# Exploring Middle School Students' Mathematical Modeling Competencies Through LLM-Powered Virtual Student Agents

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## Ⅰ. Introduction

Mathematical modeling (MM) is a key STEM competency that enables students to apply mathematics to real-world problems through inquiry, reasoning, and collaboration. Middle school is a critical stage for developing modeling and mathematical discourse skills, yet access to high-quality modeling experiences remains inequitable, especially in under-resourced settings. This study investigates how middle school students engaged in collaborative mathematical modeling within MathVC, an online platform powered by large language models (LLMs) developed by the research team.

## Ⅱ. Theoretical Background

As educators continue to predict the future skills that students need to prepare for an ever-changing world, one indispensable skill is collaborative problem solving (CPS). As workplaces evolve, employees are frequently required to work in teams—both in-person and virtually across geographic boundaries—to tackle complex, non-routine challenges. This shift reflects a broader recognition that effective problem solving often demands diverse perspectives, shared expertise, and coordinated action. Collaborative problem-solving can be defined as the "capacity to effectively engage in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution, and pooling their knowledge, skills, and efforts to reach that solution", according to the Programme for International Student Assessment (PISA, 2015, p. 72). To build on this definition, Hesse et. al.(2015), offers a framework for teachable collaborative problem solving (CPS) skills, developed by conceptualizing CPS as a joint activity where individuals work together.. It distinguishes between two broad categories of skills: social skills, which include participation, perspective taking, and social regulation and cognitive skills, which encompass task regulation, flexibility, execution and monitoring, and learning. These skills are not only essential for effective collaboration but are also teachable, observable, and accessible in educational settings. In our work, we focus on the cognitive domain of mathematical modeling which is a form of problem solving but mirrors more of the real world with ill-structured situations that students have to mathematize. The GAIMME Report (Guidelines for Assessment and Instruction in Mathematical Modeling Education, Garfunkel, S., & Montgomery, 2019), defines mathematical modeling as: a process that uses mathematics to represent, analyze, make predictions, or otherwise provide insight into real-world phenomena. GAIMME Modeling Process (6 Components): 1) Define the Problem – Understand the context and formulate a clear question; 2) Formulate a Model – Identify variables, relationships, and assumptions; 3) Compute – Use mathematics to derive results; 4) Interpret Results – Translate mathematical outcomes back into the real-world context; 5) Validate – Check the model's accuracy and limitations; 6) Report and Refine – Communicate findings and revise the model as needed Though the GAIMME report emphasizes the importance of mathematical modeling, teaching mathematical modeling is an ambitious practice for teachers as students' solution paths are varied and open-ended and challenging for students who may have not engaged in inquiry-based learning. Providing opportunities for students to engage in this practice with virtual peers allows for a low-risk, high-yield learning experience.

## Ⅲ. Methods

This study used a design-based implementation research approach (Fishman et al., 2004) to explore how middle school students engaged in collaborative mathematical modeling within MATHVC, a virtual classroom powered by LLMs. More specifically, the **MathVC platform** is designed as a multi-persona, LLM-simulated virtual classroom that enables middle school students to engage in collaborative mathematical problem solving with AI-generated peers, Alex and Blake. It integrates a **meta-planning module** that orchestrates dialogue stages (e.g., sense-making, planning, execution, validation) and a **persona simulation module** that models diverse student thinking, including common misconceptions, to foster authentic peer-like interactions. Through structured dialogue acts and evolving persona schemas, MathVC creates a realistic, scaffolded environment that supports engagement, critical reasoning, and confidence-building in mathematics.

We used the social skills and the cognitive skills of the modeling process to analyze the collaborative mathematical modeling processes of middle school students with computer AI student agents. Our research questions were: 1) What modeling competencies and social skills are exhibited by middle school students in mathematical modeling tasks with AI peers? and (2) What types of student-AI peers' interactions support the middle school students' mathematical reasoning and model refinement?

Five students participated in lab study sessions where they solved a real-world problem comparing cell phone plans. Dialogue transcripts captured student input, agent responses, and group interactions during think-aloud interviews and collaborative modeling.

For data analysis, we applied the modified Integrated Collaborative Problem Solving Rubric (Hesse et al., 2015), assessing social skills (Participation, Perspective Taking, Social Regulation) and cognitive skills of mathematical modeling(Garfunkel & Montgomery, 2019), such as Defining the Problem, Formulating a Model by identifying important variables, Computing, Interpreting, Validating, and Refining the model. Each student's contributions were evaluated using the rubric (Miles & Huberman, 1994). Dialogue was coded to examine how students identified key components on the rubric (RQ1) and how AI agents supported reasoning (RQ2). Agent responses were categorized as affirmations, procedural guidance, or conceptual corrections. We mapped dialogue sequences to visualize agent influence on student reasoning (Mercer, 2000). Student profiles were compiled to identify patterns in modeling competency. We analyzed misconceptions—such as miscalculations and formula errors—and how AI agents (i.e., Alex and Blake) helped correct them, highlighting both challenges and growth in collaborative modeling.

## IV. Results and Discussions

This study examined the collaborative mathematical modeling competencies of five middle school students—Avi, Lee, Sara, Maurio, and Jan—interacting with virtual student agents via MathVC.

**RQ1: Identifying and Applying Key Variables in Modeling and Exhibiting Social Skills**

Three students—Lee, Sara, and Avi—explicitly identified the number of text messages as a critical variable influencing plan selection. For example, Lee stated, *"It depends on how many text messages the customer does,"* while Sara noted, *"If a customer does not text a lot, it would be best to choose Plan A."* Avi added nuance by incorporating budget constraints into her reasoning. These students demonstrated a clear understanding of the problem context and formulated models that reflected relevant variables and relationships. Students also interpreted the final solution in contextually appropriate ways. Lee provided a segmented analysis based on break-even points, while Sara and Avi categorized plan suitability by texting volume. Maurio revised his initial assumption after AI peer input, and Jan affirmed group conclusions based on a specific scenario. These interpretations reflect meaningful connections between mathematical outcomes and real-world decision-making.

All five students demonstrated key social skills outlined in the Integrated Collaborative Problem Solving Rubric. Lee consistently initiated modeling steps and sustained engagement, stating, *"Let's call the number of texts x, the total cost y,"* and coordinating transitions with, "*We have Plan A down, so let's do Plan B."* Avi contributed ideas and responded to peers, noting, *"But it also has 10 cent PER text messages so it will be the most expensive if u think about it,"* and prompting next steps with, *"Okay so now we can compare Plan B now."* Sara initiated modeling and prompted further analysis, saying, *"Yes, should we also solve for the break-even point between B and C?"* Maurio engaged with realistic scenarios and sought clarification, asking, *"Ok Blake, so what would the total monthly cost be?"* Jan participated by asking questions and affirming group conclusions, such as, "Which will result?" and

*"Yeah, Plan A is the cheapest."* These excerpts illustrate varying levels of participation, perspective taking, and social regulation among the students, with most demonstrating advanced collaborative behaviors and a few requiring moderate support.

**RQ2: Peer Support and Model Refinement**

In the collaborative problem-solving dialogues, the notion that the number of text messages determines the cost of each plan is first introduced by different students across five distinct groups. Lee, Avi and Sara identify the important variable of the number of texts being important in determining the best plan and were validated by the two AI peers. In both dialogues with Maurio and Jan, Blake, the AI student was the one to suggest calculating the total monthly cost based on the number of texts about the middle of the process, framing it as the key factor in comparing plans. This contribution then allowed Maurio to build on their group thinking by stating, *"Let's say the customer sends about 6 text messages a day, which plan would be the best?"* Jan heavily relied on Alex and Blake than all other students asking often, "*How do we do that?"* and later *"Which will result to?"* requesting the AI peers for ideas and for step-by-step calculations.

Three students—Sara, Maurio, and Jan—exhibited mathematical misconceptions during the task. Sara underestimated the break-even point between Plan A and Plan B, which was corrected by Alex, "*Actually, Sara, the number of texts where Plan A and Plan B cost the same is 602.5 texts, not 10.*" Maurio incorrectly assumed Plan C was unlimited, prompting Alex and Blake to clarify plan features. Jan required support in applying formulas, which Alex and Blake provided through step-by-step guidance. Alex and Blake played complementary roles in supporting their peers. Alex offered gentle corrections and affirmations, helping students refine their reasoning. Blake provided clarity and often reinforced mathematical procedures. Their responses were consistently supportive, though speech mannerism was distinct—Alex was more conversational, while Blake was more procedural. These interactions contributed to students' ability to revise models, validate reasoning, and communicate findings effectively.

## V. Conclusion

By using the Integrated Collaborative Problem Solving Rubric as an analysis tool, we were better able to differentiate among the five students along the social dimensions and the modeling competencies. They actively contributed to the modeling process, showing proficiency in defining the problem, formulating models, computing break-even points, interpreting results, validating reasoning, and refining conclusions. Socially, they sustained participation, demonstrated perspective taking, and regulated group dynamics effectively. These findings suggest that LLM-powered virtual agents can foster deeper mathematical reasoning and enhance student engagement in collaborative modeling tasks. The support provided by AI agents enabled students to navigate ill-structured problems, apply mathematical concepts, and refine their thinking through dialogue.

## References


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