Chapter V Homework

1. Joe has a small factory in which he makes three types of toys - fine, fancy, and super. Joe seeks to determine the amount of each toy he should make to maximize net returns. The scarce resources Joe must allocate are 120 board feet of lumber and 320 hours of labor. From previous experience, Joe has developed the following information:

	Fine	Fancy	Super
Lumber Use (bd. ft.)	1	1.05	0.8
Labor Use (hours)	3	4	2
Net Return ($)	32	40	25

   a) Formulate a linear programming model for Joe's firm.
   b) Write the dual to the problem formulated in part a.
   c) Give an economic interpretation of
      i) The dual variables
      ii) The dual objective function
      iii) The dual constraints

2. Chi has a small factory in which she makes three types of prepared foods - basic, nutritious, and super. Chi seeks to determine the amount of each food she should make to maximize net returns. The scarce resources Chi must allocate are 120 hours of preparation time and 320 hours of labor. From previous experience, Chi has developed the following information

	Basic	Nutritious	Super
Preparation time	1	1.05	0.8
Labor Use (hours)	3	4	2
Net Return ($)	132	140	125

   a) Formulate a linear programming model for Joe's firm.
   b) Write the dual to the problem formulated in part a.
   c) Give an economic interpretation of
      i) The dual variables
      ii) The dual objective function
      iii) The dual constraints

3. Chips Potato Chips makes three lines of potato chips: Regular, Ruffles, and Bar-B-Que. At current prices, the net margin per unit on the three lines exclusive of labor cost is $1.20, $1.70, and $2.00, respectively. Plant capacity limits production to 10,000 units daily. The company has a labor contract which requires that at least 40 employees work 8 hrs/day every day. There are at most 75 employees available. A laborer costs $64 per day. Labor requirements are 0.05, 0.08, and 0.10 hours per unit for the three product lines, respectively.
   a) Formulate an LP problem that will determine the optimal production plan.
   b) Write the dual to the problem formulated in part a).

c) Give an economic interpretation to
   i) the dual variables
   ii) the dual objective function
   iii) the dual constraints

4. The Haul-It-Yourself Company is faced with an oversupply of trailers in some cities and shortages in other cities. The numbers of trailers and cities are given below.

City	Number	Status
Miami	100	Oversupply
Atlanta	50	Oversupply
Chicago	125	Shortage
St. Louis	95	Shortage
Houston	70	Oversupply

The cost of shipping trailers between cities is given below (on a per-trailer basis):

	Miami	Atlanta	Chicago	St. Louis	Houston
Miami	0	20	30	32	10
Atlanta	20	0	25	20	25
Chicago	30	25	0	5	20
St. Louis	32	20	5	0	12
Houston	10	25	20	12	0

a) Formulate a LP problem to determine the least cost method of moving the trailers from the oversupply cities to the shortage cities.
b) Write the dual of the problem formulated in part a.
c) Give an economic interpretation of
   i) one of each major type of the dual variables
   ii) the dual objective
   iii) one of the dual constraints

5. Funny Transport wishes to price the goods it currently sells. Funny has 4 plants that ship to 5 demand regions. The regions overlap, i.e., Plant 1 is in Demand Region 1, Plant 2 is in Demand Region 2, Plant 3 is in Demand Region 3, and Plant 4 is in Demand Region 4. Based on their computerized records, Funny Transport has the following transportation rates:

		Cost per Unit				Number of Units		
		Plant				Region	Units Supplied	Units Demanded
		1	2	3	4			
Demand Region	1	0	10	20	5	1	10	8
	2	10	0	20	4	2	5	4
	3	20	20	0	3	3	18	14
	4	5	4	3	0	4	6	5
	5	25	2	25	3	5		8

Funny Transport wishes to set up minimum prices so that the price differentials in regions are related to the historical transportation cost. Furthermore, assume all goods cost the

same amount to produce in all regions. Funny Transport wants supply and demand prices to be equal in a region.
Set up a LP to price Funny Transport's goods and tell how you would find the prices.

6. Ralph's Transport Conglomerate is trying to devise a pattern of sales, storage, and deliveries over two months. Ralph faces the following data from his suppliers:

Supply Point	Cost/Unit Purchased	Quantity Available	
		Month 1	Month 2
Phoenix	30	1000	3000
Portland	25	2000	3500
Chicago	28	4000	1000

and his demanders

	Minimum Demand	
Demand Point	Month 1	Month 2
Los Angeles	1000	1000
Cleveland	4000	4000
Dallas	500	3000

Items bought in the first month are transported or stored for use in the second month. Thus items transported in month 2 come from either month 2 purchases or storage. Ralph wishes to minimize costs. The relevant transport costs are:

	Transport costs in both months to		
From	Los Angeles	Cleveland	Dallas
Phoenix	20	80	30
Portland	50	100	75
Chicago	70	20	60

Storage costs are $1 per unit stored from month 1 to month 2 at each location.
Formulate a LP problem so that your formulation would tell how much to ship in both months and the amount to store from month to month.

7. Easy Shipment Company wishes to ship goods from four supply locations to two demand locations. The distances between regions and the amount of goods available at each supply point are given below:

Supply	Goods in Inventory	Distance to Demand Region	
		A	B
1	50	20	25
2	30	15	30
3	20	10	15
4	10	17	19
Cost of shipping is $.50 per mile			

Easy Shipment has had its analysts make demand projections and has obtained three

estimates. Analyst One has said that he knows the quantities of goods that will be sold and the quantities are as follows:

Demand Region	Quantity to be Sold
A	55
B	45

Analyst Two has said that the quantities of each good are impossible to estimate but the prices are easy.

Demand Region	Price at which Goods Would be Sold
A	$19
B	$28

Analyst Three has said that price depends on quantity and quantity depends on price. Therefore, he says Region A will pay $25 for the first ten units and $2 less for each five units thereafter and region B will pay $30 for the first 15 units and $5 less for each ten units thereafter.

a) Formulate a linear program for the demand projection of each analyst.
b) How would you expect the answers to be different?
c) How would you attempt to reconcile the three answers?

8. Steve's Deluxe Strawberry Pies, Inc. (known as SDS) buys fresh strawberries from three supply areas, transports the fruit to one of two pie factories, and ships pies to four demand areas. The supply areas and fruit availability are

Location	Annual Availability (tons)
California	50
Florida	25
Texas	45

SDS owns two processing plants. The larger plant is newer and exhibits lower processing costs. SDS prides itself that each pie contains 4 pounds of fresh strawberries. Plant locations, processing costs, and capacities are

Location	Processing Cost per Pie ($)	Annual Capacity (Pies)
Reno, Nevada	1.00	20,000
Atlanta	0.90	40,000

The four major markets supplied by SDS and expected sales requirements are

Location	Requirements (Pies)
New York	15,000
New Orleans	12,000
Los Angeles	20,000
Chicago	13,000

Transport costs per ton of fresh strawberries are

From	To: Reno	Atlanta
California	5	25
Florida	30	10

Texas	15	12

Transport costs per case of pies (1 case = 10 pies) are

From	To: New York	New Orleans	Los Angeles	Chicago
Reno	0.05	0.08	0.02	0.04
Atlanta	0.04	0.02	0.09	0.03

a) Formulate a LP model to determine the least cost movement of fruit and pies.
b) Briefly discuss the information contained in the dual.

9. Delicious Ice Cream Company (DICC) wishes to develop a formula for its Delightful Chocolate flavor. In doing so, DICC realizes that the prices of ingredients it uses in ice cream are variable, so it wishes to develop a least-cost formula given a price situation. DICC also knows its ice cream must exhibit certain characteristics, including butterfat content, solids content, sweetness, flavor and test weight. The raw ingredients that can be used are butter, whey, dry whey, nonfat dry milk, whole milk, cream, sugar, chocolate, and skim milk.
The ice cream formulated must exhibit the following characteristics:

Item	Minimum	Maximum
butterfat	20%	25%
solids	20%	35%
sweetness	2 units	3 units
test weight	5 lbs.	6 lbs.
volume	1 gallon	1 gallon
flavor	0.1 units	0.3 units

The ingredients and their contents are as follows:

Ingredients	cost/unit ($)	butterfat (%)	solids (%)	sweetness (units)	flavor (units)	test weight (lbs.)	volume (gallons)
butter	5.00	60	50	0.01	0	6	1
whey	0.05	2	2	0	0	3	1
dry whey	2.00	0.5	80	0	0	10	1
nonfat dry	3.00	0.5	80	0	0	10	1
whole milk	1.00	4	10	0.005	0	3.5	1
cream	2.00	40	12	0	0	4	1
sugar	1.50	0	80	20	0	15	1
skim milk	0.90	0	8	0	0	3.3	1
chocolate	1.20	0	1	0.20	6	0.1	1

Set up a model to minimize the cost while staying within the ingredient limit.

10. Fred's Feed Mixing Farm feeds animals with several ingredients. Fred owns 1,000 animals which he wishes to feed. He has ingredients of corn, soybeans, and wheat along with protein supplement, energy supplement and filler. Fred may also purchase several premixed feeds. The ingredient breakdown is

	Corn	Soy beans	Wheat	Protein Supplement	Energy Supplement	Filler	Mix 1	Mix 2	Mix 3
Cost/lb.	0.05	0.07	0.08	0.45	.032	0.01	0.20	0.21	0.30
% Protein	5	25	20	80	0	2	10	13	15
% TDN	25	25	25	0	0	5	20	19	17
% Energy	25	5	20	5	80	3	10	10	5
% Fats	10	20	10	0	0	10	10	15	15
% Bulk	35	25	25	20	20	80	50	43	48

The animals currently sell for $125 per cwt. live weight. Fred has historically fed animals 3 diets.

	Diets		
	High Protein	Balanced	High Fat-Energy
Max Protein %	20	18	17
Min Protein %	15	14	13
Max TDN %	25	25	30
Min TDN %	10	15	17
Max Energy %	17	19	30
Min Energy %	12	15	19
Max Fat %	17	15	20
Min Fat %	5	7	12
Max Bulk %	45	50	48
Avg. daily rate of gain by animal	2.2 lbs./day	2.1 lbs./day	2.2 lbs./day
Avg. weight of ration to get daily gain	10 lbs./day	11 lbs./day	11 lbs./day

Formulate a profit maximization LP problem which tells how to feed the animals.

11. Fine Food Specialties, Inc. purchases 3 raw ingredients and combines them to produce 2 products. The two products must meet certain specifications regarding fat content, protein, and fiber. The relevant data are

	Supply		Composition		
Ingredient	Cost ($/cwt)	Availability (cwt)	Fat %	Protein %	Fiber %
INGR1	4.50	18,000	10	25	18
INGR2	5.00	20,000	9	24	18
INGR3	3.75	17,000	11	29	17

	Formulation			Demand	
Product	Max Fat %	Min Protein %	Max Fiber	Price ($/cwt)	Sales Potential (cwt)
PROD1	10.0	24.8	18	7.15	Max 25,000
PROD2	10.1	25.2	17.8	7.5	Min 22.000 Max 32.000

a) Formulate a linear programming model that will determine the profit-maximizing production level.
b) Solve the problem using a computerized linear programming code.
c) Interpret the optimal primal and dual solution.
d) Interpret the optimal types of duality information you would get from part a.

12. Donald the tree miller is developing a plan on how to deal with today's delivery of logs. Donald wishes to figure out the way that logs can be processed to make maximum profits. Donald has several processes that can be used, the result of which is 2x4's, plywood, milling residue, and 1X2's. Each process uses energy, logs, saw time, bundling and holding capacity. The processes resource usages and yields are

Yield in thousand board feet of:	**Process 1**	**Process 2**	**Process 3**
2x4's	0.5	-	0.6
1x2's	0.3	-	0.25
plywood	-	0.6	-
mill reside	0.1	0.2	0.07
Use inputs of:	**Process 1**	**Process 2**	**Process 3**
Energy	4kwh	3.9kwh	3.5kwh
logs	1	1	1
Saw time	4 min	12 min	8 min
Bundling	3 min	1 min	6 min

In addition, each of the products produced used the following amounts of cost above the process cost and holding capacity.

	Cost per 100 board feet	**Holding Capacity per 100 board feet**
2x4	$0.5	2.5 cu. Ft.
1x2	$0.2	2.1 cu. Ft.
plywood	$0.6	6 cu ft
mill residue	$0.01	0.1 cu ft

The sale price for 2x4's is $8.00 for 4 bd ft, 1x2's $3 for 1 bd ft, Plywood $22 for 32 bd ft, and mill residue $0.095 per bd ft.
The Firm has unlimited Energy at $60 MW/hour, 500 logs, 40 hours of Saw time, 40 hours of bundling capacity, and 10,000 bd. ft. of holding capacity (although more can be rented at 0.10/unit).
Formulate a profit maximizing LP.

13. Darius of Darius' Delicious Dairy is developing a direct action directive on how to deal with incoming daily delivery of milk. Darius of Darius' Delicious Dairy wishes to figure out the way that milk can be processed so as to make maximum profits. Darius has several processes that can be used, the result of which is cream, skim milk, 2% milk, and whole milk. Each process uses whole milk energy packaging and holding capacity. The processes are resource usages and:

Yield of:	**Process 1**	**Process 2**	**Process 3**
Cream	.03	0.02	0.0
Skim Milk	.97	-	-
2% Milk	-	.98	-

Homog Milk	-	-	1.00
Use inputs of:	**Process 1**	**Process 2**	**Process 3**
Energy	4	3.9	3.5
Whole Milk	1 gal.	1 gal.	1 gal.

In addition, each of the products produced used the following amounts of packaging time and holding capacity.

	Time	**Holding Capacity**
Cream	5 seconds	2.5 cu. ft.
All Milks	10 seconds	2 cu. ft.

The sale price for cream is $3.00, skim milk $1.85, 2% milk $1.90, and homog milk $1.95. The Firm has 12 hours of packaging time, 10,000 cu. ft. of holding capacity (although more can be rented at 0.10/unit). Energy costs 0.10/unit. Whole milk is available for $1.00/gallon up to 5,000 gallons and for $1.25 for as much more as required.
Formulate a profit maximizing LP.

14. Solve one of the above models in GAMS

15. Set up your own version of one of the problem structures in Chapter 5
Do the following:
   a) formulate a Microsoft Word version of the problem
   b) set up and solve in GAMS.
   c) explain the answer.

16. Take one of the existing models in the text.
   a) Type it in and solve it
   b) Modify the data and solve again
   c) Change the context (i.e. if it makes chairs convert it to make fertilizer)
      i) change all the item names of the variables, parameters, sets and equations
      ii) alter the set element names to another context with extra elements, and solve
      iii) Solve with twice as many items in each set and explain what you had to change