FREEHOLD REGIONAL HIGH SCHOOL DISTRICT
OFFICE OF CURRICULUM AND INSTRUCTION
INTERNATIONAL BACCALAUREATE PROGRAM

IB Mathematics: Applications and Interpretation SL & HL, Year 2

Grade Level: 12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE: August 26, 2021

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

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IB Mathematics: Applications and Interpretation SL & HL, Year 2

Course Description

From the International Baccalaureate Organization: "Mathematics: applications and interpretation is for students who are interested in developing their mathematics for describing our world and solving practical problems. They will also be interested in harnessing the power of technology alongside exploring mathematical models. Students who take Mathematics: applications and interpretation will be those who enjoy mathematics best when seen in a practical context.

This course recognizes the increasing role that mathematics and technology play in a diverse range of fields in a data-rich world. As such, it emphasizes the meaning of mathematics in context by focusing on topics that are often used as applications or in mathematical modelling. To give this understanding a firm base, this course also includes topics that are traditionally part of a pre-university mathematics course such as calculus and statistics.

The course makes extensive use of technology to allow students to explore and construct mathematical models. Mathematics: applications and interpretation will develop mathematical thinking, often in the context of a practical problem and using technology to justify conjectures.

Students who choose Mathematics: applications and interpretation at SL or HL should enjoy seeing mathematics used in real-world contexts and to solve real-world problems. Students who wish to take Mathematics: applications and interpretation at higher level will have good algebraic skills and experience of solving real-world problems. They will be students who get pleasure and satisfaction when exploring challenging problems and who are comfortable to undertake this exploration using technology."

Course Sequence and Pacing

Unit Title	SL Pacing (sessions)	+HL Pacing (sessions)
4: Functions	7	n/a
5: Matrices (in Year 1, HL only)	n/a	n/a
6: Calculus	34	n/a
7: Sampling & Inference	42	+58
8: Graph Theory	7	+10

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 4 Overview: Functions (SL only in Year 2) **Duration: SL 7 sessions**

IB Topic Essential Understandings:
Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and/or tables represents different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.

The parameters in a function or equation may correspond to notable geometrical features of a graph and can represent physical quantities in spatial dimensions.

Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.

Our spatial frame of reference affects the visible part of a function and by changing this “window” can show more or less of the function to best suit our needs.

Changing the parameters of a trigonometric function changes the position, orientation and shape of the corresponding graph.

Different representations facilitate modelling and interpretation of physical, social, economic and mathematical phenomena, which support solving real-life problems.

Technology plays a key role in allowing humans to represent the real world as a model and to quantify the appropriateness of the model.

Section and IB Sub-Topics	SL Duration (sessions)	HL Duration (sessions)
C: Modeling with Functions (SL only) SL 2.6, SL 2.5	7	n/a

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 4: Functions (SL only in Year 2)
Section 4C: Modeling with Functions **Duration: SL 7 sessions**

IB Topic Essential Understandings:
Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and/or tables represents different ways to communicate mathematical ideas.

IB Content-Specific Conceptual Understandings

Different representations facilitate modelling and interpretation of physical, social, economic and mathematical phenomena, which support solving real-life problems.

Technology plays a key role in allowing humans to represent the real world as a model and to quantify the appropriateness of the model.

Standards-Aligned Objectives: Instruction and assessment will align to the following objectives, with IB command terms in bold:

SL 2.6	**Use** the modelling process described in the "mathematical modelling" section to create, fit and use the theoretical models in section SL2.5 and their graphs. **Develop** and **fit** the model: Given a context **recognize** and **choose** an appropriate model and possible parameters. **Determine** a reasonable domain for a model. **Find** the parameters of a model. **Test** and **reflect** upon the model. **Comment** on the appropriateness and reasonableness of a model. **Justify** the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation. **Use** the model to **read, interpret,** and **make** predictions based on the model.	**TOK:** What is it about models in mathematics that makes them effective? Is simplicity a desirable characteristic in models? **International-mindedness:** The Babylonian method of multiplication: $ab = \frac{(a+b)^2 - a^2 - b^2}{2}$. Sulba Sutras in ancient India and the Bakhshali Manuscript contained an algebraic formula for solving quadratic equations.
SL 2.5	**Model** mathematical phenomena with: ● linear function: $f(x) = mx + c$ ● quadratic function: $f(x) = ax^2 + bx + c;\ a \neq 0$ ○ **Identify** axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y -axis ● cubic function: $f(x) = ax^3 + bx^2 + cx + d$ ● exponential growth and decay functions ○ $f(x) = ka^x + c$ ○ $f(x) = ka^{-x} + c$ for $a > 0$ ○ $f(x) = ke^{rx} + c$ ○ **Identify** the equation of a horizontal asymptote ● direct/inverse variation: $f(x) = ax^n$, n∈ℤ ○ **Identify** the y-axis as a vertical asymptote when $n < 0$	**TOK:** What role do models play in mathematics? Do they play a different role in mathematics compared to their role in other areas of knowledge?

	• sinusoidal function: $f(x) = a\sin(bx) + d,\ \ f(x) = a\cos(bx) + d$	

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 4: Functions (SL only in Year 2)
Support Resources / NJSLS Companion Standards / NJSLS College, Career, and Ed Tech Standards

Support Resources

Resources to support the standards-aligned performance expectations and objectives are available in District Resource Sharing. These include a resource catalog of standards-aligned activities and common formative assessment items for the performance and objectives in this course.

The following supporting appendices for this curriculum are available:

- Appendix A: Accommodations and Modifications for Various Student Populations
- Appendix B: Assessment Evidence
- Appendix C: Interdisciplinary Connections

NJSLS Life Literacies & Key Skills		**4C**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze and use creative skills and ideas.	X
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.	X
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities and utility for accomplishing a specified task	X
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.	X

IB Mathematics: Applications & Interpretations, HL only, Year 1 **Unit 5: Matrices**		**Duration (Sessions): HL:15**
IB Topic Essential Understandings: Geometry and trigonometry allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This branch provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships**.**		
IB Content-Specific Conceptual Understandings		
HL Different measurement systems can be used for angles to facilitate ease of calculation. Matrices are a form of notation which allow us to show the parameters or quantities of several linear equations simultaneously. Representation of probabilities using transition matrices enables us to efficiently predict long-term behaviour and outcomes.		
Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:		
HL 1.14	**Define** a matrix: the terms element, row, column and order for m×n matrices. **Perform** algebra of matrices: equality; addition; subtraction; multiplication by a scalar for m×n matrices. **Describe** properties of matrix multiplication: associativity, distributivity and non-commutativity. **Generate** identity and zero matrices. **Generate** determinants and inverses of $n \times n$ matrices with or without technology. **Write** systems of equations in the form $Ax = b$. **Find** the solutions of the systems of equations using inverse matrices.	**TOK:** Given the many applications of matrices in this course, consider the fact that mathematicians marvel at some of the deep connections between disparate parts of their subject. Is this evidence for a simple underlying mathematical reality? Mathematics, sense, perception and reason–if we can find solutions of higher dimensions, can we reason that these spaces exist beyond our sense perception?
HL 3.9	**Perform** geometric transformations of points in two dimensions using matrices: reflections, horizontal and vertical stretches, enlargements, translations and rotations, composition of functions. **Interpret** geometrically the determinant of a transformation matrix.	**TOK:** When mathematicians and historians say that they have explained something, are they using the word “explain” in the same way**?**
HL 4.19	**Calculate** transition matrices, powers of transition matrices. **Construct** regular Markov chains; (Link to Probability). **Construc**t initial state probability matrices **Calculate** steady state and long-term probabilities by repeated multiplication of the transition matrix or by solving a system of linear equations. **Use** transition matrices to represent transitions in discrete dynamic systems. **Use** transition matrices to determine the probability for moving from state j to state i, given initial state s_0.	n/a

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 5: Matrices
Support Resources / NJSLS Companion Standards / NJSLS College, Career, and Ed Tech Standards

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NJSLS Life Literacies & Key Skills		**5A**
9.4.12.CI.1:	Demonstrate the ability to reflect, analyze and use creative skills and ideas.	x
9.4.12.CT.2:	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.	x
9.4.12.TL.1:	Assess digital tools based on features such as accessibility options, capacities and utility for accomplishing a specified task	x
9.4.12.IML.3:	Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions.	x

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 6 Overview: Calculus (SL only in Year 2) **Duration: SL 34 sessions**

IB Topic Essential Understandings:
Calculus describes rates of change between two variables and the accumulation of limiting areas. Understanding these rates of change allows us to model, interpret and analyze real-world problems and situations. Calculus helps us understand the behaviour of functions and allows us to interpret the features of their graphs.

IB Content-Specific Conceptual Understandings

Students will understand the links between the derivative and the rate of change and interpret the meaning of this in context.

Students will understand the relationship between the integral and area and interpret the meaning of this in context.

Finding patterns in the derivatives of polynomials and their behavior, such as increasing or decreasing, allows a deeper appreciation of the properties of the function at any given point or instant.

Calculus is a concise form of communication used to approximate nature.

Numerical integration can be used to approximate areas in the physical world.

Optimization of a function allows us to find the largest or smallest value that a function can take in general and can be applied to a specific set of conditions to solve problems.

Maximum and minimum points help to solve optimization problems.

The area under a function of a graph has a meaning and has applications in space and time.

Section and IB Sub-Topics	**SL Duration (sessions)**	**HL Duration (sessions)**
A: Limits and Derivations SL 5.1, SL 5.2, SL 5.3, SL 5.4, SL 5.6, SL 5.7	20	n/a
B: Integration SL5.5, SL 5.7, SL 5.8	14	n/a

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 6: Calculus (SL only in Year 2)
Section 6A: Limits and Derivatives **Duration: SL 20 sessions**

IB Topic Essential Understandings: Calculus describes rates of change between two variables and the accumulation of limiting areas. Understanding these rates of change allows us to model, interpret and analyze real-world problems and situations. Calculus helps us understand the behaviour of functions and allows us to interpret the features of their graphs.

IB Content-Specific Conceptual Understandings

Students will understand the links between the derivative and the rate of change and interpret the meaning of this in context.

Finding patterns in the derivatives of polynomials and their behavior, such as increasing or decreasing, allows a deeper appreciation of the properties of the function at any given point or instant.

Calculus is a concise form of communication used to approximate nature.

Optimization of a function allows us to find the largest or smallest value that a function can take in general and can be applied to a specific set of conditions to solve problems.

Maximum and minimum points help to solve optimization problems.

Standards-Aligned Objectives: Instruction and assessment will align to the following objectives, with IB command terms in bold:

SL 5.1	**Understand** the concept of a limit. **Calculat**e and **interpret** limits. **Interpret** a derivative as gradient function and as rate of change.	**TOK:** What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life? Is intuition a valid way of knowing in mathematics?
SL 5.2	**Recognize** a function as increasing or decreasing. **Interpret** the graphs of $f'(x) > 0$, $f'(x) = 0$, $f'(x) < 0$.	n/a
SL 5.3	**Understand** the derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}$, $n \in \mathbb{Z}$ **Write** the derivatives of functions of the form $f(x) = ax^n + bx^{n-1} + \ldots$ where all exponents are integers.	**TOK:** The seemingly abstract concept of calculus allows us to create mathematical models that permit human feats such as getting a man on the Moon. What does this tell us about the links between mathematical models and reality?
SL 5.4	**Identify** tangents and normals at a given point, and their equations.	**TOK:** In what ways has technology impacted how knowledge is produced and shared in mathematics? Does technology simply allow us to arrange existing knowledge in new and different ways, or should this arrangement itself be considered knowledge?
SL 5.6	**Determine** values of *x* where the gradient of a curve is zero; solution of $f'(x) = 0$. **Determine** local maximum and minimum points.	**TOK:** Is it possible for an area of knowledge to describe the world without transforming it**?**

SL 5.7	**Solve** optimisation problems in context.	**TOK:** How can the rise in tax for plastic containers, for example plastic bags, plastic bottles etc be justified using optimization**?**

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 6: Calculus (SL only in Year 2)
Section 6B: Integration with Applications **Duration: SL 14 sessions**

IB Topic Essential Understandings: Calculus describes rates of change between two variables and the accumulation of limiting areas. Understanding these rates of change allows us to model, interpret and analyze real-world problems and situations. Calculus helps us understand the behaviour of functions and allows us to interpret the features of their graphs.

IB Content-Specific Conceptual Understandings

Students will understand the relationship between the integral and area and interpret the meaning of this in context.

Calculus is a concise form of communication used to approximate nature.

Numerical integration can be used to approximate areas in the physical world.

The area under a function on a graph has a meaning and has applications in space and time.

Standards-Aligned Objectives: Instruction and assessment will align to the following objectives, with IB command terms in bold:

SL 5.5	**Understand** integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \ldots$, where $n \in \mathbb{Z}, n \neq -1$. **Determine** the constant term using anti-differentiation with a boundary condition. **Determine** the value of definite integrals using technology. **Calculate** the area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.	**TOK:** Is it possible for an area of knowledge to describe the world without transforming it?
SL 5.7	**Solve** optimisation problems in context.	**TOK:** How can the rise in tax for plastic containers, for example plastic bags, plastic bottles etc be justified using optimization?
SL 5.8	**Approximate** areas using the trapezoidal rule.	n/a

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 6: Calculus (SL only in Year 2)
Support Resources / NJSLS Companion Standards / NJSLS College, Career, and Ed Tech Standards

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NJSLS Life Literacies & Key Skills		**6A**	**6B**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze and use creative skills and ideas.	X	X
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.	X	X
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities and utility for accomplishing a specified task	X	X

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 7 Overview: Sampling & Inference **Duration: SL 42 sessions, +HL 58 sessions**

IB Topic Essential Understandings:
Statistics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses and predict the occurrence of events. Statistical representations and measures allow us to represent data in many different forms to aid interpretation.

Probability enables us to quantify the likelihood of events occurring and so evaluate risk. Both statistics and probability provide important representations which enable us to make predictions, valid comparisons and informed decisions. These fields have power and limitations and should be applied with care and critically questioned, in detail, to differentiate between the theoretical and the empirical/observed. Probability theory allows us to make informed choices, to evaluate risk and to make predictions about seemingly random events.

IB Content-Specific Conceptual Understandings

Organizing, representing, analysing and interpreting data, and utilizing different statistical tools facilitates prediction and drawing of conclusions.

Different statistical techniques require justification and the identification of their limitations and validity.

Different probability distributions provide a representation of the relationship between the theory and reality, allowing us to make predictions about what might happen.

AHL
Statistical literacy involves identifying reliability and validity of samples and whole populations in a closed system.

A systematic approach to hypothesis testing allows statistical inferences to be tested for validity.

Section and IB Sub-Topics	SL Duration (Sessions)	HL Duration(Sessions)
A: Sampling Distributions SL 4.7, SL 4.8, SL 4.9, AHL 4.12, AHL 4.14, AHL 4.15, AHL 4.17, AHL 4.18	15	20
B: Inference SL 4.11, AHL 4.12, AHL 4.16, AHL 4.18	27	38

IB Mathematics: Applications & Interpretations, SL & HL, Year 2
Unit 7: Sampling & Inference
Section 7A: Sampling Distributions **Duration: SL 15 sessions, +HL 20 sessions**

IB Topic Essential Understandings:
Statistics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses and predict the occurrence of events. Statistical representations and measures allow us to represent data in many different forms to aid interpretation.

Probability enables us to quantify the likelihood of events occurring and so evaluate risk. Both statistics and probability provide important representations which enable us to make predictions, valid comparisons and informed decisions. These fields have power and limitations and should be applied with care and critically questioned, in detail, to differentiate between the theoretical and the empirical/observed. Probability theory allows us to make informed choices, to evaluate risk and to make predictions about seemingly random events.

IB Content-Specific Conceptual Understandings

Organizing, representing, analysing and interpreting data, and utilizing different statistical tools facilitates prediction and drawing of conclusions.

Different statistical techniques require justification and the identification of their limitations and validity.
Modelling and finding structure in seemingly random events facilitates prediction.

Different probability distributions provide a representation of the relationship between the theory and reality, allowing us to make predictions about what might happen.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

SL 4.7	**Identify** discrete random variables. **Construct** probability distributions for discrete random variables. **Calculate** the Expected value (mean), $E(X)$ for discrete data. **Applications** of discrete random variables and their probability distributions.	**TOK Connections:** What do we mean by a "fair" game? Is it fair that casinos should make a profit?
SL 4.8	**Identify** a binomial distribution. **Verify** that a distribution meets the criteria to be considered binomial. **Calculate** mean and variance of the binomial distribution.	**TOK Connections:** What criteria can we use to decide between different models?
SL 4.9	**Recognize** the normal distribution and curve. **Identify** properties of the normal distribution. **Construct** a diagrammatic representation for data that is normally distributed. **Calculate** mean and standard deviation of Normal probability distribution . **Calculate** and interpret z-scores. **Calculate** and interpret probabilities associated with z-scores.	**TOK Connections:** To what extent can we trust mathematical models such as the normal distribution? How can we know what to include, and what to exclude, in a model?

	Use inverse normal calculations to find z-scores associated with probabilities	
HL 4.14	**Perform** linear transformation of a single random variable. **Calculate** and **interpret** the expected value of linear combinations of n random variables. **Calculate** the **interpret** variance and standard deviation of linear combinations of n independent random variables. **Recognize** $\overline{x}$ as an unbiased estimate of μ. **Recognize** s_{n-1}^2 as an unbiased estimate of σ^2.	**TOK Connections:** Mathematics and the world: In the absence of knowing the value of a parameter, will an unbiased estimator always be better than a biased one?
HL 4.15	**Recognize** a linear combination of n independent normal random variables is normally distributed. In particular, $X \sim N(\mu, \sigma^2) \Rightarrow \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$. **Demonstrate** understanding of central limit theorem.	**TOK Connections:** The central limit theorem can be proved mathematically (formalism), but its truth can be confirmed by its applications (empiricism). What does this suggest about the nature and methods of mathematics?
HL 4.18	**Determine** critical values and critical regions. Test for population mean for normal distribution. Test for proportion using binomial distribution. Test for population mean using Poisson distribution.	**TOK Connections:** Mathematics and the world. In practical terms, is saying that a result is significant the same as saying that it is true? Mathematics and the world. Does the ability to test only certain parameters in a populations affect the way knowledge claims in the human sciences are valued? When is it more important not to make a Type I error and when is it more important not to make a Type II error?

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 7: Sampling & Inference
Section 7B: Inference **Duration: SL 27 sessions, +HL 38 sessions**

IB Topic Essential Understandings:
Statistics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses and predict the occurrence of events. Statistical representations and measures allow us to represent data in many different forms to aid interpretation.

Probability enables us to quantify the likelihood of events occurring and so evaluate risk. Both statistics and probability provide important representations which enable us to make predictions, valid comparisons and informed decisions. These fields have power and limitations and should be applied with care and critically questioned, in detail, to differentiate between the theoretical and the empirical/observed. Probability theory allows us to make informed choices, to evaluate risk and to make predictions about seemingly random events.

IB Content-Specific Conceptual Understandings

Different probability distributions provide a representation of the relationship between the theory and reality, allowing us to make predictions about what might happen.

AHL
Statistical literacy involves identifying reliability and validity of samples and whole populations in a closed system.

A systematic approach to hypothesis testing allows statistical inferences to be tested for validity.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

SL 4.11	**Formulate** null and alternative hypotheses, H_0 and H_1. **Understand** and **apply** significance levels, p-values, expected and observed frequencies. **Perform** hypothesis test and interpret the results.	**TOK Connections:** Why have some research journals "banned" p-values from their articles because they deem them too misleading? In practical terms, is saying that a result is significant the same as saying it is true? How is the term "significant" used differently in different areas of knowledge?
SL 4.11	**Perform** and **interpret** results of the $\chi 2$ test for independence. **Utilize** contingency tables, degrees of freedom, critical value. **Perform** and interpret results of the $\chi 2$ goodness of fit test. Know the limitations of $\chi 2$ tests. The t -test. Use of the p -value to compare the means of two populations. Using one-tailed and two-tailed tests.	
HL 4.12	**Design** valid data collection methods, such as surveys and questionnaires. **Select** relevant variables from many variables. **Choose** relevant and appropriate data to analyse.	**TOK Connections:** What are the strengths and limitations of different methods of data collection, such as questionnaires?

	Categorize numerical data in a $\chi 2$ table and justifying the choice of categorisation. **Choose** an appropriate number of degrees of freedom when estimating parameters from data when carrying out the $\chi 2$ goodness of fit test. Definition of reliability and validity. Reliability tests. Validity tests.	
HL 4.16	**Construct** confidence intervals for the mean of a normal population.	**TOK Connections:** Mathematics and the world. Claiming brand A is "better" on average than brand B can mean very little if there is a large overlap between the confidence intervals of the two means.
HL 4.18	**Determine** critical values and critical regions. Test for population mean for normal distribution. Test for proportion using binomial distribution. Test for population mean using Poisson distribution. **Use** technology to test the hypothesis that the population product moment correlation coefficient (ρ) is 0 for bivariate normal distributions. **Calculate** and **interpret** the probabilities Type I and II errors.	**TOK Connections:** Mathematics and the world. In practical terms, is saying that a result is significant the same as saying that it is true? Mathematics and the world. Does the ability to test only certain parameters in a population affect the way knowledge claims in the human sciences are valued? When is it more important not to make a Type I error and when is it more important not to make a Type II error?

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 7: Sampling & Inference
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NJSLS Life Literacies & Key Skills		**7A**	**7B**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze and use creative skills and ideas.	X	X
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.	X	X
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities and utility for accomplishing a specified task	X	X
9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.	X	X
9.4.12.IML.3	Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions.	X	X

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 8 Overview: Graph Theory **Duration: SL 7 sessions, +HL 10 sessions**

IB Topic Essential Understandings:
Geometry and trigonometry allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This branch provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

In two dimensions, the Voronoi diagram allows us to navigate, path-find or establish an optimum position.

AHL
Graph theory algorithms allow us to represent networks and to model complex real-world problems.

Section and IB Sub-Topics	SL Duration (Sessions)	HL Duration(Sessions)
A: Graph Theory SL 3.6, AHL 3.14, AHL 3.15, AHL 3.16l	7	+10

IB Mathematics: Applications & Interpretations, SL & HL, Year 2
Unit 8 : Graph Theory
Section 8A: Graph Theory **Duration: SL 7 sessions, +HL 10 sessions**

IB Topic Essential Understandings:
Geometry and trigonometry allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This branch provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

IB Content-Specific Conceptual Understandings

In two dimensions, the Voronoi diagram allows us to navigate, path-find or establish an optimum position.

AHL
Graph theory algorithms allow us to represent networks and to model complex real-world problems.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

SL 3.6	**Construct** and **interpret** Voronoi diagrams identifying sites, vertices, edges, cells. **Apply** nearest neighbour algorithm. **Add** a site to an existing Voronoi diagram and interpret in the context of the problem. **Apply** Voronoi diagrams to real world problems such as the "toxic waste dump" problem.	**TOK:** Is the division of knowledge into disciplines or areas of knowledge artificial?
AHL 3.14	**Understand and apply** the terminology of graph theory: vertices, edges, walk, connected, adjacent vertices, adjacent edges, degree of a vertex. **Construct** a graph from given information. **Apply** graph theory algorithms to represent networks and model real world problems. **Identify, interpret, apply** or **construct** simple graphs; complete graphs; unweighted and weighted graphs. **Identify, interpret, apply** or **construct** directed graphs; in degree and out degree of a directed graph; subgraphs; trees.	**TOK:** Mathematics and knowledge claims. Proof of the four-colour theorem. If a theorem is proved by computer, how can we claim to know that it is true?
AHL 3.15	**Find** the adjacency matrix or a given graph. **Identify** the number of walks given an adjacency matrix.(Number of k -length walks (or less than k -length walks) between two vertices.) **Construct** and interpret weighted adjacency tables. **Construct** the transition matrix for a strongly-connected, undirected or directed graph.	n/a
AHL 3.16	**Utilize** tree and cycle algorithms with undirected graphs; **identify** walks, trails, paths, circuits, cycles.	**TOK:** What practical problems can or does mathematics try to solve? Why are problems

	Apply: Eulerian trails and circuits, Hamiltonian paths and cycles, Minimum spanning tree (MST) graph algorithms: Kruskal's and Prim's algorithms for finding minimum spanning trees. **Solve:** ● Chinese postman problem and algorithm for solution, to determine the shortest route around a weighted graph with up to four odd vertices, going along each edge at least once. ● Travelling salesman problem to determine the Hamiltonian cycle of least weight in a weighted complete graph. **Apply** nearest neighbour algorithm for determining an upper bound for the travelling salesman problem. **Apply** deleted vertex algorithm for determining a lower bound for the travelling salesman problem.	such as the travelling salesman problem so enduring? What does it mean to say the travelling salesman problem is "NP hard"?

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 8: Graph Theory
Support Resources / NJSLS Companion Standards / NJSLS College, Career, and Ed Tech Standards

Support Resources

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- Appendix C: Interdisciplinary Connections

NJSLS Life Literacies & Key Skills		**8A**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze and use creative skills and ideas.	x
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.	x
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.	x
9.4.12.CT.4	Enlist input from a variety of stakeholders (e.g., community members, experts in the field) to design a service learning activity that addresses a local or global issue (e.g., environmental justice).	x
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.	x

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 9 Overview: Complex Numbers (HL only) **Duration: +HL 10 sessions**

IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables to solve mathematical problems.

IB Content-Specific Conceptual Understandings

AHL
Different representations of numbers enable quantities to be compared and used for computational purposes with ease and accuracy.

Numbers and formulae can appear in different, but equivalent forms, or representations, which can help us to establish identities.

Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples

Utilizing complex numbers provides a system to efficiently simplify and solve problems.

Representing abstract quantities using complex numbers in different forms enables the solution of real-life problems.

Section and IB Sub-Topics	SL Duration (Sessions)	HL Duration(Sessions)
A: Complex Numbers AHL 1.12, AHL 1.13	n/a	5

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 9: Complex Numbers (HL only)
Section 9A: Complex Numbers **Duration: HL 5 sessions**

IB Topic Essential Understandings: Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables to solve mathematical problems.

IB Content-Specific Conceptual Understandings

AHL
Different representations of numbers enable quantities to be compared and used for computational purposes with ease and accuracy.

Numbers and formulae can appear in different, but equivalent forms, or representations, which can help us to establish identities.

Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples

Utilizing complex numbers provides a system to efficiently simplify and solve problems.

Representing abstract quantities using complex numbers in different forms enables the solution of real-life problems.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

AHL 1.12	**Identify** the number i such that $i^2 = -1$, and every complex number has the form $z = a + bi$ **Demonstrate** understanding of the terms real part, imaginary part, conjugate, modulus and argument. **Calculate** sums, differences, products, quotients, by hand and with technology. **Calculate** powers of complex numbers, in Cartesian form, with technology. **Represent** complex numbers on the Argand diagram (complex plane). **Recognize** complex numbers as solutions to quadratic equations of the form $ax^2 + bx + c = 0$, $a \neq 0$, with real coefficients where $b^2 - 4ac < 0$.	**TOK Connections:** How does language shape knowledge? For example do the words "imaginary" and "complex" make the concepts more difficult than if they had different names?
AHL 1.13	**Write** the Modulus argument (polar) form: $z = r(\cos\theta + i\sin\theta) = \text{rcis}\theta$. **Write** Exponential form: $z = re^{i\theta}$ (Euler form). **Convert** between Cartesian, polar and exponential (Euler) forms, by hand and with technology. **Calculate** products, quotients and integer powers in polar or exponential forms. **Add** sinusoidal functions with the same frequencies but different phase shift angles. **Interpret** complex numbers geometrically: addition and subtraction of complex numbers can be represented as vector addition and subtraction; multiplication of complex numbers can be represented as a rotation and a stretch in the Argand diagram.	**TOK Connections:** Why might it be said that $e^{i\pi}+1=0$ is beautiful? What is the place of beauty and elegance in mathematics? What about the place of creativity?

IB Mathematics: Applications & Interpretation SL & HL, Year 2
Unit 9: Complex Numbers (HL only)
Support Resources / NJSLS Companion Standards / NJSLS College, Career, and Ed Tech Standards

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NJSLS Life Literacies & Key Skills		**9A**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze and use creative skills and ideas.	X
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.	X

IB Mathematics: Applications & Interpretations, HL, Year 2
Unit 10: Vectors and Differential Equations **Duration: HL 35 sessions**

IB Topic Essential Understandings: Geometry and trigonometry allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This branch provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships**.**

IB Content-Specific Conceptual Understandings

HL
Different measurement systems can be used for angles to facilitate ease of calculation.

Vectors allow us to determine position, change of position (movement) and force in two and three-dimensional space.

Kinematics allows us to describe the motion and direction of objects in closed systems in terms of displacement, velocity and acceleration.

Many physical phenomena can be modelled using differential equations and analytic and numeric methods can be used to calculate optimum quantities.

Phase portraits enable us to visualize the behavior of dynamic systems.

Section and IB Sub-Topics	**SL Duration**	**HL Duration (Sessions)**
A: Vectors HL 3.10, 3.11, 3.12, 3.13, 1.13, 1.15, 3.15,	NA	23
B: Differential Equations HL 5.14, 5.15, 5.16, 5.17, 5.18	NA	12

IB Mathematics: Applications & Interpretations, HL, Year 2
Unit 10 : Vectors and Differential Equations
Section 10A: Vectors **Duration (Sessions): HL: 23**

IB Topic Essential Understandings: Geometry and trigonometry allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This branch provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships**.**

IB Content-Specific Conceptual Understandings

HL
Different measurement systems can be used for angles to facilitate ease of calculation.

Vectors allow us to determine position, change of position (movement) and force in two and three-dimensional space.

Kinematics allows us to describe the motion and direction of objects in closed systems in terms of displacement, velocity and acceleration.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

HL 3.10	**Understand** the concept of a vector and a scalar. **Represen**t vectors using directed line segments. **Identify** unit vectors; base vectors $i,\ j\ and\ k$. **Identify** components of a vector; utilize column representation: $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$ **Identify** the zero vector 0, the vector $-\ v$. **Determine** position vectors $\overrightarrow{\mathrm{OA}} = a.$ **Rescale** and normalize vectors.	**TOK:** Vectors are used to solve many problems in position location. This can be used to save a lost sailor or destroy a building with a laser-guided bomb. To what extent does possession of knowledge carry with it an ethical obligation?
HL 3.11	**Write** the vector equation of a line in two and three dimensions: $r\ =\ a\ +\ \lambda b$, where $b$is a direction vector of the line.	**TOK:** Mathematics and the knower: Why are symbolic representations of three-dimensional objects easier to deal with than visual representations? What does this tell us about our knowledge of mathematics in other dimensions?
HL 3.12	**Apply** vectors to kinematics. **Mode**l linear motion with constant velocity in two and three dimensions. **Model** motion with variable velocity in two dimensions.	**n/a**
HL 3.13	**Define** and **calculate** the scalar product of two vectors.	**TOK:** What counts as understanding in mathematics? Is it more than just getting the right answer?

	Determine the measure of angle between two vectors; the acute angle between two lines. **Define** and **calculate** the vector product of two vectors. **Interpret** geometrically: $\|v \times w\|$. **Identify** components of vectors: ● Vector *a* acting in the direction of vector *b* is: $\frac{a \cdot b}{\|b\|} = \|a\| \cos\theta$. ● Vector *a* acting perpendicular to vector *b*, in the plane formed by the two vectors, is: $\frac{\|a \times b\|}{\|b\|} = \|a\| \sin\theta$.	
HL 1.13 (Link to unit 9)	**Interpret** complex numbers geometrically.	
HL 1.15	**Define** eigenvalues and eigenvectors geometrically: an eigenvector, corresponding to a real nonzero eigenvalue, points in a direction in which it is stretched by the transformation and the eigenvalue is the factor by which it is stretched. If the eigenvalue is negative, the direction is reversed. **Calculate** eigenvalues and eigenvectors. **Identify** the characteristic polynomial of square matrices. **Perform** diagonalization of 2×2 matrices (restricted to the case where there are distinct real eigenvalues).	
HL 3.15	**Identify** and **utilize** applications to powers of 2×2 matrices.	**n/a**

IB Mathematics: Applications & Interpretations HL, Year 2
Unit 10: Vectors and Differential Equations
Section 10B: Differential Equations **Duration (Sessions): HL: 12**

IB Topic Essential Understandings: Geometry and trigonometry allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This branch provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships**.**

IB Content-Specific Conceptual Understandings

HL
Different measurement systems can be used for angles to facilitate ease of calculation.

Many physical phenomena can be modelled using differential equations and analytic and numeric methods can be used to calculate optimum quantities.

Phase portraits enable us to visualize the behavior of dynamic systems.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

HL 5.14	**Model** differential equation(s) from a context. **Solve** differential equations by separation of variables.	n/a
HL 5.15	**Construct** slope fields and their diagrams.	n/a
HL 5.16	**Use** Euler's method for finding the approximate solution to first order differential equations. **Determine** the numerical solution of dy/dx = f(x ,y). **Determine** the numerical solution of the coupled system dx/dt = f_1(x, y, t) and dy/dt= f_2(x, y, t).	n/a
HL 5.17	**Utilize** phase portrait for the solutions of coupled differential equations of the form: dx/dt= ax + by; dy/dt= cx + dy. **Apply** qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues. **Sketch** trajectories and use phase portraits to identify key features such as equilibrium points, stable populations and saddle points.	n/a
HL 5.18	**Calculate** solutions of $\frac{d^2x}{dt^2} = f(x, \frac{dx}{dt}, t)$ by Euler's method.	n/a

IB Mathematics: Applications & Interpretations HL, Year 2
Unit 10: Vectors and Differential Equations
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NJSLS Life Literacies & Key Skills		**10A**	**10B**
9.4.12.CT.2:	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.	x	x
9.4.12.TL.1:	Assess digital tools based on features such as accessibility options, capacities and utility for accomplishing a specified task		x
9.4.12.TL.4:	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.		x