example 2

Find the sum of the 20th term of the series 16 + 9 + 2 + ...

a = 16 d = 9 – 16 = -7 n = 20

$S = \frac{n(2a + (n-1)d)}{2}$

$S = \frac{20 (2x16) + (20-1)(-7)}{2}$

$= \frac{20 (32 + 19(-7)}{2}$

S =10 (32 - 133) = 10(-101)

S = -1010

EVALUATION

1. Find the sum of the arithmetic series with 16 and -117 as the first and 20th term respectively.
2. The salary scale for a clerical officer starts at N55, 200 per annum. A rise of N3, 600 is given at the end of each year; find the total amount of money earned in 12 years.

GENERAL EVALUATION /REVISION QUESTION

1. An A. P. has 15 terms and a common difference of -3, find its first and last term if its sum is 120.
2. On the 1st of January, a student puts N10 in a box, on the 2nd she puts N20 in the box, on the 3rd she puts N30 and so on putting on the same no. of N10 notes as the day of the month. How much will be in the box if she keeps doing this till 16th January?
3. The salary scale for a clerical officer starts at N55, 200 per annum. A rise of N3, 600 is given at the end of each year, find the total amount of money earned in 12 years.
4. Find the 7th term and the nth term of the progression 27,9,3,…
5. If 8, x, y, - 4 are in A.P, find x and y.

Name____ Date_______________

WEEKEND ASSIGNMENT

1. Find the 4th term of an A. P. whose first term is 2 and the common difference is 0.5 (a) 4 (b) 4.5 (c) 3.5 (d) 2.5
2. In an A. P. the difference between the 8th and 4th term is 20 and the 8th term is $1^1/_2$ times the 4th term, find the common difference (a) 5 (b) 7 (c) 3 (d) 10
3. Find the first term of the sequence in no. 2 (a) 70 (b) 45 (c) 25 (d) 5
4. The next term of the sequence 18, 12, 60 is (a) 12 (b) 6 (c) -6 (d) -12
5. Find the no. of terms of the sequence $^1/_2$, ¾, 1, ……………….. 51/2 (a) 21 (b) $4^3/_4$ (c) 1 (d) 22

THEORY

1. Eight wooden poles are to be used for pillars and the length of the poles form an arc Arithmetic Progression (A. P.) if the second pole is 2m and the 6th pole is 5m, give the lengths of the poles in order and sum up the lengths of the poles.

2 a. Write down the 15th term of the sequence.

$\frac{2}{1\times3}, \frac{3}{2\times4}, \frac{4}{3\times5}, \frac{5}{4\times6}$

b. An arithmetic progression (A. P.) has 3 as its term and 4 as the common difference.

c. Write an expression in its simplest form for the nth term.

d. Find the 10th term and the sum of the first

Reading Assignment

New General Mathematics SSS2

Name____ Date_______________

WEEK FOUR

TOPIC: GEOMETRIC PROGRESSION

CONTENT

- Definition of Geometric Progression
- Denotations of Geometric progression
- The nth term of a G. P.
- The sum of Geometric series
- Sum of G. P. to infinity
- Geometric mean

<u>Definition of G. P</u>

The sequence 5, 10, 20, 40 has a first term of 5 and the common ratio
Between the term is 2 e.g. ($^{10}/_{5}$ or $^{40}/_{20}$ = 2).
A sequence in which the terms either increase or decrease in a common ratio is called a Geometric Progression
(G. P)
G. P: a, ar, ar^2, ar^3 ………………

Denotations in G. P
a = 1^{st} term
r = common ratio
U_n = nth term
S_n = sum

<u>The nth term of a G. P</u>

The nth term = Un
Un = ar^{n-1}
1^{st} term = a
2^{nd} term = a x r =ar
3^{rd} term = a x r x r = ar^2
4^{th} term = a x r x r x r = ar^3
8^{th}term = a x r x r x r x r x r x r x r = ar^7
nth term = a x r x r x r x ………. $\mathbf{ar^{n-1}}$

<u>Example</u>

Given the GP 5, 10, 20, 40. Find its (a) 9^{th} term (b) nth term
Solution
a = 5 r = 10/5 = 2
U_9 = ar^{n-1}
U_9 = $5 (2)^{9-1}$
= $5 (2)^8$
= 5 x 256 = 1,280
(b) $U_n = ar^{n-1}$
= $5(2)^{n-1}$

<u>Example 2</u>

The 8^{th} term of a G.P is -7/32. Find its common ratio if it first term is 28.
U_8 = -7/32 U_n = ar^{n-1}
-7/32 = $28 (r)^{8-1}$
-7/32 = $28r^7$
-7/32 x 1/28 =
$^{-7}/_{32}$ x $^{1}/_{28}$ = r^7

$$r = \sqrt[7]{\frac{-7}{32 \times 28}} = \sqrt[7]{\frac{-7}{896}}$$

r = - 0.5

<u>Evaluation</u>

1. The 6^{th} term of a G.P is 2000. Find its first term if its common ratio is 10.

Name____ Date________________

2. Find the 7^{th} term and the nth term of the progression 27, 9 , 3, …

THE SUM OF A GEOMETRIC SERIES

$a + ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots\ldots\ldots ar^{n-1}$

represent a general geometric series where the terms are added.

$S = a + ar + ar^2 \ldots\ldots\ldots\ldots ar^{n-1}$ eqn 1

Multiply through r

$rs = ar + ar^2 + ar^3 \ldots\ldots\ldots\ldots ar^n \ldots\ldots\ldots$ eqn 2

subtracteqn 2 from 1

$S - rs = a - ar^n$

$S \frac{(1 - r)}{1 - r} = \frac{a(1-r^n)}{1-r}$

$$\mathbf{S. = \frac{a(1 - r^n)}{1 - r}} \qquad r < 1$$

Multiply through by -1 or subs. eqn. 1 from e.g. 2

$rs - s = ar^n - a$

$S \frac{(r - 1)}{r - 1} = \frac{a(r^n - 1)}{r - 1}$

$$\mathbf{S = \frac{a(r^n - 1)}{r - 1}} \qquad \text{for } r > 1$$

Example:

Find the sum of the series.

a. ½ + ¼ + 1/8 + …………………… as far as 6^{th} term

b. 1 + 3 + 9 + 27 + ………………… 729

Solution

$a = ½$

$r = ½ \quad (r = ¼ \div ½ = ½)$

$\therefore r < 1$

$S = \frac{a(1-r^n)}{1 - r}$

$S_6 = \left[\frac{½(1 - (½)^6}{1 - ½}\right]$

$S_6 = \frac{½(1 - {}^1/_{64})}{½}$

$S_6 = 1 - \frac{1}{64} = \frac{64 - 1}{64} = \frac{63}{64}$

2. $a = 1, r = 3, n = ?\ U_n = 729$

$U_n = ar^{n-1}$

$729 = 1 \times 3^{n-1} \qquad (3^{n-1} = 3^n \times 3^{-1})$

$729 = \frac{3n}{3}$

$3^n = 3 \times 729$

$3^n = 31 \times 36$

$3^n = 3^7$

$\therefore n = 7$

$S = \frac{a(r^n-1)}{r - 1}$

$S = \frac{a(3^7 - 1)}{3 - 1} = \frac{2187 - 1}{2}$

$\frac{2186}{2} = 1093$

Evaluation: Find the sum of the series 40, -4, 0.4 as far as the 7^{th} term.

SUM OF G. P. TO INFINITY

Name____ Date_______________

Sum of G. P to infinity is only possible where r is < 1.

Where r is > 1 there is no sum to infinity.

Example:

1. Find the sum of G. P. 1 + ½ + ¼ + …………………… (a) to 10 terms (b) to 100 terms. Hence deduce the sum of the series (formula) if it has a very large no. of term or infinity.

(a) a = 1 r = ½

n = 10

$S = \frac{a(1-r^n)}{1-r}$

$S = \frac{1(1-(^1/_2)^{10})}{1-½} = \frac{1(1-0.0001)}{1/2}$

2 (1 – 0.001)

2 – 0.002 = 1.998.

b. n = 100.

$S = \frac{a(1-r^n)}{1-r}$

$S = \frac{1(1-(^1/_2)^{100})}{1-½} = \frac{1(1-(^1/_2)^{10})^{10}}{½}$

$\frac{1(1-(0.001)^{10}}{½}$

$\frac{1(1)}{½}$ = **2**

Therefore $(1/2)^{100}$ tend to 0 (infinity).

In general,

$S = \frac{a(1-r^n)}{1-r} = \frac{a(1-0)}{1-r} = \frac{a}{1-r}$

$\therefore S_\infty = \frac{\mathbf{a}}{\mathbf{1-r}}$ = $n \to \infty$

Example 2:

Find the sum of the series 45 + 30 + 20 + …………….. to infinity.

a = 45, r = $^2/_3$, n = infinity

$S\infty = \frac{a}{1-r}$ $S = \frac{45}{1-{}^2/_3}$

$S\infty = 45 \div {}^1/_3$

$45 \times {}^3/_1$

= **135**

Evaluation

1. The sum to infinity of a Geometric Series is 100. Find the first term if the common ratio is $-^1/_2$.
2. The 3rd and 6th term of a G. P. are 48 and $14^2/_9$ respectively, write down the first four terms of the G. P.
3. The sum of a G. P. is 100 find its first term if the common ratio is 0.8.

GEOMETRIC MEAN

If three numbers such as x , y and z are consecutive terms of a G.P then their common ratio will be

$\frac{y}{x} = \frac{z}{y}$

$y^2 = xz$

$y = \sqrt{xz}$

The middle value , y is the geometric mean (GM). We can conclude by saying that the GM of two numbers is the positive square root of their products.

Example

Calculate the geometric mean of I. 3 and 27 II. 49 and $\frac{25}{4}$

Solution

I. G.M of 3 and 27 II. G.M of 49 and $\frac{25}{4}$

$= \sqrt{3 \times 27}$

$= \sqrt{81}$

$= 9$

$\sqrt[4]{\;} = 49 \times \frac{25}{4}$

$= 7 \times \frac{5}{2}$

$= \frac{35}{2} = 17\ 1/2$

Example

The first three terms of a GP are k + 1, 2k – 1, 3k + 1. Find the possible values of the common ratio.

Solution

The terms are k + 1, 2k – 1, 3k + 1

$\frac{2k-1}{k+1} = \frac{3k+1}{2k-1}$

$(2k-1)(2k-1) = (k+1)(3k+1)$

$4k^2-2k-2k+1 = 3k^2+k+3k+1$

$4k^2-4k+1 = 3k^2+4k+1$

$4k^2-3k^2-4k-4k+1-1 = 0$

$k^2-8k = 0$

$k(k-8) = 0$

$k = 0$ or $k - 8 = 0$

$k = 0$ or 8

The common ratio will have two values due to the two values of k

When k=0

K+1 = 0+1 =1

2k- 1= 2x0 – 1 = -1

3k+ 1= 3x0+ 1 = 1

terms are 1 , -1 , 1

common ratio, r = -1/1

r = -1

when k= 8

k+1 = 8+1 = 9

2k- 1 = 2x8 – 1 = 15

3k+1 = 3x8 +1 = 25

terms are 9,15,25

common ratio,r = 15/9

EVALUATION

The third term of a G.P. is 1/81. Determine the first term if the common ratio is 1/3.

GENERAL EVALUATION /REVISION QUESTION

1. p - 6, 2p and 8p + 20 are three consecutive terms of a GP. Determine the value of (a) p (b) the common ratio
2. If $\frac{1}{16}$, x , $\frac{1}{4}$, y , ….are in GP , find the product of x and y
3. The third term of a G.P is 45 and the fifth term 405.Find the G.P. if the common ratio r is positive.
4. Find the 7^{th} term and the nth term of the progression 27,9,3,…
5. In a G.P, the second and fourth terms are 0.04 and 1 respectively. Find the (a) common ratio (b) first term

WEEKEND ASSIGNMENT

1. In the 2^{nd} and 4^{th} term of a G.P are 8 and 32 respectively, what is the sum of the first four terms. (a) 28 (b) 40 (c) 48 (d) 60
2. The sum of the first five term of the G.P. 2, 6, 18, is (a) 484 (b) 243 (c) 242 (d) 130
3. The 4^{th} term of a GP is -2/3 and its first term is 18 what is its common ratio. (a) ½ (b) $^1/_3$ (c) $^{-1}/_3$ (d) $^{-1}/_2$
4. If the 2^{nd} and 5^{th} term of a G. P. are -6 and 48 respectively, find the sum of the first four terms: (a) -45 (b) -15 (c) 15 (d) 33
5. Find the first term of the G.P. if its common ratio and sum to infinity – $^3/_3$ and respectively (a) 48 (b) 18 (c) 40 (d) -42

THEORY

1.The 3^{rd} term of a GP is 360 and the 6^{th} term is 1215. Find the

(i) Common ratio (ii)First term (iii) Sum of the first four terms

1b. If (3- x) + (6) + (7- 5x) is a geometric series, find two possible values for

(i) x (ii) the common ratio, r (iii) the sum of the G.P

2.The first term of a G. P. is 48. Find the common ratio between its terms if its sum to infinity is 36.

Reading Assignment

New General Mathematics SSS2

Name____ Date_______________

WEEK FIVE

QUADRATIC EQUATIONS

CONTENT

- Construction of Quadratic Equations from Sum and Product of Roots.
- Word Problem Leading to Quadratic Equations.

CONSTRUCTION OF QUADRATIC EQUATIONS FROM SUM AND PRODUCT OF ROOTS

We can find the sum and product of the roots directly from the coefficient in the equation. It is usual to call the roots of the equation α and β If the equation

$ax^2 + bx + C = 0$ …………….. I

has the roots α and β then it is equivalent to the equation

$(x - \alpha)(x - \beta) = 0$

$x^2 - \beta x - \beta x + \alpha\beta = 0$ ………… 2

Divide equation (i)by the coefficient of x^2

$\frac{ax^2}{a} + \frac{bx}{a} + \frac{C}{a} = 0$ ………… 3

Comparing equations (2) and (3)

$x^2 + \frac{b}{a}x + \frac{C}{a} = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

then

$\alpha + \beta = \frac{-b}{a}$

and $\alpha\beta = \frac{C}{a}$

For any quadratic equation, $ax^2 + bx + C = 0$ with roots α and β

$\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{C}{a}$

Examples

1. If the roots of $3x^2 - 4x - 1 = 0$ are αand β, find α + β and αβ
2. if α and βare the roots of the equation
   $3x^2 - 4x - 1 = 0$, find the value of

(a) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(b) α - β

Solutions

1. Since $\alpha + \beta = \frac{-b}{a}$

Comparing the given equation $3x^2 - 4x - 1 = 0$ with the general form

$ax^2 + bx + C = 0$

a = 3, b = -4, C = 1.

Then

$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{3}$

$= +\frac{4}{3} = +1\,^{1}/_{3}$

$\alpha\beta = \frac{C}{a} = \frac{-1}{3} = \frac{-1}{3}$

2.a $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

Here, comparing the given equation, with the general equation,

a = 3, b = -4, C = - 1

from the solution of example 1 (since the given equation are the same),

$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{3} = +\frac{4}{3}$

$\alpha\beta = \frac{C}{a} = -\frac{1}{3}$

then

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(4/3)^2 - 2(-1/3)}{-1/3}$

$= \frac{\frac{16}{9} \pm \frac{2}{3}}{-\frac{1}{3}}$

$= \frac{16 + 6}{9} \div -1/3$

$\frac{22}{9} \times \frac{-3}{1}$

$= \frac{-22}{3}$

or $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{22}{3} = -7\ 1/3$

b) Since

$(\alpha-\beta)^2 = \alpha^2 + \beta^{-2}\alpha\beta$

but

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\therefore (\alpha-\beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\therefore (\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$(\alpha - \beta) = \sqrt{(4/3)^2 - 4(-^{1}/3)}$

$= \sqrt{16/9 + ^{4}/3}$

$= \sqrt{\frac{16 + 12}{9}}$

$= \sqrt{\frac{28}{9}} = \frac{\sqrt{28}}{3}$

$\therefore \alpha - \beta = \frac{\sqrt{28}}{3}$

Evaluation

If α and β are the roots of the equation

$2x^2 - 11x + 5 = 0$, find the value of

a. α - β

b. $\frac{1}{\alpha + 1} + \frac{1}{\beta + 1}$

c. $\alpha^2 + \beta^2$

WORD PROBLEM LEADING TO QUADRATIC EQUATIONS

Examples

1. Find two numbers whose difference is 5 and whose product is 266.

Solution

Let the smaller number be x.
Then the smaller number be x+5.
Their product is x(x+5) .

Hence,
x(x+5) = 266
x^2+5x- 266 = 0
(x-14)(x+19)=0
x=14 or x= -19

The other number is 14+5 or -19+5 i.e 19 or -14
:. The two numbers are 14 and 19 or -14 and -14.

2. Tina is 3 times older than her daughter. In four years time, the product of their ages will be 1536. How old are they now?

Solution

Let the daughter's age be x.
Tina's age = 3x
In four years' time,
Daughter's age = (x+4)years
Tina's age = (3x+4)years
The product of their ages :
(x+4)(3x+4)= 1536
$3x^2$+ 16x – 1520 = 0
(x-20)(3x+76) = =0
x=20 or x=-25.3

Since age cannot be negative, x=20years.
:. Daughter's age = 20years.
Tina's age = 20x3=60years.

Evaluation

1. Think of a number, square it, add 2 times the original number. The result is 80. Find the number.
2. The area of a square is 144cm^2 and one of its sides is (x+2)cm. Find x and then the side of the square.
3. Find two consecutive odd numbers whose product is 224.

GENERAL EVALUATION/REVISION QUESTIONS

1. The area of a rectangle is 60cm^2. The length is 11cm more than the width. Find the width.
2. A man is 37years old and his child is 8. How many years ago was the product of their ages 96?
3. If α and β are the roots of the equation $2x^2$ – 9x+4=0, find

a) α + β (b) αβ (c) α – β (d) αβ/ α + β

WEEKEND ASSIGNMENT

If α and β are the roots of the equation $2x^2$ + 9x+9=0:

1. Find the product of their roots. A. 4 B. 4.5 C. 5.5 B. -4.5
2. Find the sum of their roots. A. 4 B. 4.5 C. 5.5 B. -4.5

Name____ Date_______________

3. Find $\alpha^2+\beta^2$ A. 11.5 B. -11.25 C. 11.25 D. -11.5

5. Find αβ/ α + β A. 1 B.-1 C. 1.5 D. 4.5

THEORY

1. The base of a triangle is 3cm longer than its corresponding height. If the area is $44cm^2$, find the length of its base.
2. Find the equation in the form $ax^2+bx+c=0$ whose sum and products of roots are respectively:

a) 3,4 (b) -7/3 , 0 (c) 1.2,0.8

Reading Assignment

Essential Mathematics for SSS2, pages 50-54, exercise 4.6 and

WEEK SIX
REVIEW OF FIRST HALF TERM LESSONS

WEEK SEVEN
TOPIC: SIMULTANEOUS EQUATIONS

CONTENT

- Solving Simultaneous Equations Using Elimination and Substitution Method
- Solving Equations Involving Fractions.
- Word problems.

SIMULTANEOUS LINEAR EQUATIONS

Methods of solving Simultaneous equation

i. Elimination method
ii. Substitution method
iii. Graphical method

ELIMINATION METHOD

One of the unknowns with the same coefficient in the two equations is eliminated by subtracting or adding the two equations. Then the answer of the first unknown is substituted into either of the equations to get the second unknown.

Example

Solve for x and y in the equations 2x + 5y = 1 and 3x – 2y = 30

Solution

To eliminate x multiply equation 1 by 3 and equation 2 by 2

2x + 5y = 1 .... eqn 1 (x (3)

3x – 2y = 30 .... eqn 2 (x (2)

Resulting into,

6x + 15 y = 3 .... eqn 3

6x – 4y = 60 .... eqn 4

Subtract eqn 3 from eqn 4

6x – 6x + 15y – (- 4y) = 3 – 60

$\frac{19y}{19} = \frac{-57}{19}$ 3

y = -3

Substitute y = - 3 into eqn 1

2x + 5 (-3) = 1

2x = 1 + 15

$\frac{2x}{2} = \frac{16}{2}$

x = 8

∴ y = -3 and x = 8

Evaluation

Using elimination method to solve the simultaneous equations.

Name____ Date_______________

1. $5x - 4y = 38$ and $x + 3y = 22$
2. $2c-3d= -4$ and $4c-3d= -14$

SUBSTITUTION METHOD

One of the unknowns (preferably the one having 1 has its coefficient) is made the subject of the formula in one of the equations and substituted into the other equation to obtain the value of the first unknown which is then substituted into either of the equations to get the second unknown.

Example: Solve the simultaneous equation $2x + 5y = 1$ and $3x - 2y = 30$

Solution

$2x + 5y = 1$…………… eq 1

$3x - 2y = 30$ …………. eq 2

Make x the subject in eqn 1

$\frac{2x}{2} = \frac{1 - 5y}{2}$

$x = \frac{1 - 5y}{2}$ ……….. eqn 3

Substitute eq3 into eqn 2

$\frac{3(1-5y)}{2} - 2y = 30$

Multiple through by 2 or find the LCM and cross multiply.

$\frac{3 - 15y - 4y}{2} = 30$

$3 - 15y - 4y = 60$

$3 - 19y = 60$

$-19y = 60 - 3$

$\frac{-19y}{-19} = \frac{57}{-19}$ 3

$y = -3$

Substitute $y = -3$ into eq 3

$x = \frac{1 - 5y}{2}$

$x = \frac{1 - 5(-3)}{2} = \frac{1 + 15}{2} = \frac{16}{2}$

$x = 8$

$\therefore x = 8, y = -3$

Evaluation

Solve for x and y in the equations

1. $x + 2y = 10$ and $4x + 3y = 20$
2. $4x-y=8$ and $5x+y=19$

SIMULTANEOUS EQUATIONS INVOLVING FRACTIONS

Example

1. Solve the following equations simultaneously

$\frac{2}{x} - \frac{1}{y} = 3$ and $\frac{4}{x} + \frac{3}{y} = 16$

Solution

$\frac{2}{x} - \frac{1}{y} = 3$

$\frac{4}{x} + \frac{3}{y} = 16$

Instead of using x and y as the unknown, let the unknown be $(^1/_x)$ an $(^1/_y)$.

$2(^1/_x) - (^1/_y) = 3$ ……………. eqn 1

$4(^1/_x) - 3(^1/_y) = 16$ ………….. eqn 2

Using elimination method, multiply equation 1 by 2 to eliminate x.

$4(^1/_x) - 2(^1/_y) = 6$ …………….. eqn 3

$4(^1/_x) + 3(^1/_y) = 16$ ……………. eqn 4

$\underline{-5(^1/_y)} = \underline{-10}$

$-5 \qquad -5$

$\frac{1}{y} = 2$

∴ y = ½

Substitute (1/y) = 2 into eqn 1

$2(^1/_x) - (^1/_y) = 3$

$2(^1/_x) - (2) = 3$

$2(^1/_x) = 3 + 2$

$2(^1/_x) = 5$

$\frac{1}{x} = \frac{5}{2}$

∴ $x = ^2/_5$

∴ $y = ½, \; x = ^2/_5$

Evaluation

I. Solve for x and y simultaneously,

$\frac{x}{2} + \frac{y}{2} = 1$

$\frac{x}{2} - \frac{y}{6} = 1½$

II. Solve the pair of equations for x and y respectively.

$2x^{-1} - 3y^{-1} = 4$

$4x^{-1} + y^{-1} = 1$

FURTHER EXAMPLES

Solve for x and y simultaneously: 2x – 3y + 2 = x + 2y – 5 = 3x + y.

Solutions

2x – 3y + 2 = x + 2y – 5 = 3x + y

Form two equations out of the question

2x – 3y + 2 = 3x + y

x + 2y – 5 = 3x + y

OR

2x – 3y + 2 = x + 2y – 5 ------------- eq 1

x + 2y – 5 = 3x + y -------------- eq 2

Rearrange the equations to put the unknown on one side and the constant at the other side.

2x – 3y – x – 2y = - 5 – 2

2x – x – 3y – 2y = -7

x – 5y = -7 ---------------- eq 3

From eqn 2

x – 3x + 2y – y – 5

- 2x + y = 5 ------------- eq 4

Using substitution method solve eq 3 & 4

x – 5y = -7 ---------------- eq 3

-2x + y = 5 --------------- eq 4

Make y the subject in eq 4.

y = 5 + 2x --------------- eq 5

Substitute eqn 5 into eqn 3.

x – 5 (5 + 2x) = -7

x – 25 – 10x = -7

-9x – 25 = -7

-9x = -7 + 25

-9x = 18

$x = \frac{18}{-9}$

X = -2

Substitute x = - 2 into eqn 5

y = 5 + 2x

y = 5 + 2(-2)

y = 5 – 4

y = 1

∴ x = -2, y = 1

Example

Solve the equations

$5^{x - y/2} = 1$ $\frac{81^x}{9} = 27^{3x - y}$

Solution

$5^{x - y/2} = 1$ ----------- eq 1

$\frac{81^x}{9} = 27^{3x - y}$ ---------- eq 2

From eq 1 (using the law of indices)

$5^{x - y/2} = 5^0$

$x - y/2 = 0$

$2x - y = 0$ ------------ eq 3

From eq 2.

$\frac{81^x}{9} = 27^{3x - y}$

$\frac{3^{4x}}{3^2} = 3^{3(3x-y)}$

$3^{4x-2} = 3^{3(3x-y)}$

By comparison

$4x - 2 = 9x - 3y$

$4x - 9x + 3y = 2$

$-5x + 3y = 2$ --------- eq 4

Solve equation 3 and 4 simultaneously

$2x - y = 0$ --------- eq 3

$-5x + 3y = 2$ ---------- eq 4

Using elimination method: multiply equation 3 by 3

$6x - 3y = 0$ -------- eq 3

$-5x + 3y = 2$ ---------- eq 4

eq 3 + eq 4

$x = 2$

Substitute x = 2 into eq 3

$2x - y = 0$

$2(2) - y = 0$

$4 - y = 0$

$4 = 0 + y$

$4 = y$

∴ x = 2, y=4

WORD PROBLEMS LEADING TO SIMULTANEOUS EQUATIONS

Examples

1.Seven cups and eight plates cost N1750, eight cups and seven plates cost N1700. Calculate the cost of a cup and a plate

solution

Let a cup be x and plate be y

$7x + 8y = 1750$ -------------- eq 1

$8x + 7y = 1700$ -------------- eq 2

Multiply eq 1 by 8 and eq 2 by 7 to eliminate x (cups).

$56x + 64y = 14000$ ---------- eq 3

$56x + 49y = 11900$ ---------- eq 4

Subtracting eq 4 from eq 3

$15y = 2100$

$y = \frac{2100}{15}$

$Y = 140$

Substitute y = 140 into eq 2

$8x + 7y = 1700$

$8x + 7(140) = 1700$

$8x + 980 = 1700$

$8x = 1700 - 980$

$8x = 720$

$x = \frac{720}{8}$

$x = 90$

∴ Each cup cost ₦90 and each plate cost ₦140

2. Find a two digit number such that two times the tens digit is three less than thrice the unit digit and 4 times the number is 99 greater than the number obtained by reversing the digit.

Solution

Let the two digit number be ab, where a is the tens digit and b is the unit digit

From the first statement,

$2a + 3 = 3b$

$2a – 3b = -3$ …………eq1

From the second statement,

$4(10a + b) – 99 = 10b + a$

$40a + 4b – 99 = 10b + a$

$40a – a + 4b – 10b = 99$

$39a – 6b = 99$

Dividing through by 3

$13a – 2b = 33$ …………eq2

Solving both equations simultaneously,

$a = 3$, $b = 3$

Hence, the two digit number is 33

EVALUATION

1.The sum of two numbers is 110 and their difference is 20. Find the two numbers.

2.A pen a ruler cost #30.If the pen costs #8 more than the ruler, how much does each item cost ?

GENERAL EVALUATION AND REVISION QUESTION

1. Solve the following simultaneous equation: $3(2x – y) = x + y + 5$ & $5(3x - 2y) = 2 (x –y) + 1$
2. Five years ago, a father was 3 times as old as his son. Now, their combined ages amount to 110years. How old are they?
3. A doctor and three nurses in a hospital together earn #255 000 per month, while three doctors and eight nurses together earn #720 000 per month. Calculate (a) how much a doctor earns per month. (b) How much a nurse earns per month.
4. Solve simultaneously, $2^{x + 2y} = 1$; $3^{2x+y} = 27$
5. Solve: $2x – 2y + 5 = 3x – 4y + 2 = -1$

WEEKEND ASSIGNMENT

1. If $(x-y) \log_{10}6 = \log_{10} 216$ and $2^{x+y} =32$, calculate the values of x and y

a. x=1 , y=4 b. x= 4 , y =1 c. x=-4 , y= 1 d. x=4, y= -1

2. The point of intersection of the lines $3x- 2y =-12$ and $x + 2y = 4$ is …

a. (5, 0) b. (3, 4) c. (-2, 5) d. (-2, 3)

3. Find the value of (x - y), if $2x + 2y =16$ and $8x – 2y = 44$ a. 2 b. 4 c. 5 d. 6

4. If $5^{(p+2q)} =5$ and $4^{(p+3q)} =16$, the value of $3^{(p+q)}$ is ….. a.0 b. -1 c.2 d. 1

5. Given $\frac{4x – 3y}{7x – 4y} = \frac{11}{23}$ evaluate $\frac{y^2 – 3x}{3}$ a. -2 b. 3 c. -3 d. 2

THEORY

1. Given that $2^{1- x/y} = 1/32$, find x in terms of y, and hence solve the simultaneous equations $2x + 3y – 30 = 0$ and $2^{1- x/y} = 1/32$ (WAEC)
2. A number is made up of two digits. The sum of the digits is 11. If the digits are interchanged, the original number is increased by 9. Find the original number. (WAEC)

Reading assignment

Essential Mathematics for SSS2, pages 55-59, exercise 5.2

Name____ Date______________

WEEK EIGHT
TOPIC: SIMULTANEOUS EQUATIONS

CONTENT

- Solving Simultaneous Equations Involving One linear and One quadratic.
- Solving Simultaneous Equations Using Graphical Method

SIMULTANEOUS EQUATIONS INVOLVING ONE LINEAR AND ONE QUADRATIC

One of the equations is in linear form while the other is in quadratic form.

Note: One linear, one quadratic is only possible analytically using substitution method.

Examples:

1. Solve simultaneously for x and y (i.e. the points of their intersection)

$3x + y = 10$ & $2x^2 + y^2 = 19$

Solution

$3x + y = 10$ ----------- eq 1

$2x^2 + y^2 = 19$ --------- eq 2

Make y the subject in eq 1 (linear equation)

$y = 10 – 3x$ ---------- eq 3

Substitute eq 3 into eq 2

$2x^2 + (10-3x)^2 = 19$

$2x^2 + (10 – 3x)(10 – 3x) = 19$

$2x^2 + 100 – 30x – 30x + 9x^2 = 19$

$2x^2 + 9x^2 - 30x – 30x + 100 – 19 = 0$

$11x^2 - 60x + 81 = 0$

$11x^2 - 33x – 27x + 81 = 0$

$11x(x-3) – 27(x – 3) = 0$

$(11x – 27)(x – 3) = 0$

$11x – 27 = 0$ or $x-3 = 0$

$11x = 27$ or $x = 3$

$\therefore x = 27/11$ or 3

Substitute the values of x into eq 3.

When $x = 3$

$y = 10 – 3(x)$

$y = 10 - 3(3)$

$y = 10 – 9 = 1$

When $x = 27/11$

$y = 10 – 3(27/11)$

$y = 10 - 51/11$

$y = \frac{110 - 51}{11}$

$y = 59/11$

$\therefore$ when $x = 3$, $y = 1$

$x = \frac{27}{11}$, $y = \frac{59}{11}$

2. Solve the equations simultaneously $3x + 4y = 11$ & $xy = 2$

solution

$3x + 4y = 11$ -------- eq 1

$xy = 2$ -------- eq 2

Make y the subject in eq 1

$4y = 11 – 3x$

$y = \frac{11 – 3x}{4}$ ………… eq3

substituteeq 3 into eq 2

$x y = 2$

$x\left(\frac{11- 3x}{\ }\right) = 2$

4 2

$x(11-3x) = 2 \times 4$

$11x - 3x^2 = 8$

$-3x^2 + 11x - 8 = 0$

$-3x^2 + 3x + 8x - 8 = 0$

$-3x(x-1) + 8(x-1) = 0$

$(-3x + 8)(x-1) = 0$

$-3x + 8 = 0$ or $x - 1 = 0$

$3x = 8$ or $x = 1$

$x = 8/3$ or 1

Substitute the values of x into eq 3

$y = \frac{11 - 3x}{4}$

when $x = 1$

$y = \frac{11 - 3(1)}{4} = \frac{11-3}{4} = \frac{8}{2}$

$y = 4$

when $x = 8/3$

$y = \frac{11 - 3(8/3)}{4}$

$y = \frac{33 - 24}{12} = \frac{9}{12} = \frac{3}{4}$

$\therefore x = 1, y = 2$

$x = 8/3, y = 3/4.$

Evaluation

Solve for x and y

1. $3x^2 - 4y = -1$
   $2x - y = 1$
2. $4x^2 + 9y^2 = 20$
   $2x - 9y = -2$

MORE EXAMPLES

Solve simultaneously for x and y.

$3x - y = 3$ -------- eq 1

$9x^2 - y^2 = 45$ --------- eq 2

Solution

From eq 2

$(3x)^2 - y^2 = 45$

$(3x-y)(3x+y) = 45$ ---------- eq 3

Substitute eq 1 into eq 3

$3(3x + y) = 45$

$3x + y = 15$ ……………..eq4

Solve eq 1 and eq 4 simultaneously.

$3x - y = 3$ --------- eq 1

$3x + y = 15$ -------- eq 4

eq 1 + eq 4

$6x = 18$

$x = 18/6$

$x = 3$

Substitute $x = 3$ into eq 4.

$3x + y = 15$

$3(3) + y = 15$

$9 + y = 15$

$y = 15 - 9$

$y = 6$

$\therefore x = 3, y = 6$

Evaluation

Name____ Date_______________

Solve for x and y in the following pairs of equations

1. (a) $4x^2 - y^2 = 15$ (b) $3x^2 + 5xy - y^2 = 3$

$2x - y = 5$ $x - y = 4$

WORD PROBLEMS LEADING TO LINEAR AND QUADRATIC EQUATIONS

Example

The product of two numbers is 12. The sum of the larger number and twice the smaller number is 11. Find the two numbers.

Solution

Let x = the larger number

y = the smaller number

Product, x y = 12 …………….eq1

From the last statement,

x + 2y = 11 …………. eq2

From eq2, x = 11 – 2y …………...eq3

Sub. Into eq1

y(11 – 2y) = 12

11y – 2y2 = 12

2y2 -11y + 12 = 0

2y2 – 8y – 3y + 12 = 0

2y(y-4) – 3(y-4) = 0

(2y-3)(y-4) =0

2y-3 =0 or y-4 =0

2y = 3 or y = 4

y= 3/2 or 4

when y = 3/2

x = 11 – 2y

x = 11 – 2(3/2)

x = 11 – 3

x = 8

when y=4

x = 11- 2y

x = 11 – 2(4)

x = 11 – 8

x = 3

Therefore, (8 , 3/2)(3 , 4)

Evaluation

Solve the following simultaneous equation

1. (a) $2^{2x-3y} = 32$, $3^{x-2y} = 81$ (b) $2^{x+2y}=1$, $3^{2x+y} = 27$

2. Bisi's and Fibie's ages add up to 29. Seven years ago Bisi was twice as old as Fibie. Find their present ages.

SOLVING SIMULTANEOUS EQUATIONS USING GRAPHICAL METHOD

Examples

Using the scale 2cm to 1 units on x-axis and 2cm to 2 unit on y-axis, draw the graph of $y = x^2 - x - 1$ and y = 2x – 1 (on the same scale and axis for values of x: $-3 \leq x \leq 4$

Solution

Table of values for $y = x^2 - x - 1$

X	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
x^2	**9**	**4**	**1**	**0**	**1**	**4**	**9**	**16**
-x	**+3**	**+2**	**+1**	**0**	**-1**	**-2**	**-3**	**-4**
-1	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**
Y	**11**	**5**	**1**	**-1**	**-1**	**1**	**5**	**11**

X	-3	-2	- 1	0	1	2	3	4
Y	11	5	1	-1	-1	1	5	11

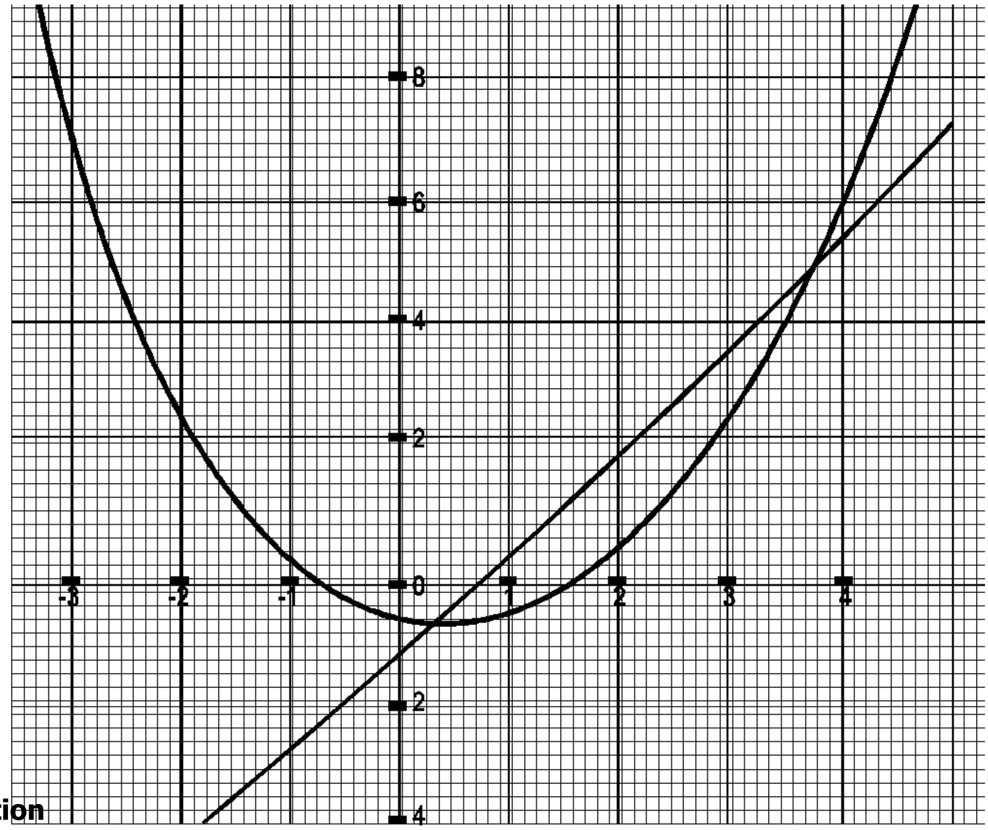
Table of values for y = 2x – 1

X	**-3**	**-2**	**-1**	**0**	**1**	**2**
2x	**-6**	**-4**	**-2**	**0**	**2**	**4**

Name____ Date________________

-1	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**
Y	**-7**	**-5**	**-3**	**-1**	**1**	**3**

X	-3	- 2	- 1	0	1	2	3
Y	-7	-5	- 3	- 1	1	3	5



Evaluation

a. Copy and complete the table below of values for the relation $y = 2x^2 - 3x - 7$

x	-2	-1	0	1	2	3	4	5
y								

b.Using a scale of 2cm to 1 unit on x-axis and 2cm to 5 unit on y-axis, draw the graph of the relation $y = 2x^2-3x-7$ for $-3 \leq x \leq 5$

c.Using the same scale and axis, draw the graph of $y = 2x-1$

d. Use your graph to find the values of x and y.

GENERAL EVALUATION AND REVISION QUESTIONS

1. Solve the simultaneous equation: $3x^2 - 4y = -1$ & $2x - y = 1$
2. Five years ago, a father was 3 times as old as his son, now their combined ages amount to 110 years. How old are they?
3. Solve: $4x^2 - y^2 = 15$ & $2x - y = 5$
4. Seven cups and eight plates cost # 1750. Eight cups and seven plates cost #1700. Calculate the cost of a cup and of a plate.

WEEKEND ASSIGNMENT

Solve each of the following pairs of equations simultaneously,

1. $xy = -12$; $x - y = 7$ a. (3 , -4)(4 ,-3) b. (-2 ,4)(-3, -4) c.(-4, 5)(-2 , 3) d.(3 ,-3)(4,-4)
2. $x - 5y = 5$; $x^2 - 25y^2 = 55$ a (-8, 0)(3/5 , 0) b. (0, 0)(-8 , 3/5) c. (8 , 3/5) d. (0, 8)(0, 3/5)
3. $y = x^2$ and $y = x + 6$ (a).(0,6) (3,9) (b)(-3,0) (2,4) (c) (-2,4) (3,9) (d).(-2, 3), (-3,2)
4. $x - y = -3/2$; $4x^2 + 2xy - y^2 = 11/4$: a. (-1, 1/2)(1, 5/2). b. (3, 2/5) (1, 1/2) c.(3/2 , -1) (4,2) d.(-1 , -1/2)(-1 , 5/2)
5. $m^2 + n^2 = 25$; $2m + n - 5 = 0$: a. (0,5)(4, -3) b.(5,0)(-3,4)c.(4,0)(-3,5) d(-5,3)(0,4)

THEORY

Name____ Date________________

1a. Find the coordinate of the points where the line $2x - y = 5$ meets the curve $3x^2 - xy - 4 = 10$

b. Solve the simultaneous equation: $2^{2x+4y} = 4$, $3^{3x + 5y} - 81 = 0$

2. A woman is q years old while her son is p years old. The sum of their ages is equal to twice the difference of their ages. The product of their ages is 675.

Write down the equations connecting their ages and solve the equations in order to find the ages of the woman and her son. (WAEC)

WEEK NINE
TOPIC: STRAIGHT LINE GRAPHS

CONTENT

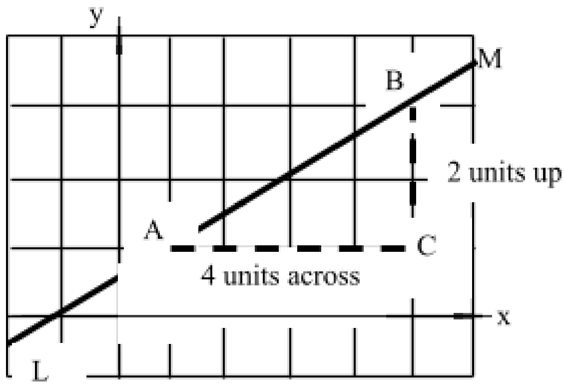
- Gradient of a Straight Line.
- Gradient of a Curve.
- Drawing of Tangents to a Curve.

GRADIENT OF A STRAIGHT LINE

The gradient (or slope) of a straight line is a measure of the steepness of the line.
The gradient of a line may be positive or negative.

Positive gradient (uphill slope)

Consider line LM shown in the diagram below. The line slopes upwardsto the right and it makes an acute angle of θwith the positive x-axis, so tan θ is positive. The gradient of the line can be found by choosing any two convenient points such as A and B on the line. In moving from A to B, x increases (→) and y also incrases (↑).



i.e. increase in x = horizontal distance = AC
increase in y = vertical distance = BC
the gradient of a line is represented by letter m.
the gradient of a line LM is given by:

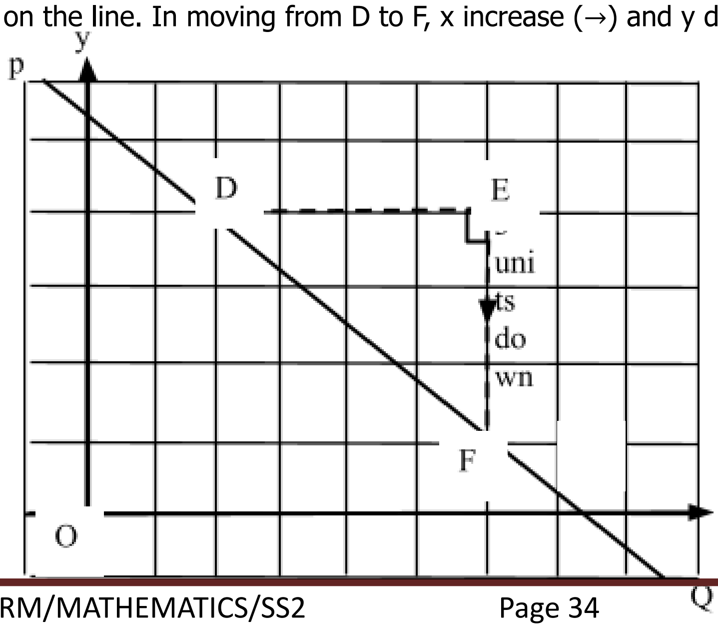
$m = \frac{increase\ in\ y\ from\ A\ to\ B}{increase\ in\ x\ from\ A\ to\ B} = \frac{BC}{AC} = \frac{2}{4} = \frac{1}{2}$

also in ABC, tan $\theta = \frac{BC}{AC} = \frac{2}{4} = \frac{1}{2}$

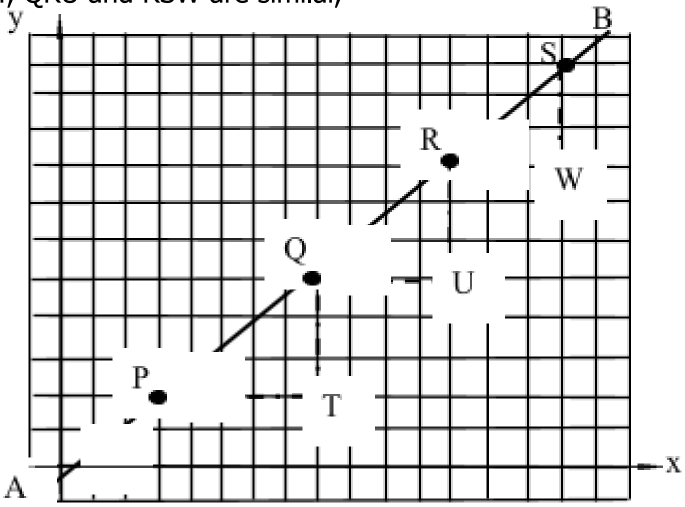
it follows that the gradient of line AB = tan θ. When a line slopes upwards (uphill) to the right, the gradient of the line is positive.

Negative gradient (downhill slope)

In the diagram below, line PQ slopes downwards and it makes an obtuse angle β with positive x-axis, so tan βis negative. Again, to find the gradient of the line, we choose two convenient points such as D and F on the line. In moving from D to F, x increase (→) and y decreases (↓).



i.e. increase in x = horizontal distance = DE and decrease in y = vertical distance = EF.
The gradient, m of line PQ is given by:
m = $\frac{increase\ in\ y\ from\ D\ to\ F}{increase\ in\ x\ from\ D\ to\ F} = \frac{EF}{DE} = \frac{-3}{4} = -\frac{3}{4}$
Also the gradient of line PQ = tanβ
When a line slopes downwards to the right (i.e. downhill) the gradient is negative.
For example, in the diagram below, the slope goes up 3 units for every 4 units across. Since triangles PQT, QRU and RSW are similar,

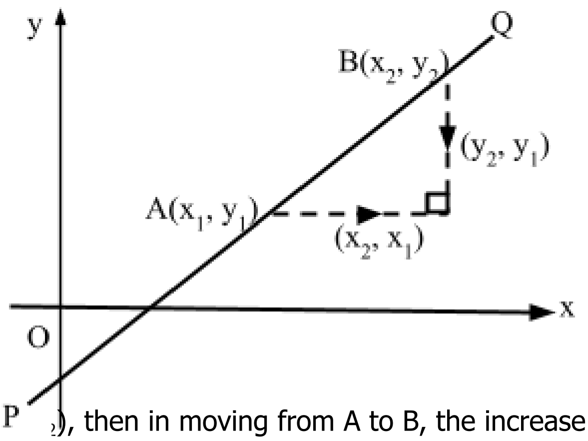


We have: $\frac{QT}{PT} = \frac{RU}{QU} = \frac{SW}{RW} = \frac{3}{4}$
This means the gradient of the line is given by:

$$m = \frac{QT}{PT}\ or\ m = \frac{RU}{QU}\ or\ m = \frac{SW}{RW}$$

Where the letter 'm' represents gradient.

Calculating the Gradient of a Line
The gradient of a straight line can be calculated from any two points on the line.
Let the two points on line PQ be A and B. if the coordinates of point A are (x_1, y_1) and the coordinates
Gradients of lines and curves

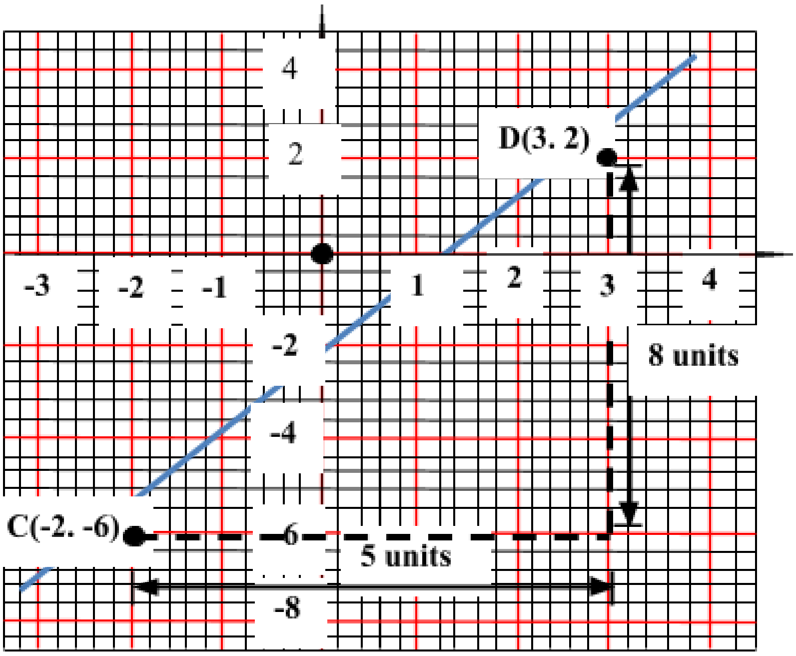


of point B are $_2$), then in moving from A to B, the increase in x (or change in x) is AC and the increase in y (or change in y) is CB, i.e. AC = $x_2 - x_1$ and CB = $y_2 - y_1$,
Thus, the gradient, m of the line PQ is given by:
m = $\frac{increase\ in\ y}{increase\ in\ x} = \frac{difference\ in\ y\ coordinate}{difference\ in\ x\ coordinate}$

$= \frac{CB}{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

Exercise
Calculate the gradient of the line joining the points C(-2, -6) and D(3, 2) and.
Solution
Method 1
Plot the points C(-2, -6) and D(3, 2).

Name____ Date________________



Draw a straight line to pass through the points.

Gradient = $\frac{increase\ in\ y}{increase\ in\ x} = \frac{ED}{CE} = \frac{8}{5}$

Method 2

We can calculate the gradient in the following 2 ways.

a) In moving from C to D

$(x_1, y_1) = (-2, -6)$ and $(x_2, y_2) = (3, 2)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-6)}{3-(-2)} = \frac{2+6}{3+2} = \frac{8}{5}$

b) In moving from D to C

$(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (-2, -6)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-2}{-2-3} = \frac{-8}{-5} = \frac{8}{5}$

Notice that the answer is the same in obht cases, therefore, it does not matter which point we call the first or the second.

Example

Find the gradient of the line joining (-4, 6) and (3, 0)

Solution

Let m = gradient,

$(x_1, y_1) = (-4, 6)$ and $(x_2, y_2) = (3, 0)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-6}{3-(-4)} = \frac{-6}{3+4} = \frac{-6}{7} = -\frac{6}{7}$

Evaluation

Find the gradients of the line joining the following pairs of points.

1. (9,7) , (2,5)
2. (2,5) , (4,5)
3. (2,3) , (6,-5)

Drawing the Graphs of Straight Lines

Example

(a) Draw the graph of 3x + 2y = 8

(b) Find the gradient of the line.

Solution

(a) First make y the subject.

$3x + 2y = 8$

$2y = 8 - 3x$

$y = \frac{8-3x}{2}$

Choose three easy values and then make a table of values as shown below.

When x = 0, $y = \frac{8-0}{2} = 4$

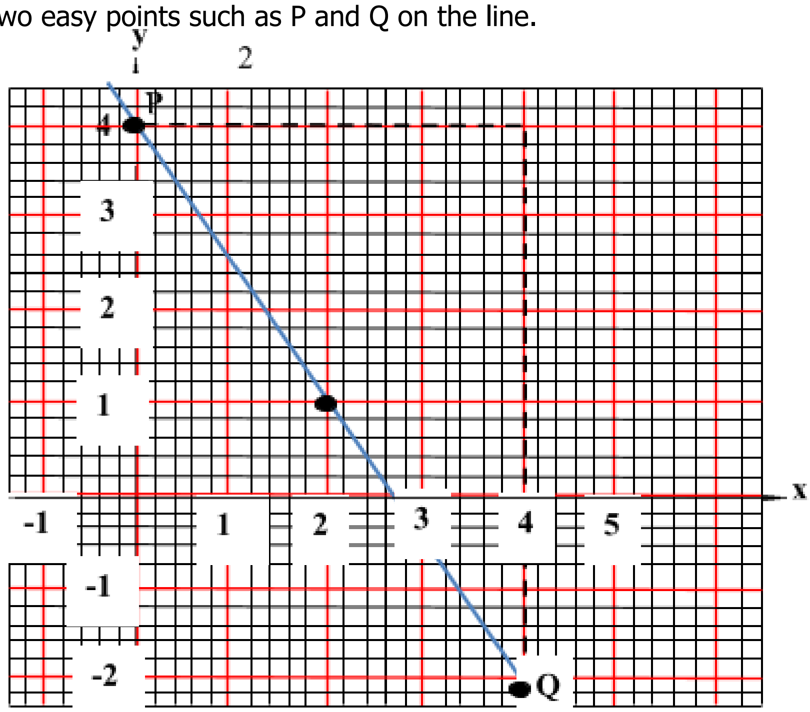
When x = 2, $y = \frac{8-6}{2} = \frac{2}{2} = 1$

When x = 4, $y = \frac{8-12}{2} = \frac{-4}{2} = -2$

x	0	2	4
y	4	1	-2

The graph of 3x + 2y = 8 is shown below.

(b) Choose two easy points such as P and Q on the line.



Gradient of PQ = $\frac{increase\ in\ y}{increase\ in\ x} = \frac{-RQ}{PR}$

$\frac{-6}{4} = -\frac{3}{2}$

Evaluation

Using three convenient points, draw the graph of the following linear equations and then find their gradients.

1. 2x-y-6=0 2.) 5y+4x=20 3.) 3x-2y=9

GRADIENT OF A CURVE

Finding the gradient of a straight line is constant at any point on the line. However, the gradient of a curve changes continuously as we move along the curve. In the diagram below, the gradient at P is not equal to the gradient at S. to find the gradient of a curve, draw a tangent to the curve, draw a tangent to the curve at the point your require to find the gradient. For example, the gradient of curve at point P is the same as the gradient of the tangent PQ. Also the gradient of the curve at S is the same as the gradient of the tangent ST.

The diagram above represents the graph of the function $y = 2x^2 + x - 5$.

The gradients at P and S can be found as follows:

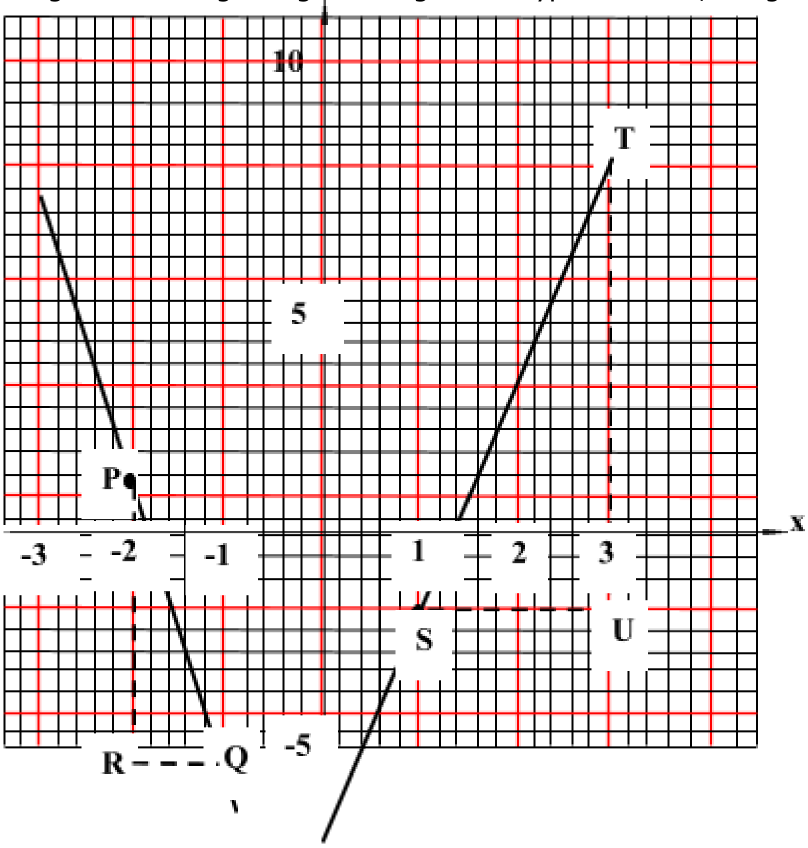
Name____ Date________________

Gradient at P = gradient of tangent PQ. By constructing a suitable right-angled triangle with hypotenuse PQ, the gradient is Gradient = $\frac{PR}{PQ} = \frac{-7}{1} = -7$

Remember that the gradient is negative because the tangent slopes downwards from left to right.

Gradient at S = gradient of tangent ST.

By constructing a suitable right-angled triangle with hypotenuse ST, the gradient is



Gradient = $\frac{TU}{SU} = \frac{10}{2} = 5$

Remember that the gradient is positive because the tangent slopes upwards from left to right.

Note: This method only gives approximate answer. However, the more accurate your graphs are, the more accurate your answers will be.

Evaluation

Draw the graphs of the following functions and use the graphs to find the gradients at indicated points.

1) $y= x^2 - x-2$ at $x= -1$

2) $y= x^2-3x-4=0$ at $x = 4$

GENERAL EVALUATION/ REVISION QUESTIONS

1. A straight line passes through the points (3,k) and (-3,2k). If the gradient of the line is -2/3, find the value of k. What is the equation line?
2. Sketch the following graphs using gradient-intercept method.

a) y= 0.5x - 3 b) y= 5x c) y = x/4 - 3 d) 2y-10 = 2x

3. Find the gradients of the curves at the points indicated.

a) $y= 6x - x^2$ at x= 3 b) $x^2 - 6x + 5$

WEEKEND ASSIGNMENT

1. Find the gradient of the equation of line 2y – 10 = 2x A. 1 B. 2 C. 3 D. 4
2. Find the gradient of the line joining (7,-2) and (-1,2) A. ½ B. – ½ C. 1/3 D. -1/3
3. Find the equation of a straight line passing through (-3,-5) with gradient 2.

A. y =3x-1 B. y=2x-1C. y=2x-1 D. y=3x+1

Given that 3y-6x +15=0, use the information to answer questions 4 and 5.

4. Find the gradient of the line. A. 5 B. -5 C. 2 D. -2
5. Find the intercept of the line. A. 5 B. -5 C. 2 D. -2

Name____ Date_______________

THEORY

1. Draw the graph of y= 2x-3 using convenient points and scale. Hence , find the gradient of the line at any convenient point.

2a) Copy and complete the following table of values for the relation $y= 2x^2 – 7x-3$.

X	-2	1-	0	1	2	3	4	5
Y	19		-3		-9			

b) Using 2cm to 1unit on the x-axis and 2cm to 5units on the y-axis, draw the graph of $y= 2x^2 -7x-3$ for $-2≤x≤5$.

c) From your graph, find the:

i. minimum value of y.

ii. the equation of the line of symmetry.

iii. the gradient of the curve at x=1.

Reading Assignment

New General Mathematics for SSS2, pages 190-192, exercise 16d.