MAT313S23
Sample Quiz 7

Determinants

MAT313 SAMPLE **Quiz 7** Prof Sormani

Find the following determinant using the Method of Minors:

$$\det\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 8 & 1 & 1 \\ 8 & 0 & 0 & 0 \\ 2 & 0 & 8 & 0 \end{pmatrix}$$

HW: Find the same determinant using row reduction following our algorithm.

Scoring Rubrik:

Timed Portion:
+1 for circling a column or row in the original 4x4 matrix
and circling the corresponding column or row in the 4x4 + - + grid
+1 for writing the first line with 3x3 minors
+1 for circling for circling a column or row in the 3x3 matrices
and circling the corresponding column or row in the 3x3 + - + grid
+1 for writing the first line with 2x2 minors
+1 for finding the 2x2 determinants correctly
and doing the final arithmetic correctly.
Homework:
+1 for correct first row action and equality
+1 for correct second row action and equality
+1 for correct third row action and equality
+1 for correct fourth row action and equality
+1 for correct finals row actions and equality
See sample grading in the solutions.

There are many correct answers for the Method of Minors!

One correct answer with a video:

Find the determinant using the Method of Minors

Can use any column or row that we wish. Choose one with lots of zeroes

$$\det\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 8 & 1 & 1 \\ 8 & 0 & 0 & 0 \\ 2 & 0 & 8 & 0 \end{pmatrix}$$

Write the + - + - grid and circle the same column:

$$\begin{matrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{matrix}$$

starts with a minus

$$= -0 \det(\ldots) + 1 \det\begin{pmatrix} 1 & 0 & 2 \\ 8 & 0 & 0 \\ 2 & 0 & 8 \end{pmatrix} - 0 \det(\ldots) + 0 \det(\ldots)$$

Choose a column or row with lots of zeroes. Draw

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

starts with a minus

$$= -0(\ldots) + 1\left(-8 \det\begin{pmatrix} 0 & 2 \\ 0 & 8 \end{pmatrix} + 0 \det(\ldots) - 0 \det(\ldots)\right) - 0 + 0$$

$$= -0 + 1\left(-8\left(0 \cdot 8 - 2 \cdot 0\right) + 0 - 0\right) - 0 + 0 = 1(-8)(0-0)$$

$$= 1(-8)(0) = 0$$

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Another correct answer:

Using the Method of Minors

$$\det\begin{pmatrix}1&0&2&0\\0&8&1&1\\8&0&0&0\\2&0&8&0\end{pmatrix} \qquad \begin{matrix}+&-&+&-\\-&+&-&+\\+&-&+&-\\-&+&-&+\end{matrix}$$

$$= 1\det\begin{pmatrix}8&1&1\\0&0&0\\0&8&0\end{pmatrix} - 0\det\begin{pmatrix}0&2&0\\0&0&0\\0&8&0\end{pmatrix} + 8\det\begin{pmatrix}0&2&0\\8&1&1\\0&8&0\end{pmatrix} - 2\det\begin{pmatrix}0&2&0\\8&1&1\\0&0&0\end{pmatrix}$$

$$= 1\left[8\det\begin{pmatrix}0&0\\8&0\end{pmatrix} - 0\det\begin{pmatrix}1&1\\8&0\end{pmatrix} + 0\det\begin{pmatrix}1&1\\0&0\end{pmatrix}\right]$$

$$- 0\left[\underbrace{\sim\sim\sim}_{\text{doesn't matter because multiplied by 0}}\right] + 8\left[0\det\begin{pmatrix}1&1\\8&0\end{pmatrix} - 8\det\begin{pmatrix}2&0\\8&0\end{pmatrix} + 0\det\begin{pmatrix}2&0\\1&1\end{pmatrix}\right]$$

$$-2\left[0(\sim) - 8\det\begin{pmatrix}2&0\\0&0\end{pmatrix} + 0(\sim)\right]$$

$$= 1\left[8(0\cdot 0 - 8\cdot 0) - 0 + 0\right] + 8\left(0 - 8(2\cdot 0 - 0\cdot 8) + 0\right)$$

$$-2\left[0 - 8(2\cdot 0 - 0\cdot 0) + 0\right]$$

$$= 0$$

(+1)

A third correct answer:

Better Solution → much quicker if we choose a row or column with many zeroes!

$$\det\begin{pmatrix}1&0&2&0\\0&8&1&1\\8&0&0&0\\2&0&8&0\end{pmatrix} \qquad \begin{matrix}+&-&+&-\\-&+&-&+\\+&-&+&-\\-&+&-&+\end{matrix}$$

$$= 8\det\begin{pmatrix}0&2&0\\8&1&1\\0&8&0\end{pmatrix} - 0(\sim) + 0(\sim) - 0(\sim) = 8\begin{pmatrix}0\det\begin{pmatrix}1&1\\8&0\end{pmatrix}\\ -2\det\begin{pmatrix}8&1\\0&0\end{pmatrix}\\ +0\det\begin{pmatrix}8&1\\0&4\end{pmatrix}\end{pmatrix}$$

$$= 8\left(0 - 2(8\cdot 0 - 1\cdot 0) + 0\right) = 8(0 - 0 + 0) = 0$$

(+1)

For the homework there is only one correct answer. **You must follow row reduction as taught in Lessons 2-5.**

One point for each correctly completed row action up to five points:

HW

skew
$\rho_3 \to \rho_3 - 8\rho_1$
$\rho_4 \to \rho_4 - 2\rho_1$

$$\det\begin{pmatrix} \boxed{1} & 0 & 2 & 0 \\ 0 & 8 & 1 & 1 \\ 8 & 0 & 0 & 0 \\ 2 & 0 & 8 & 0 \end{pmatrix} = (1)\det\begin{pmatrix} \boxed{1} & 0 & 2 & 0 \\ 0 & \boxed{8} & 1 & 1 \\ 0 & 0 & -16 & 0 \\ 0 & 0 & 4 & 0 \end{pmatrix}$$

must skew by leaders row to 0

must scale 8 to 1

+1

scale
$\rho_2 \to \frac{1}{8}\rho_2$

$$= (8)(1)\det\begin{pmatrix} \boxed{1} & 0 & 2 & 0 \\ 0 & \boxed{1} & 1/8 & 1/8 \\ 0 & 0 & \boxed{-16} & 0 \\ 0 & 0 & 4 & 0 \end{pmatrix}$$

Must scale -16 to 1

+1

scale
$\rho_3 \to \frac{-1}{16}\rho_3$

$$= (-16)(8)(1)\det\begin{pmatrix} \boxed{1} & 0 & 2 & 0 \\ 0 & \boxed{1} & 1/8 & 1/8 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 4 & 0 \end{pmatrix}$$

must skew by leaders row to 0

+1

skew
$\rho_4 \to \rho_4 - 4\rho_3$

+1

$$= (1)(-16)(8)(1)\det\begin{pmatrix} \boxed{1} & 0 & 2 & 0 \\ 0 & \boxed{1} & 1/8 & 1/8 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Echelon Form is not I! Row of Zeroes.

$$= (1)(-16)(8)(1)(0) = 0$$

+1