Further Reading and Video Sources

for: https://youtu.be/5M2RWtD4EzI

Contents

Further Reading

1. The History of Calculus
2. Infinity
3. The Question of Teaching Calculus

The History of Calculus

Orlin, Ben, and Ben Orlin. Change Is the Only Constant the Wisdom of Calculus in a Madcap World. Black Dog & Leventhal Publishers, 2019. (https://www.google.com/books/edition/Change_Is_the_Only_Constant/sIaGDwAAQBAJ?hl=en&gbpv=0)

This is an easy-to-read, very beginner-friendly comic-book-style collection of stories about calculus. I recommend this to anyone who is just starting their journey into the world of calculus, though anyone of any math level can appreciate the humor and stories told in it.

Strogatz, Steven. Infinite Powers: How Calculus Reveals the Secrets of the Universe. United States, HarperCollins, 2019. (https://www.google.com/books/edition/Infinite_Powers/alZkDwAAQBAJ?hl=en&gbpv=0)

Written in layman's terms and an inspiring tone, this book tells an engaging narrative of the development of calculus by delving into the goals and stories of different mathematicians and highlighting the real-life applications of the big ideas of calculus. This book helped me choose the mathematicians and ideas to narrow down on and develop a clear narrative for my video. I recommend this to anyone who is interested in learning about more of the main ideas found in the narrative of calculus.

Bressoud, David M. Calculus Reordered: A History of the Big Ideas. Princeton University Press, 2019. (https://www.google.com/books/edition/Calculus_Reordered/IB-EDwAAQBAJ)

This book thoroughly follows the concepts of calculus based on the historical order of its development, with detailed descriptions into the lives of philosophers and their thinking processes as they experimented with and discovered the ideas that would lead to calculus. My video loosely followed the ordering of ideas laid out in this book, and used its description of Archimedes' derivation of the area of the circle (which can be found in chapter 1.2). I recommend this to anyone who is interested in going past just the "main ideas" of the narrative of calculus, and is willing and patient enough to read and reread what may be unfamiliar notation or ways of thinking.

"My hope is that readers of this book will find inspiration in its story. I assume some knowledge of the tools of calculus, though, in truth, most of what I have written requires little more than mathematical curiosity."

Timeline of Calculus: https://www.desmos.com/calculator/txixcghvyc

World Map of Calculus: https://www.desmos.com/calculator/cdvheuweg4

These are resources I made to organize my notes after reading Bressoud's book. It may help with visualizing the many names, places, time periods and ideas of the millenia of calculus, and show how they overlapped and interacted with each other.

Non-Western contributions to calculus:

https://www.cbc.ca/news/science/calculus-created-in-india-250-years-before-newton-study-1.632433

Almeida, Dennis F., and George G. Joseph. "Eurocentrism in the History of Mathematics: the Case of the Kerala School." Race & Class 45.4 (2004): 45-59.

These are articles about how some of calculus can be attributed to Indian mathematicians in the Kerala School. I link this to show some non-western sources of the development of calculus, as inspired by a comment from Rishabh Khatri.

https://mathshistory.st-andrews.ac.uk/Biographies/Madhava/

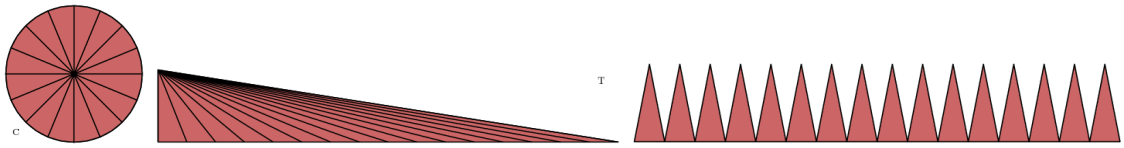
This link explores Madhava of Sangamagramma's discovery of what are known as Taylor/Maclaurin expansions of trigonometric functions and others, several hundred years before they were discovered in Europe.

https://mathshistory.st-andrews.ac.uk/Biographies/Al-Khwarizmi/

This link goes into more depth about the historical context of al-Khwarizimi's life and some of his contributions to algebra.

Archimedes' derivation of the area of a circle:
http://www.ams.org/publicoutreach/feature-column/fc-2012-02

In chapter 2.1 of my video, I presented a slightly modified version of Archimedes' derivation of the area of a circle. If that one confused you, here is a link with a derivation that is much more faithful to Archimedes' full line of reasoning.



This is a bit of a tangent, but the visual in this link (shown above) made me realize that there is a slight connection between Archimedes' circle proof and Kepler's second law, in that the area of "pizza slices" of the circle above are equal to the area of the slices in the triangle to its right. This was a pleasant "aha" moment for me I wanted to quickly share!

Another "history of science" channel you may also enjoy (recommended by Volbla in the comments): https://www.youtube.com/c/KathyLovesPhysicsHistory

Infinity

Rucker, Rudy v. B. 1946-. Infinity and the Mind: The Science and Philosophy of the Infinite. Boston: Birkhäuser, 1982 (https://www.google.com/books/edition/Infinity_and_the_Mind/MD0UAwAAQBAJ?hl=en&gbpv=0)

Infinity. Everything about it, from the Greeks' apeirophobia to Godel's Incompleteness Theorem. This book prances around with existentially terrifying concepts in a nonchalant and tongue-in-cheek tone. Although I have personally only read the first two chapters, I recommend it to anyone who is interested in the human search for meaning in infinity and is willing to have a few existential crises while reading!

Wildberger, Norman. "Set theory: Should you believe." (https://web.maths.unsw.edu.au/~norman/papers/SetTheory.pdf)

This is a controversial opinion essay by a professor at UNSW Sydney that modern set theory is flawed. I am (very, very) not qualified to comment on this, but from the internet forums I've read, the arguments in this paper don't seem to be taken seriously. I link this to show the diverse takes that modern mathematicians have about infinity.

The Question of Teaching Calculus

Tall, David. "Students' difficulties in calculus." proceedings of working group. Vol. 3. 1993. (https://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1993k-calculus-wg3-icme.pdf)

> This is a review-of-literature surrounding concerns with how calculus is taught, and brings up the calculus reform movement of the early 1990s.

Articles by David Bressoud:

> https://www.utdanacenter.org/blog/decades-later-problematic-role-calculus-gatekeeper-opportunity-persists (on inequalities in access to quality calculus education)
>
> https://www.mathvalues.org/masterblog/launchings201906-z45y4-fhkpj (on the calculus reform movement)
>
> https://www.mathvalues.org/masterblog/launchings201906-z45y4 (on his book, *Calculus Reordered*, which I quoted in the video)

3Blue1Brown podcast (which I quoted in the video): https://www.youtube.com/watch?v=SUMLKweFAYk

Lockhart, Paul. A mathematician's lament: How school cheats us out of our most fascinating and imaginative art form. Bellevue literary press, 2009. (https://www.maa.org/external_archive/devlin/LockhartsLament.pdf)

> An opinion essay written in fiery language on how modern math education fails to capture what math is. Recommended to me by John Chessant in the comments.
>
> *"if I had to design a mechanism for the express purpose of destroying a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done—I simply wouldn't have the imagination to come up with the kind of senseless, soulcrushing ideas that constitute contemporary mathematics education."*
>
> *"Mathematics is the music of reason. To do mathematics is to engage in an act of discovery and conjecture, intuition and inspiration; to be in a state of confusion—not because it makes no sense to you, but because you gave it sense and you still don't understand what your creation is up to; to have a breakthrough idea; to be frustrated as an artist; to be awed and overwhelmed by an almost painful beauty; to be alive, damn it."*

Video Sources

1. Music Sources
2. Sound Effect Sources
3. Image Sources

Music Sources

chapter 2.1

ancient greek music: https://www.youtube.com/watch?v=elERNFoEf3Y

chapter 2.2

arabian music: https://www.youtube.com/watch?v=-h7fr_QUAv4

french baroque music: https://www.youtube.com/watch?v=SjZb-ZkS2Sg

chapter 2.3

claudio monteverdi: https://www.youtube.com/watch?v=RajAq0Yd-s4

mozart: https://www.youtube.com/watch?v=JTc1mDieQI8

chapter 2.4

beethoven: https://www.youtube.com/watch?v=_mVW8tgGY_w

chapter 3

debussy: https://www.youtube.com/watch?v=CvFH_6DNRCY

Sound Effect Sources

wrestling bell: https://www.youtube.com/watch?v=eAcDaC1o5nA

cheering: https://www.youtube.com/watch?v=yLNALmt6KFs

poof: https://www.youtube.com/watch?v=fxI4toJa67E

thud: https://www.youtube.com/watch?v=4OUa62jBA6Y

Image Sources

CHAPTER 1

observable universe: https://commons.wikimedia.org/wiki/File:Observable_universe_logarithmic_illustration.png

god the father: https://commons.wikimedia.org/wiki/File:Cima_da_Conegliano,_God_the_Father.jpg

CHAPTER 2.1

world map: https://en.wikipedia.org/wiki/File:Afro-Eurasia_location_map_with_borders.svg

archimedes: https://commons.wikimedia.org/wiki/File:Archimedes,_printer%27s_sample_for_the_World%27s_Inventors_souvenir_album_%28A25%29_for_Allen_%26_Ginter_Cigarettes_MET_DP838812.jpg

aristotle: https://commons.wikimedia.org/wiki/File:Aristotle_Bust_White_Background_Transparent.png

plato: https://commons.wikimedia.org/wiki/File:Bust_of_Plato,_Vatican_Museum,_Rome.jpg

zeno: https://www.worldhistory.org/image/14138/bust-of-zeno-of-elea/

clapping hands: https://commons.wikimedia.org/wiki/File:Twemoji12_1f44f.svg

archimedes method: http://www.archimedespalimpsest.org/

CHAPTER 2.2

baghdad house of wisdom: https://commons.wikimedia.org/wiki/File:Maqamat_hariri.jpg

al-khwarizimi: https://picryl.com/media/1983-cpa-5426-d3d166

al-jabr: https://commons.wikimedia.org/wiki/File:Image-Al-Kit%C4%81b_al-mu%E1%B8%ABta%E1%B9%A3ar_f%C4%AB_%E1%B8%A5is%C4%81b_al-%C4%9Fabr_wa-l-muq%C4%81bala.jpg

fibonacci: https://ndla.no/subject:1:4200b774-5b7b-4900-bf1d-e0b298b9cb97/topic:2:165921/topic:2:165924/resource:1:115155

robert recorde: https://commons.wikimedia.org/wiki/File:Robert_Recorde_%28306312390%29.jpg

cardano: https://commons.wikimedia.org/wiki/File:Girolamo_Cardano.jpg

viete: https://picryl.com/media/viete-francois-0b7b42

descartes: https://commons.wikimedia.org/wiki/File:091717-34-Descartes-Philosophy.jpg

captain hook: https://disney.fandom.com/wiki/Captain_Hook

conic sections: https://en.wikipedia.org/wiki/File:Conic_Sections.svg

marin mersenne: https://commons.wikimedia.org/wiki/File:Marin_Mersenne._Line_engraving._Wellcome_L0019806.jpg

fermat: https://commons.wikimedia.org/wiki/File:Pierre_de_Fermat.jpg

minecraft arrow: https://minecraft.fandom.com/wiki/Arrow

aryabhata: https://commons.wikimedia.org/wiki/File:Aryabhata-5_%281%29.jpg

CHAPTER 2.3

solid of revolution: https://commons.wikimedia.org/wiki/File:Solid_of_revolution-Cone.svg

ap test problem: https://apcentral.collegeboard.org/media/pdf/ap21-frq-calculus-ab.pdf

mosque of ibn tulun: https://commons.wikimedia.org/wiki/File:Mosque_of_Ibn_Tulun,_Cairo,_Egypt4.jpg

taj mahal: https://commons.wikimedia.org/wiki/File:Taj_Mahal_Front.JPG

cavalieri: https://commons.wikimedia.org/wiki/File:Bonaventura_Cavalieri._Fusinati_sculp.jpg

cavalieri's principle: https://commons.wikimedia.org/wiki/File:Cavalieri%27s_Principle_in_Coins.JPG

torricelli: https://commons.wikimedia.org/wiki/File:Evangelista_Torricelli._Line_engraving_by_Tomba_after_himsel_Wellcome_V0005863.jpg

gabriel's horn: https://mathworld.wolfram.com/GabrielsHorn.html

newton: https://www.flickr.com/photos/124561666@N02/14217990329

leibniz: https://jenikirbyhistory.getarchive.net/media/leibniz-hannover-59b7c6

galileo: https://commons.wikimedia.org/wiki/File:1635_Justus-Suttermans_Galileo-Galilei.jpg

kepler: https://www.flickr.com/photos/esoastronomy/25623161844

earth: https://commons.wikimedia.org/wiki/File:The_Blue_Marble_4463x4163.jpg

solar system: https://openclipart.org/detail/219865/solar-system-scale

principia: https://commons.wikimedia.org/wiki/File:Prinicipia-title.png

christmas: https://commons.wikimedia.org/wiki/File:Christmas-1869902_1920.jpg

man silhouette:
https://publicdomainvectors.org/en/free-clipart/Male-head-profile-silhouette/73221.html

newton's mom: https://totallyhistory.com/isaac-newtons-mother-hannah-ayscough/

newton college:
https://commons.wikimedia.org/wiki/File:Trinity_College_Cambridge_1690.jpg

random people profiles: https://this-person-does-not-exist.com/en

clavis mathematicae:
https://www.maa.org/press/periodicals/convergence/mathematical-treasure-oughtreds-clavis-mathematicae

discourse on the method:
https://commons.wikimedia.org/wiki/File:Descartes_Discours_de_la_Methode.jpg

arithmetica infinitorum:
https://www.maa.org/press/periodicals/convergence/mathematical-treasures-walliss-1655-56-works-on-quadrature

apple: https://commons.wikimedia.org/wiki/File:Honeycrisp-Apple.jpg

ftc:
https://www.khanacademy.org/math/in-in-grade-12-ncert/xd340c21e718214c5:definite-integrals/xd340c21e718214c5:fundamental-theorem-of-calculus-chain-rule/a/fundamental-theorem-of-calculus-review

huygens:
https://commons.wikimedia.org/wiki/File:Christiaan_Huygens_by_Caspar_Netscher_1671,_oil_painting,_Museum_Boerhaave,_Leiden_%28375796327 5%29.jpg

john collins: https://mathshistory.st-andrews.ac.uk/Biographies/Collins/pictdisplay/

leibniz infinite series: https://books.google.com/books?id=Mtt1EAAAQBAJ, page 88

jacob bernoulli: https://www.maa.org/book/export/html/654543

johann bernoulli: https://commons.wikimedia.org/wiki/File:Johann_Bernoulli2.jpg

CHAPTER 2.4

euler: https://commons.wikimedia.org/wiki/File:Leonhard_Euler.jpg

convergence tests: https://sites.google.com/site/calculusproject13/

d'alembert: https://en.wikipedia.org/wiki/Jean_le_Rond_d%27Alembert

lagrange: https://commons.wikimedia.org/wiki/File:Joseph_Louis_Lagrange2.jpg

taylor: https://commons.wikimedia.org/wiki/File:Brook_Taylor._Line_engraving_after_R._Earlom._Wellcome_V0005740.jpg

maclaurin: https://commons.wikimedia.org/wiki/File:Colin_Maclaurin._Coloured_stipple_engraving_by_R._Page,_1814_Wellcome_V0003765.jpg

niels abel: https://commons.wikimedia.org/wiki/File:Niels_Henrik_Abel_%281802_-_1829%29.jpg

megaphone: https://freesvg.org/vector-graphics-of-megaphone

cauchy: https://commons.wikimedia.org/wiki/File:Augstin_Louis,_Baron_Cauchy._Lithograph_by_Z._Belliard_after_Wellcome_V0001035.jpg

CHAPTER 3

calculus reordered: https://press.princeton.edu/books/hardcover/9780691181318/calculus-reordered

infinite powers: https://www.stevenstrogatz.com/books/infinite-powers