ALPHAGEOMETRY SYNTAX USED IN REPRESENTING GEOMETRIC ELEMENTS

1. Points: Represented as lowercase letters (e.g., a, b, c).

2. Segments: Represented as `segment a b`, `segment b c`, etc.

3. Lines: Represented as `on_line a1 b c`.

4. Circles: Represented as `circle o a b c`.

5. Triangles: Represented as `triangle a b c`.

6. Midpoints: Represented as `midpoint o b c`.

7. Foot of Altitude: Represented as `foot h1 a b c`.

8. Reflect: Represented as `reflect pa p b c`.

9. Incenter: Represented as `incenter i a b c`.

10. Parallel Line: Represented as `on_bline e o a`, `on_bline e o b`.

11. Equidistant: Represented as `eqdistance d a b c`.

12. Equiangle: Represented as `on_line a1 b c`, `on_line a1 c ? eqangle e c e j e j e f`.

13. Parallel: Represented as `circle o a b c ? cong o p o q`.

14. Isosceles Triangle(AB=AC): Represented as `iso_triangle a b c`.

15. Circle on a Line: Represented as `circle o1 b n w`.

16. Angle Bisector: Represented as `angle_bisector r b a c`, `angle_bisector r m o n`.

17. Cyclic Quadrilateral: Represented as `circle o1 a1 b1 c1`, `on_circle x o a`, `on_circle x o1 a1`.

18. Orthocenter: Represented as `orthocenter h a b c`.

19. On Diameter: Represented as `on_dia q a h`, `on_circle q o a`.

20. Circle with Radius on Line: Represented as `circle o1 p c e`, `circle o2 p b f`.

21. Collinearity: Represented as `on_line x f k`, `on_line x l g ? coll x o a`.

22. Angle Bisector and Angle Mirror: Represented as `triangle a b z`, `angle_bisector f b a z, on_bline f a b`.

23. Mirror and Circle: Represented as `mirror t r s`, `on_bline o r s`, `on_circle j o s`, `circle o1 j s t`.

24. Parallel: Represented as `on_line a e`, `on_bline d a c`, `angle_mirror e c a d, on_bline e a d`.

25. Cyclic Quadrilateral: Represented as `circle o a b c`, `on_circle d o a`, `on_aline p b c a b d`, `on_aline p d c a d b`.

26. Circle with Parallel Lines: Represented as `triangle a b c`, `circle o a b c`, `on_line d a b`, `on_line e a c, on_circle e a d`, `on_bline f b d, on_circle f o a`.

27. Circle with Parallel Lines: Represented as `triangle a b c`, `circle o a b c`, `on_line d a b`, `on_line e a c, on_circle e a d`, `on_bline f b d, on_circle f o a`.

28. Parallel: Represented as `triangle a b z`, `angle_bisector f b a z, on_bline f a b`, `on_tline c b f b, on_line c a f`.

29. Circle with Orthocenter and Midpoints: Represented as `triangle a b c`, `circle o a b c`, `angle_bisector r b a c, angle_bisector r m o n`, `circle o1 b m r`, `circle o2 c n r`.

30. Circle with Equidistant Points: Represented as `triangle a b c`, `circle o a b c`, `on_line d b c`, `on_line e b c, on_circle e a d`, `on_circle f o a, on_circle f a d`.

ALPHAGEOMETRY TRANSLATIONS OF IMO PROBLEMS INTO ALPHAGEOMETRY CODE

IMO 2000 Problem 1:
English Statement:
In triangle ABC, the bisector of angle A meets side BC at D. Let G1 be the centroid of triangle ABD, and G2 the centroid of triangle ACD. Prove that G1G2 is parallel to BC.

AlphaGeometry Syntax:
a b c = triangle a b c; d = on_line d a b; g1 = centroid g1 a b d; g2 = centroid g2 a c d;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `on_line d a b`: Defines point D as the intersection of the bisector of angle A with side BC.
- `centroid g1 a b d`: Computes the centroid G1 of triangle ABD.
- `centroid g2 a c d`: Computes the centroid G2 of triangle ACD.

IMO 2000 Problem 6:
English Statement:
Let ABC be a triangle, H its orthocenter, and T1, T2, T3 the feet of the altitudes from A, B, C, respectively. Let H1, H2, H3 be the reflections of H in T1, T2, T3, respectively. Prove that the circumcircles of triangles T1H1T2, T2H2T3, T3H3T1 have a common point.

AlphaGeometry Syntax:
a b c = triangle a b c; h = orthocenter h a b c; t1 t2 t3 = foot t1 a b c;
h1 = reflect h1 h t1 t2; h2 = reflect h2 h t2 t3; h3 = reflect h3 h t3 t1;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `orthocenter h a b c`: Finds the orthocenter H of triangle ABC.
- `foot t1 a b c`: Computes the feet of altitudes T1, T2, T3.
- `reflect h1 h t1 t2; h2 = reflect h2 h t2 t3; h3 = reflect h3 h t3 t1;`: Reflects H to obtain H1, H2, and H3.

IMO 2002 Problem 2a:
English Statement:
Given a circle, a point A inside it, and a point B outside it, construct a circle tangent to the given circle at C and passing through B.

AlphaGeometry Syntax:
b c = segment b c; o = midpoint o b c; a = on_circle a o b;
d = on_circle d o b, on_bline d a b; e = on_bline e o a, on_circle e o b;
f = on_bline f o a, on_circle f o b; j = on_pline j o a d, on_line j a c ? eqangle e c e j e j e f;

Explanation:
- `segment b c`: Defines the segment BC.
- `midpoint o b c`: Finds the midpoint O of BC.
- `on_circle a o b`: Places point A on the circle with center O.
- `on_circle d o b, on_bline d a b`: Constructs the circle with center D on line AB.
- `on_bline e o a, on_circle e o b`: Constructs the circle with center E tangent to the given circle at C and passing through B.
- `on_bline f o a, on_circle f o b`: Constructs another circle with center F on line AO.

IMO 2002 Problem 2b:
English Statement:
Given a circle, a point A inside it, and a point B outside it, construct a circle tangent to the given circle at C and passing through B.

AlphaGeometry Syntax:
b c = segment b c; o = midpoint o b c; a = on_circle a o b;
d = on_circle d o b, on_bline d a b; e = on_bline e o a, on_circle e o b;
f = on_bline f o a, on_circle f o b; j = on_pline j o a d, on_line j a c ? eqangle c e c j c j c f;

Explanation:
- The syntax is the same as for Problem 2a, as the problems are identical.

IMO 2003 Problem 4:
English Statement:
In triangle ABC, let O be the circumcenter. Point B1 lies on the circumcircle such that B1C is parallel to BC. Similarly, points C1 and A1 are defined. Prove that the circumcenters of triangles A1BC, B1CA, C1AB, and ABC form a parallelogram.

AlphaGeometry Syntax:

a b c = triangle a b c; o = circle o a b c; b1 = on_circle b1 o a, on_bline b1 c a;
d1 = on_circle d1 o a, on_bline d1 c a; x = on_line x b b1, on_line x a c;
d = on_line d d1 x, on_circle d o a; p = foot p d b c; q = foot q d c a; r = foot r d a b ? cong p q q r;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `circle o a b c`: Defines the circumcircle with center O.
- `on_circle b1 o a, on_bline b1 c a`: Constructs point B1 on the circumcircle with B1C parallel to BC.
- `on_circle d1 o a, on_bline d1 c a`: Constructs point D1 on the circumcircle with D1C parallel to BC (similar construction for A1).
- `on_line x b b1, on_line x a c`: Construct the intersection point X.
- `on_line d d1 x, on_circle d o a`: Constructs the circumcenters of triangles A1BC, B1CA, C1AB, and ABC.
- `cong p q q r`: Shows that the circumcenters form a parallelogram.

IMO 2004 Problem 1:
English Statement:
Let ABC be a triangle, O its circumcenter, M the midpoint of BC, and N the midpoint of AC. Angle bisectors of angles A and C meet the perpendicular bisectors of sides BC and AB at P and Q, respectively. Prove that P, Q, and O are collinear.

AlphaGeometry Syntax:
a b c = triangle a b c; o = midpoint o b c; m = on_circle m o b, on_line m a b;
n = on_circle n o b, on_line n a c; r = angle_bisector r b a c, angle_bisector r m o n;
o1 = circle o1 b m r; o2 = circle o2 c n r;
p = on_circle p o1 r, on_circle p o2 r ? coll p b c;

Explanation:
- The syntax captures the construction of points and the collinearity relationship.

IMO 2004 Problem 5:
English Statement:
Let ABC be a triangle with circumcenter O, and let D be a point on side BC. Points E and F lie on sides AC and AB, respectively, such that DE and DF are tangent to the circumcircle of triangle ABC. Prove that AE, BF, and the perpendicular bisector of segment BC are concurrent.

AlphaGeometry Syntax:
a b c = triangle a b c; o = circle o a b c; d = on_circle d o a;
p = on_aline p b c a b d, on_aline p d c a d b ? cong a p c p;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `circle o a b c`: Defines the circumcircle with center O.
- `on_circle d o a`: Places point D on the circumcircle.
- `on_aline p b c a b d, on_aline p d c a d b ? cong a p c p`: Constructs points P on line BC and Q on line AC such that AP is equal to AQ, showing that they are the perpendiculars.

IMO 2005 Problem 5:
English Statement:
Let ABC be a triangle, and let D be a point on side BC. Let E and F be points on sides AC and AB, respectively, such that DE and DF are parallel to the bisector of angle BAC. Prove that the circumcircles of triangles AEF and ABC are tangent.

AlphaGeometry Syntax:
a b c = triangle a b c; d = eqdistance d a b c; e = on_line e b c;
f = on_line f a d, eqdistance f d e b; p = on_line p a c, on_line p b d;
q = on_line q e f, on_line q b d; r = on_line r e f, on_line r a c;
o1 = circle o1 a p d; o2 = circle o2 b p c;
m = on_circle m o1 p, on_circle m o2 p ? cyclic p q r m;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `eqdistance d a b c`: Ensures that DE and DF are parallel to the bisector of angle BAC.
- `on_line e b c`: Places point E on side AC.
- `on_line f a d, eqdistance f d e b`: Places point F on side AB such that DE and DF are parallel to the bisector of angle BAC.
- `on_line p a c, on_line p b d`: Constructs points P on line AC and Q on line BC.
- `on_line q e f, on_line q b d`: Constructs points Q on line DE and R on line DF.
- `on_line r e f, on_line r a c`: Constructs points R on line DE and P on line AC.
- `on_circle o1 a p d; o2 = circle o2 b p c`: Constructs the circumcircles of triangles AEF and ABC.
- `on_circle m o1 p, on_circle m o2 p ? cyclic p q r m`: Demonstrates that the circumcircles are tangent.

IMO 2007 Problem 4:
English Statement:
Let ABC be a triangle, and let O be its circumcenter. The points P and Q lie on side BC such that OP = OQ. The circumcircles of triangles APO and A1QO intersect at O and another point R. Prove that AR is perpendicular to PQ.

AlphaGeometry Syntax:
a b c = triangle a b c; o = circle o a b c;
r = on_circle r o a, on_bline r a b; l = midpoint l c a; k = midpoint k c b;
p = on_line p o k, on_line p c r; q = on_line q o l, on_line q c r;
l1 = foot l1 l c r; k1 = foot k1 k c r ? eqratio k k1 l l1 r q r p;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `circle o a b c`: Defines the circumcircle with center O.
- `on_circle r o a, on_bline r a b`: Constructs point R on the circumcircle.
- `midpoint l c a; midpoint k c b;`: Finds the midpoints L and K of sides CA and CB.
- `on_line p o k, on_line p c r; on_line q o l, on_line q c r;`: Constructs points P and Q on side BC.
- `foot l1 l c r; foot k1 k c r ? eqratio k k1 l l1 r q r p`: Demonstrates that AR is perpendicular to PQ.

IMO 2008 Problem 1a:
English Statement:
Let ABC be a triangle with orthocenter H, and let D, E, and F be the feet of the altitudes from A, B, and C, respectively. The reflections of H in lines BC, AC, and AB are denoted by A1, B1, and C1, respectively. Prove that A1, B1, and C1 are collinear.

AlphaGeometry Syntax:
a b c = triangle a b c; h = orthocenter h a b c;
d = midpoint d b c; e = midpoint e a c; f = midpoint f a b;
a1 = on_circle a1 d h, on_line a1 b c; a2 = on_circle a2 d h, on_line a2 b c;
b1 = on_circle b1 e h, on_line b1 c a; b2 = on_circle b2 e h, on_line b2 c a;
c1 = on_circle c1 f h, on_line c1 a b; c2 = on_circle c2 f h, on_line c2 a b ? cyclic c1 c2 b1 b2;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `orthocenter h a b c`: Computes the orthocenter H.

- `midpoint d b c; midpoint e a c; midpoint f a b;`: Finds the midpoints D, E, and F of sides BC, AC, and AB.
- `on_circle a1 d h, on_line a1 b c; on_circle a2 d h, on_line a2 b c;`: Reflects H to obtain A1 and A2.
- `on_circle b1 e h, on_line b1 c a; on_circle b2 e h, on_line b2 c a;`: Reflects H to obtain B1 and B2.
- `on_circle c1 f h, on_line c1 a b; on_circle c2 f h, on_line c2 a b ? cyclic c1 c2 b1 b2;`: Reflects H to obtain C1 and C2.

IMO 2008 Problem 1b:
English Statement:
Let ABC be a triangle with orthocenter H, and let D, E, and F be the feet of the altitudes from A, B, and C, respectively. The reflections of H in lines BC, AC, and AB are denoted by A1, B1, and C1, respectively. Prove that A1, B1, and C1 are collinear.

AlphaGeometry Syntax:
a b c = triangle a b c; h = orthocenter h a b c;
d = midpoint d b c; e = midpoint e a c; f = midpoint f a b;
a1 = on_circle a1 d h, on_line a1 b c; a2 = on_circle a2 d h, on_line a2 b c;
b1 = on_circle b1 e h, on_line b1 c a; b2 = on_circle b2 e h, on_line b2 c a;
c1 = on_circle c1 f h, on_line c1 a b; c2 = on_circle c2 f h, on_line c2 a b ? cyclic c1 c2 b1 a1;

Explanation:
- The syntax is the same as for Problem 1a, as the problems are identical.

IMO 2008 Problem 6:
English Statement:
Let XYZ be a triangle, and let O be its circumcenter. Points A1, B1, C1 lie on sides YZ, XZ, XY, respectively, such that OA1, OB1, OC1 are perpendicular to YZ, XZ, XY, respectively. The circumcircles of triangles A1BC, AB1C, ABC1 intersect again at P, Q, R, respectively. Prove that P, Q, R are collinear.

AlphaGeometry Syntax:
x@4.96_-0.13 y@-1.0068968328888160_-1.2534881080682770
z@-2.8402847238575120_-4.9117762734006830 = triangle x y z; o = circle o x y z;
w@6.9090049230038776_-1.3884003936987552 = on_circle w o x;
a = on_tline a z o z, on_tline a x o x; b = on_tline b z o z, on_tline b w o w;
c = on_tline c y o y, on_tline c w o w; d = on_tline d x o x, on_tline d y o y;

i1 = incenter i1 a b c; i2 = incenter i2 a c d;
f1 = foot f1 i1 a c; f2 = foot f2 i2 a c;
q t p s = cc_tangent q t p s i1 f1 i2 f2; k = on_line k q t, on_line k p s ? cong o k o x;

Explanation:
- `triangle x y z`: Represents triangle XYZ.
- `circle o x y z`: Defines the circumcircle with center O.
- `on_circle w o x`: Places point W on the circumcircle.
- `on_tline a z o z, on_tline a x o x; on_tline b z o z, on_tline b w o w; on_tline c y o y, on_tline c w o w; on_tline d x o x, on_tline d y o y;`: Constructs points A, B, C, and D on the circumcircle.
- `incenter i1 a b c; incenter i2 a c d;`: Computes the incenters of triangles A1BC, AB1C, and ABC1.
- `foot f1 i1 a c; foot f2 i2 a c;`: Computes the feet F1 and F2 of altitudes.
- `cc_tangent q t p s i1 f1 i2 f2; on_line k q t, on_line k p s ? cong o k o x;`: Constructs the tangents and shows the collinearity.

IMO 2009 Problem 2:
English Statement:
Let ABC be an acute-angled triangle with circumcenter O. Let P be a point in the interior of triangle ABC, and let D, E, F be the feet of the perpendiculars from P to BC, CA, and AB, respectively. Let K, L, M be the midpoints of BC, CA, and AB, respectively. Prove that if OP = DP = EP = FP, then O is the circumcenter of triangle ABC.

AlphaGeometry Syntax:
a b c = triangle a b c; o = circle o a b c;
d = free; e = eqdistance d b c; t = on_bline b d, on_bline c e;
a = eqangle2 b t e; p = on_line a b, on_line c d, on_line b e;
k = midpoint k b c; l = midpoint l c a; m = midpoint m a b ? concyclic a b c l k m;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `circle o a b c`: Defines the circumcircle with center O.
- `eqdistance d b c`: Ensures that OP = DP = EP = FP.
- `on_bline b d, on_bline c e`: Constructs the perpendiculars from P to BC, CA, and AB.
- `eqangle2 b t e`: Ensures that OP = DP = EP = FP.
- `on_line a b, on_line c d, on_line b e`: Constructs points A, B, and C on the circumcircle.
- `midpoint k b c; midpoint l c a; midpoint m a b ? concyclic a b c l k m;`: Demonstrates that O is the circumcenter of triangle ABC.

IMO 2009 Problem 3:
English Statement:
Let ABC be a triangle with circumcenter O. The bisector of angle BAC intersects BC at D. Let E be the reflection of O in the midpoint of BC. Prove that the lines DE and BO are perpendicular.

AlphaGeometry Syntax:
a b c = triangle a b c; o = circle o a b c;
d = on_line d b c, on_line d a o; m = midpoint m b c; e = reflect e o m;
b = on_line b d e, on_line b o d ? perpendicular o b e;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `circle o a b c`: Defines the circumcircle with center O.
- `on_line d b c, on_line d a o`: Constructs point D as the intersection of the bisector of angle BAC with side BC.
- `midpoint m b c`: Finds the midpoint M of side BC.
- `reflect e o m`: Reflects O to obtain E.
- `on_line b d e, on_line b o d ? perpendicular o b e;`: Demonstrates that DE and BO are perpendicular.

IMO 2009 Problem 6:
English Statement:
Let ABC be a triangle and let M be the midpoint of BC. Denote by w1 and w2 the circumcircles of triangles ABM and ACM, respectively. Suppose that w1 and w2 intersect at two distinct points M and N. Prove that the midpoint of MN lies on the circumcircle of triangle ABC.

AlphaGeometry Syntax:
a b c = triangle a b c; m = midpoint m b c;
w1 = circle w1 a b m; w2 = circle w2 a c m;
n = on_circle n w1 w2, on_bline n a m;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `midpoint m b c`: Finds the midpoint M of side BC.
- `circle w1 a b m; circle w2 a c m;`: Constructs the circumcircles of triangles ABM and ACM.

- `on_circle n w1 w2, on_bline n a m;`: Constructs the intersection point N on the circumcircle of ABC.

IMO 2010 Problem 2:
English Statement:
Let ABC be an acute-angled triangle. The internal bisectors of angles A, B, C meet the opposite sides in D, E, F, respectively. Let P, Q, R be the feet of the altitudes from D, E, F, respectively. Prove that the circumcircles of triangles PEF, PFD, PDE are tangent to the circumcircle of triangle ABC.

AlphaGeometry Syntax:
a b c = triangle a b c;
d = angle_bisector d a b c; e = angle_bisector e b c a; f = angle_bisector f c a b;
p = foot p d a b; q = foot q e b c; r = foot r f c a;
x1 = reflect x1 p e f, on_bline x1 p a; x2 = reflect x2 q f d, on_bline x2 q b; y2 = reflect y2 q f e, on_bline y2 q b; y3 = reflect y3 r d e, on_bline y3 r c;
z = on_line z x1 x2, on_line z y2 y3 ? cong e z e d;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `angle_bisector d a b c; angle_bisector e b c a; angle_bisector f c a b;`: Constructs the internal bisectors of angles A, B, C.
- `foot p d a b; foot q e b c; foot r f c a;`: Finds the feet P, Q, R of altitudes from D, E, F.
- `reflect x1 p e f, on_bline x1 p a; reflect x2 q f d, on_bline x2 q b; reflect y2 q f e, on_bline y2 q b; reflect y3 r d e, on_bline y3 r c;`: Reflects points to obtain X1, X2, Y2, Y3.
- `on_line z x1 x2, on_line z y2 y3 ? cong e z e d;`: Demonstrates that the circumcircles are tangent.

IMO 2010 Problem 4:
English Statement:
Let ABC be a triangle with circumcenter O. The points P, Q, R are on the sides BC, CA, AB, respectively. The circumcircles of triangles AQR and BPR meet at S, different from A. Prove that SA is perpendicular to OS.

AlphaGeometry Syntax:
a b c = triangle a b c; o = circle o a b c;
p = on_line p b c, on_aline p a b b c a;
q = on_line q c a, on_aline q b c c a b;
r = on_line r a b, on_aline r c a a b c;

s = on_circle s o a, on_circle s o b ? perpendicular s a o;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `circle o a b c`: Defines the circumcircle with center O.
- `on_line p b c, on_aline p a b b c a; on_line q c a, on_aline q b c c a b; on_line r a b, on_aline r c a a b c;`: Constructs points P, Q, R on the sides BC, CA, AB.
- `on_circle s o a, on_circle s o b ? perpendicular s a o;`: Demonstrates that SA is perpendicular to OS.

IMO 2011 Problem 1:
English Statement:
In the interior of the square ABCD, point P is chosen so that AP = BP + CP. Let M and N be the midpoints of CD and BC, respectively. Prove that angle MPN is a right angle.

AlphaGeometry Syntax:
a b c d = square a b c d;
p = on_tline p a d, on_tline p b c ? eqdistance b p b c a;
m = midpoint m c d; n = midpoint n b c;
x = on_line x m p, on_line x n p ? perpendicular m x n;

Explanation:
- `square a b c d`: Represents square ABCD.
- `on_tline p a d, on_tline p b c ? eqdistance b p b c a;`: Places point P in the interior such that AP = BP + CP.
- `midpoint m c d; midpoint n b c;`: Finds the midpoints M and N of sides CD and BC.
- `on_line x m p, on_line x n p ? perpendicular m x n;`: Demonstrates that angle MPN is a right angle.

IMO 2011 Problem 2:
English Statement:
In triangle ABC, let X, Y, and Z be the feet of the perpendiculars from A, B, and C to BC, CA, and AB, respectively. Let P be the intersection of the perpendicular from Y to AB with the perpendicular from Z to AC. Define Q and R similarly. Prove that P, Q, and R are collinear.

AlphaGeometry Syntax:
a b c = triangle a b c;
x = foot x a b c; y = foot y b c a; z = foot z c a b;

p = on_pline p y b a, on_line p z c;
q = on_pline q z c b, on_line q x a;
r = on_pline r x a c, on_line r y b ? collinear p q r;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `foot x a b c; foot y b c a; foot z c a b;`: Finds the feet X, Y, Z of perpendiculars.
- `on_pline p y b a, on_line p z c; on_pline q z c b, on_line q x a; on_pline r x a c, on_line r y b ? collinear p q r;`: Demonstrates that P, Q, and R are collinear.

IMO 2011 Problem 5:
English Statement:
In triangle ABC, let D, E, and F be the midpoints of BC, CA, and AB, respectively. Let ω be the circumcircle of triangle ABC. The tangents to ω at B and C intersect at X. The line through B perpendicular to DE intersects lines AC and DF at Y and Z, respectively. Prove that the four points A, X, Y, and Z are concyclic.

AlphaGeometry Syntax:
a b c = triangle a b c;
d = midpoint d b c; e = midpoint e c a; f = midpoint f a b;
o = circle o a b c;
x = on_circle x o b, on_circle x o c ? tangent x b c;
y = on_line y b d, on_line y a c; z = on_line z d f, on_line z a b;
w = on_circle w o a, on_line w x y, on_line w x z ? cyclic a x y z;

Explanation:
- `triangle a b c`: Represents triangle ABC.
- `midpoint d b c; midpoint e c a; midpoint f a b;`: Finds the midpoints D, E, F of sides BC, CA, AB.
- `circle o a b c;`: Defines the circumcircle with center O.
- `on_circle x o b, on_circle x o c ? tangent x b c;`: Constructs point X as the intersection of tangents at B and C.
- `on_line y b d, on_line y a c; on_line z d f, on_line z a b;`: Constructs points Y and Z.
- `on_circle w o a, on_line w x y, on_line w x z ? cyclic a x y z;`: Demonstrates that A, X, Y, and Z are concyclic.

IMO 2012 Problem 2:
English Statement:

Let ABCD be a cyclic quadrilateral. Prove that the circumcircles of triangles ABC and ACD are tangent to each other if and only if AB + CD = AD + BC.

AlphaGeometry Syntax:
a b c d = cyclic_quadrilateral a b c d;
o1 = circle o1 a b c, on_circle o1 a d; o2 = circle o2 a c d, on_circle o2 b c;
t = on_circle t o1 a, on_circle t o2 a ? tangent t a c;

Explanation:
- `cyclic_quadrilateral a b c d;`: Represents cyclic quadrilateral ABCD.
- `circle o1 a b c, on_circle o1 a d; circle o2 a c d, on_circle o2 b c;`: Constructs the circumcircles of triangles ABC and ACD.
- `on_circle t o1 a, on_circle t o2 a ? tangent t a c;`: Demonstrates that the circumcircles are tangent if and only if AB + CD = AD + BC.

IMO 2012 Problem 5:
English Statement:
Let ABCD be a convex quadrilateral with ∠C = ∠D = 90°. The diagonals AC and BD intersect at P. The orthogonal projections of P onto AB, BC, CD, and DA are Q, R, S, and T, respectively. Prove that the circumcircles of triangles PQR and PST are tangent to each other.

AlphaGeometry Syntax:
a b c d = convex_quadrilateral a b c d, angle a b c ? right_angle c a d ? right_angle;
p = on_tline p a c, on_tline p b d;
q = on_line q p a, on_line q b c; r = on_line r p b, on_line r c d;
s = on_line s p c, on_line s d a; t = on_line t p d, on_line t a b;
o1 = circle o1 p q r, on_circle o1 p s ? tangent o1 q s;

Explanation:
- `convex_quadrilateral a b c d, angle a b c ? right_angle c a d ? right_angle;`: Represents convex quadrilateral ABCD with right angles at C and D.
- `on_tline p a c, on_tline p b d;`: Constructs the intersection point P of diagonals AC and BD.
- `on_line q p a, on_line q b c; on_line r p b, on_line r c d; on_line s p c, on_line s d a; on_line t p d, on_line t a b;`: Constructs points Q, R, S, and T.
- `circle o1 p q r, on_circle o1 p s ? tangent o1 q s;`: Demonstrates that the circumcircles of triangles PQR and PST are tangent.

IMO 2012 Problem 6:
English Statement:
Let ABC be an acute-angled triangle with orthocenter H. The tangent at A to the circumcircle of ABC meets the line BC at D. Let M be the midpoint of BC. Let Q be the point on line AD such that MH = MQ. Prove that BQ and CH meet on the circumcircle of ABC.

AlphaGeometry Syntax:
a b c = triangle a b c; h = orthocenter h a b c;
d = on_line d a h ? tangent d a b c; m = midpoint m b c;
q = on_line q a d, on_tline q m h ? collinear b q c;

Explanation:
- `triangle a b c; orthocenter h a b c;`: Represents triangle ABC with orthocenter H.
- `on_line d a h ? tangent d a b c;`: Constructs point D as the intersection of the tangent at A to the circumcircle with line BC.
- `midpoint m b c;`: Finds the midpoint M of side BC.
- `on_line q a d, on_tline q m h ? collinear b q c;`: Demonstrates that BQ and CH meet on the circumcircle.

IMO 2013 Problem 2:
English Statement:
Let ABC be a triangle with incenter I and circumcircle ω. Let D be the foot of the altitude from A to BC, and let E and F be the feet of the perpendiculars from B and C to CA and AB, respectively. Let X be the intersection of EF and BC. Suppose the circumcircle of AIX intersects line BC at Y, distinct from B. Prove that the circumcircles of the triangles XYD and ADE are tangent to each other.

AlphaGeometry Syntax:
a b c = triangle a b c; i = incenter i a b c;
d = foot d a b c; e = foot e b c a; f = foot f c a b;
x = on_line x e f, on_line x b c;
y = on_circle y i x, on_bline y b c;

Explanation:
- `triangle a b c; incenter i a b c;`: Represents triangle ABC with incenter I.
- `foot d a b c; foot e b c a; foot f c a b;`: Finds the feet D, E, F of perpendiculars.
- `on_line x e f, on_line x b c;`: Constructs point X as the intersection of EF and BC.

- `on_circle y i x, on_bline y b c;`: Constructs point Y as the intersection of the circumcircle of AIX with line BC.