STRAND 4: GEOMETRY
Scale Drawing

Introduction to Scale Drawing

The Scale

- Scale is a ratio that represents the relationship between the dimensions of a model and the corresponding dimensions on the actual figure or object.

The figure below shows the relative positions of Mombasa, Nairobi and Nakuru. The distance between Mombasa and Nairobi on a straight line is 450 km and that between Nairobi and Nakuru is 142 km.

(i). Measure in centimetres the distance between:

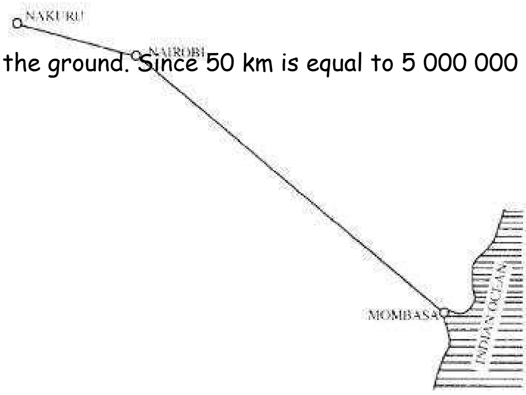
42. Mombasa and Nairobi.

43. Nairobi and Nakuru.

(ii). How many kilometres does 1 cm represent between:

20. Mombasa and Nairobi?

21. Nairobi and Nakuru?



You should notice that I cm on the map represents 50 km on the ground. Since 50 km is equal to 5 000 000 cm, this statement can be written in ratio form as 1: 5 000 000.

As a representative fraction (R.F.), 1: 5000 000 is written as $\frac{1}{5,000,000}$

- The ratio of the distance on a map to the actual distance on the ground is called ***the scale of the map.***

Example;

(1). The scale of a map is given in a statement as '1 cm represents 4 km.' Convert this ratio form.

Solution

1 cm represents 4 x 100 000 cm

Therefore, the ratio is 1: 400 000 and the R.F. is $\frac{1}{400{,}000}$

(2). The scale of a map is given as 1: 250 000. Write this as a statement.

Solution

35. *250 000 means 1 cm on the map represents 250 000 cm on the ground. km, i.e., 1 cm represents 2.5 km. Therefore, 1 cm represents* $\frac{250{,}000}{100{,}000}$

<u>EXCERCISE</u>

(1). On a map, 1 cm represents 4 kilometres. Re-write as a ratio.

(2). What distance on the ground is represented by 3.7 cm on the map if 1 cm represents 4 kilometres?

(3). Two towns A and B are 42.8 km apart on the ground. What is this distance on the map given that 1 cm represents 1.5 kilometres?

(4). A map is drawn to a scale of 1:50 000. Write this scale as a statement connecting map distance to ground distance.

(5). What is the actual distance if the distance on the map is 12.7 cm given that 1 cm represents 2.5 kilometres?

(6). On a map, 1 cm represents 5 kilometres. A railway line measures 8.3 km. What is its length on the map?

(7). The scale of a map is given in a statement as '1 cm represents 4.5 km.' Convert this to a representative fraction (R.F.).

(8). The scale of a map is given as 1: 350 000. Write this as a statement.

(9). A map is drawn to a scale of 1:500 000. What is the actual distance if the distance on the map is 14.7 cm?

(10). What is the distance on the map of Two towns P and Q which are 62.8 km apart on the ground given that 1 cm represents 2.5 kilometres?

Scale Diagrams

<u>Scale Diagrams.</u>

- A scale drawing is an enlargement of an object.
- An enlargement changes the sizes of an object by multiplying each of the lengths by a scale factor to make it larger or smaller.
- It is usually stated as a ratio.

Note:

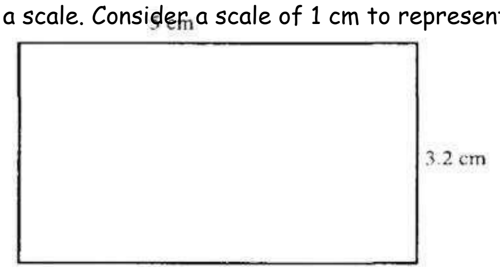
- One should be careful in choosing the right scale, so that the drawing fits on the paper without much detail being lost.

Example.

1. The length of a classroom is 10 metres and its width 6.4 metres. By scale drawing, represent this on a figure.

Solution

First look for a scale. Consider a scale of 1 cm to represent 2 m. Hence, the classroom will be 10/2 = 5 cm by 6.4/2



= 3.2 cm.

<u>Assignment</u>

(1). The length of a classroom is 10 metres and its width 6.4 metres. By scale drawing, represent this on a figure.

(2). A plot of land in form of a rectangle has dimensions 120 m by 180 m. Draw this on a paper.

(3). A rectangular field measures 40 m by 100 m. The length of the field on the map is 5 cm. What is the area of the field on the map?

(4). A rectangular field measures 40 m by 100 m. The length of the field on the map is 5 cm. Write the scale of the map as a representative fraction and hence the width of the field on the map?

(5). A plot of land in form of a rectangle measures 100 m by 80 m. Draw this on a paper.

(6). Two villages M and N are connected by a straight road 750 m long on a level ground. A third village L is 450 m from M and 650 m from N. Using a suitable scale, draw the diagram and find the shortest distance of L from the road.

(7). The length of a rectangular field on the map is 4 cm, If the rectangular field measures 50 m by 120 m write the scale of the map as a representative fraction and hence the width of the field on the map?

(8). Represent the dimensions of a plot of land in form of a rectangle measuring 150 m by 70 m. on a paper.

(9). Two Towns S and T are connected by a straight road 650 m long on a level ground. A third village R is 350 m from S and 550 m from T. Using a suitable scale, draw the diagram and find the shortest distance of R from the road.

(10). Using a suitable scale, draw the diagram of a rectangular plot of land measuring 6 km by 3 km.

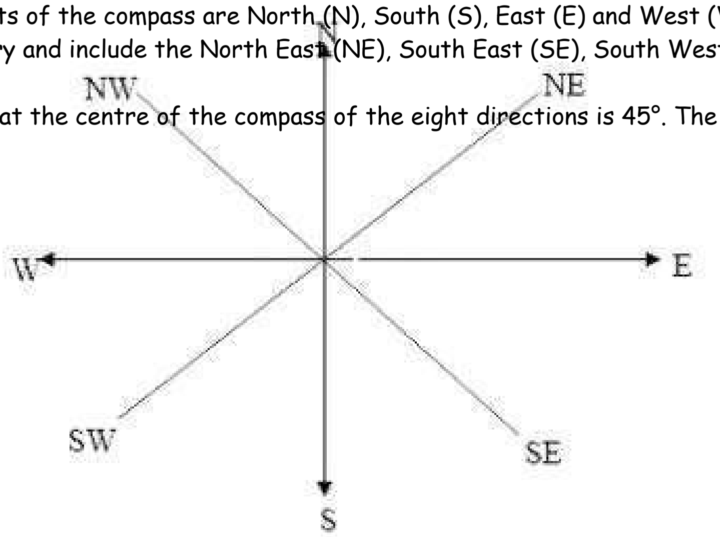
Compass Bearing

Bearing and Distance.

- **Bearing** is the angle measured in a clockwise or counterclockwise direction from the north direction or South direction to a given direction.
- **Distance** is the length between the two places. Bearing and distance can be used to locate the position of a given point from another reference point.

Points of the compass

- The figure below shows the eight points of the compass.
- The four main points of the compass are North (N), South (S), East (E) and West (W). The other four points shown are secondary and include the North East (NE), South East (SE), South West (SW) and North West (NW).
  Each angle formed at the centre of the compass of the eight directions is 45°. The angle between N and E is 90°.



Compass Bearing

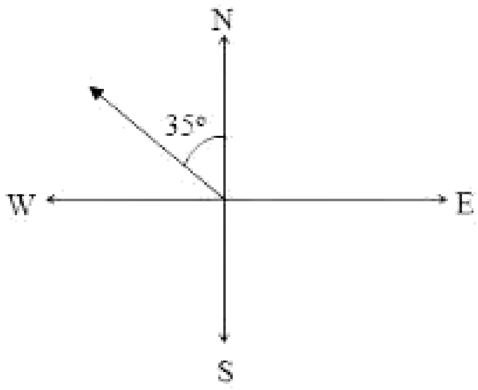
- It is measured in either clockwise direction or anti-clockwise direction from North or South and the angle is an acute angle. E.g. N 45° W, S 60° E

Example;

Draw a sketch to show the bearing marking the angle N35°W clearly.

Solution

N35°W means from N measure 35° toward the W or 35°W of N.



Assignment

(1). Three boys Isaac, Alex and Ken are standing in different parts of a football field. Isaac is 100 metres north of Alex and Ken is 120 metres east of Alex. Find the Compass bearing of Ken from Isaac.

(2). Kilo school is 12 kilometres from Sokomoko on a bearing of N 50°W. Tiba dispensary is 10 kilometres from Kilo on a bearing of S 60°E. Find the compass bearing of Sokomoko from Tiba.

(3). Survey posts R, Q and P are situated such that they form a triangle. If Q is on a bearing of S 60° W and 12 kilometres away and R is on a bearing of S 30°E and 8 kilometres away from P, find the compass bearing of Q from R.

(4). Kisumu and Nanyuki are situated in such a way that Nanyuki is on a bearing of N 75° E from Nakuru and Kisumu on a bearing of N 80° W from Nakuru. If Kisumu is 190 km and Nanyuki is 160 km from Nakuru,

Find:

a. The compass bearing of Kisumu from Nanyuki.

b. The distance of Kisumu from Nanyuki.

(5). From a meteorological weather station P on a plateau, a hill Q is 5 km on a bearing N 78° E and a railway station, R, is 1.5 km away on a bearing S20°W. Use scale drawing to find the compass bearing of Q from the railway station.

(6). A town P is 200 km West of Q. Town R is at a distance of 80 km on a bearing of N490E from P. Town S is due East of R and due North of Q. Determine the compass bearing of S from P.

(7). A route for safari rally has five sections AB, BC, CD, DE and EA. B is S 20° W km on a bearing N 50° E from A.C is 500km from B. The bearing of B from C is N 60° W. D is 400km on a bearing S 50° W from C. E is 250km on a bearing N 25° E from D. Using the scale 1cm for 50km draw the diagram representing the route for the rally. From the diagram determine:

a. The distance in km of A from E

b. The compass bearing of E from A.

(8). Manyatta village is 74 km North West of Nyangata village. Chamwe village is 42 km west of Nyangate. By using an appropriate scale drawing, find the compass bearing of Chamwe from Manyatta.

(9). Four towns R, T K and G are such that T is 84 km directly to the north of R, and K is on a distance of N 65° W from R at a distance of 60 km. G is on a bearing of N 20° W from K and a distance of 30 km. Using a scale of 1cm to represent 10km, make an accurate scale drawing to show the relative positions of the towns. Find the distance and the compass bearing of G from T.

(10). Using the scale: 1 cm represents 10 km, construct a diagram showing the positions of B, C, Q and D. Determines the distance between B and C and the compass bearing of D from B.

True Bearing

Bearing and Distance.

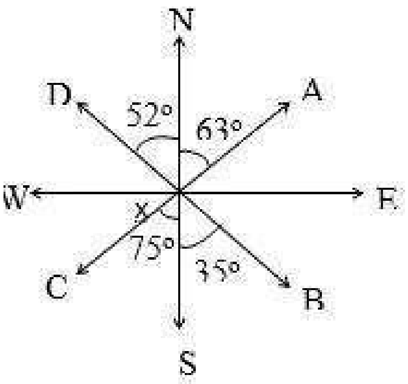
True Bearing.

- It is also known as***Three-figure bearings***.
- True bearing is given as the number of degrees from north, measured in a clockwise direction.
- Three-digit are always used for given angles but for angles less than 100° extra zero(s) is always added in front of the digits e.g. 008° , 060°, 070° up to 099°.
- The true bearings of due north is given as 000°. Due South East as 135° and due North West as 315°, etc. In figure 22.7, the true bearing of Q from P is 138°.

Example.

Find the three-figure bearings of A, B, C, and D from X.

Solution



(1). The arrow N shows the direction N, NXA = 63°. the bearing of A from X is 063°

(2). NXB = 180 - 35 = 145°. The bearing of B from X is 145°

(3). NXC clockwise = 180 + 75 = 255°. The bearing of C from X is 255° 4. NXD clockwise = 360 - 52 = 308°. The bearing of D from X is 308°.

Assignment

(1). A coastguard at a port observes two steamships approaching the harbour. The first ship P appears on a bearing 100° and the second ship Q on a bearing 020°. If the guard estimates the distances of the ships to be 120 km and 80 km respectively, find:

(a) the distance between ships P and Q

(b) the bearing of Q from P.

(2). A prison guard on a watchtower sees a bridge 120 m away on a bearing of 230° and a bus stop 80 m away on a bearing of 090°. Use scale drawing to find:

36. The bearing of the bridge from the bus stop.

37. The distance between the bus stop and the bridge

(3). From a point P, the bearing of a house is 060°. From a point Q 100m due east of P, the bearing is 330°. Draw a labelled sketch to show the positions of P, Q and the house.

(4). Three boys Isaac, Alex and Ken are standing in different parts of a football field. Isaac is 100 metres north of Alex and Ken is 120 metres east of Alex. Find the True bearing of Ken from Isaac.

(5). Kilo school is 12 kilometres from Sokomoko on a bearing of 320°. Tiba dispensary is 10 kilometres from Kilo on a bearing of 120°. Find the True bearing of Sokomoko from Tiba.

(6). Survey posts R, Q and P are situated such that they form a triangle. If Q is on a bearing of 210° and 12 kilometres away and R is on a bearing of 150° and 8 kilometres away from P, find the true bearing of Q from R.

(7). Kisumu and Nanyuki are situated in such a way that Nanyuki is on a bearing of 075° from Nakuru and Kisumu on a bearing of 280° from Nakuru. If Kisumu is 190 km and Nanyuki is 160 km from Nakuru, find:

49. The bearing of Kisumu from Nanyuki.

50. The distance of Kisumu from Nanyuki.

(8). Town A is on a bearing 050° from town C. Town B is on a bearing 020° from C. If B is 500 km from C and A is 500 km from B, find by scale drawing:

51. The distance of A from C.

52. The bearing of B form A.

(9). A route for safari rally has five sections AB, BC, CD, DE and EA. B is 200 km on a bearing 050° from A.C is 500km from B. The bearing of B from C is 300° . D is 400km on a bearing 230° from C. E is 250km on a bearing 025° from D.

Using the scale 1cm for 50km draw the diagram representing the route for the rally. From the diagram determine

a. The distance in km of A from E

b. The bearing of E from A

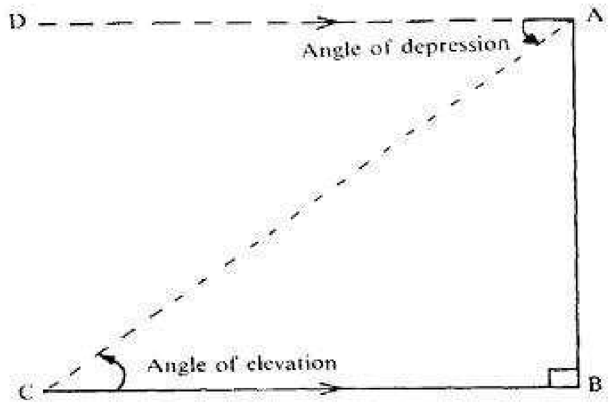
(10). A town P is 200 km West of Q. Town R is at a distance of 80 km on a bearing of 049° from P. Town S is due East of R and due North of Q. Determine the bearing of S from P.

Angle of Elevation and Depression

Angle of Elevation and the Angle of Depression.

- The angle of elevation is the angle between the horizontal line of sight and the line of sight up to an object.
- The angle of depression is the angle between the horizontal line of sight and the line of sight down to an object.

- The angle of elevation is equal to the angle of depression.



- Angles of depression and elevation can be measured by use of an instrument called clinometer. ▪ A simple clinometer can be made from a cardboard in the shape of a protractor.

N/B:

- The angle of elevation increases as the observer moves towards the object and decreases as the observer moves away from the object.

Example.

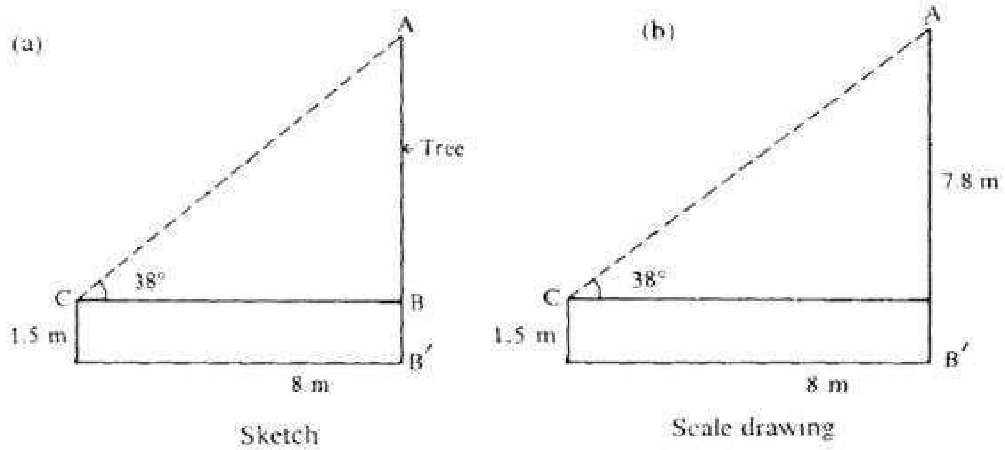
A boy 1.5 m tall and 8 m from a tree finds that the angle of elevation to the top of the tree is 38°. Find the height of the tree by scale drawing.

Solution

The figure below shows the sketch and scale drawing.

Using a scale of 1 cm to 2 m. The measurement of AB' on the scale is 3.9 cm.

(1). cm represents 2 m, therefore 3.9 cm represents (3.9 x 2) m = 7.8 m.



Assignment

(1). A boy 1.5 m tall and 8 m from a tree finds that the angle of elevation to the top of the tree is 38°. Find the height of the tree by scale drawing.

(2). The angle of elevation of the top of a flagpole from a point A, 14 metres away, is 36°. Use scale drawing to find the height of the flagpole.

(3). The angle of depression from the top of a cliff to a stationary boat is 48°. Find the horizontal distance by accurate drawing if the height of the cliff is 80 metres. Measure the angle of elevation of the top of the cliff from the boat. What to you notice?

(4). A building tower casts a shadow 33 metres away. The angle of elevation of the tower from the tip of the shadow is 21°. Find the height of the tower by scale drawing.

(5). The angle of elevation of a church tower from a point A, 50 metres away from the foot of the church, is 24°. Find the distance between A and B if the angle of elevation of the tower from B is 20°.

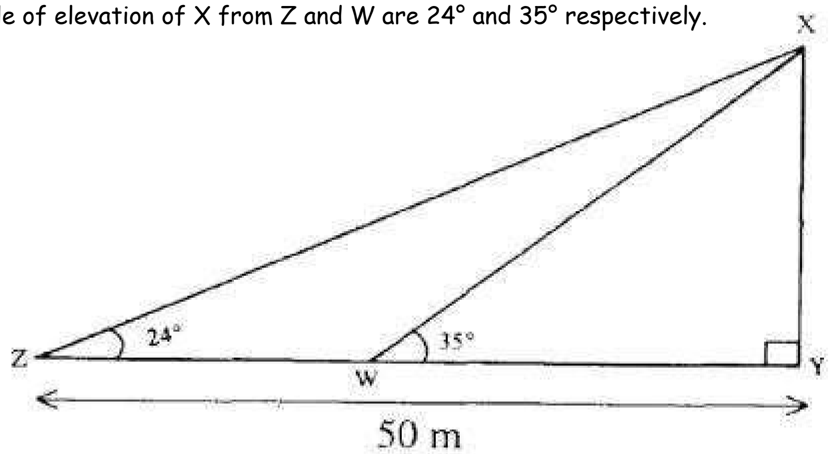
(6). From a viewing tower 15 metres above the ground, the angle of depression of an object on the ground is 30° and the angle of elevation of an aircraft vertically above the object is 42°. By choosing a suitable scale, find the height of the aircraft above the object.

(7). A soldier standing on top of a cliff 100 m high notices two enemy boats in line, whose angles of depression are 10 and 23°. Find, by scale drawing. the distance between the boats.

(8). The angles of elevation of an aircraft from two villages A and B 1 km apart are 67° and 53° respectively. Find the height of the aircraft in metres by scale drawing.

(9). The angle of elevation of a stationary hot air balloon 50 m above the ground from a man on the ground is 17°. The balloon moves vertically upwards so that the angle of elevation from the man is 30°. Find, by scale drawing, the distance the balloon moves. How far above the ground is the balloon?

(10). In figure below, Z is 50 m away from Y. Use scale drawing to find the distance from W to Y, given that the angle of elevation of X from Z and W are 24° and 35° respectively.



Survey

<u>Simple Survey Techniques.</u>

- Surveying an area of land involves taking field measurements of the area so that a map of the area can be drawn to scale. Pieces of land are usually surveyed in order to fix boundaries (land adjudication) of land for different owners, for town planning, road construction, water supplies, mineral development, etc.
- There are two simple methods used in surveying.

53. Triangulation.
54. Survey of an area by use of Compass Bearing and Distances.

(a). Triangulation.

- This is a method in which the area to be surveyed is divided into convenient geometrical figures, or is covered by a suitable geometrical figure.

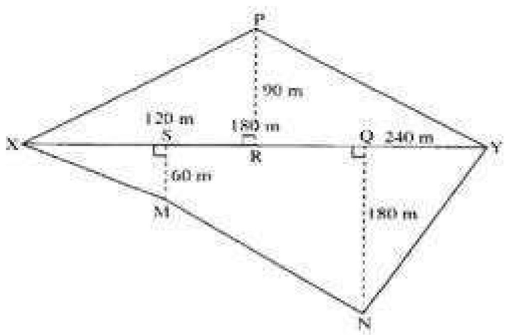
(b). Survey of an area by use of Compass Bearing and Distances.

- In this survey method, only bearings and distances from a chosen point are considered.

In this chapter, we will narrow down into Triangulation into details.

Example.

(1). A survey of a small island was drawn as shown below. Record the measurements shown in the figure in a tabular form in a field book.



Solution

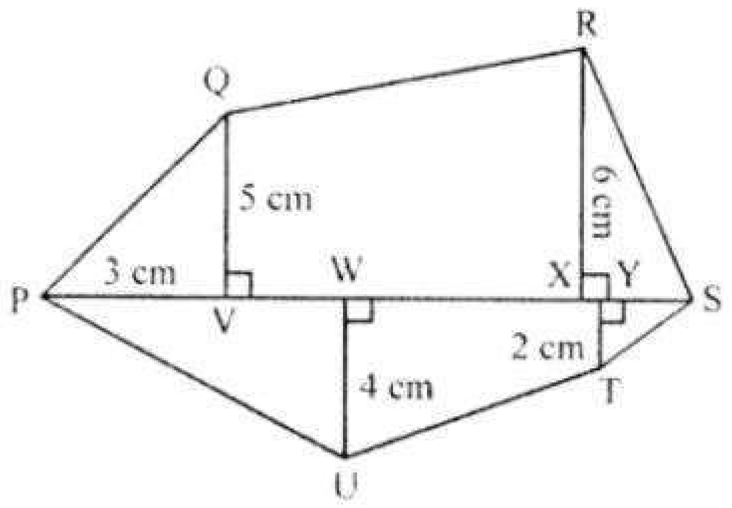
The line XY is called base lines of the survey. The lines perpendicular to the base lines and joining the points M, N, and P at the edge S, R and Q are called offsets.

The measurements are recorded in a field book in a tabular form as follows:

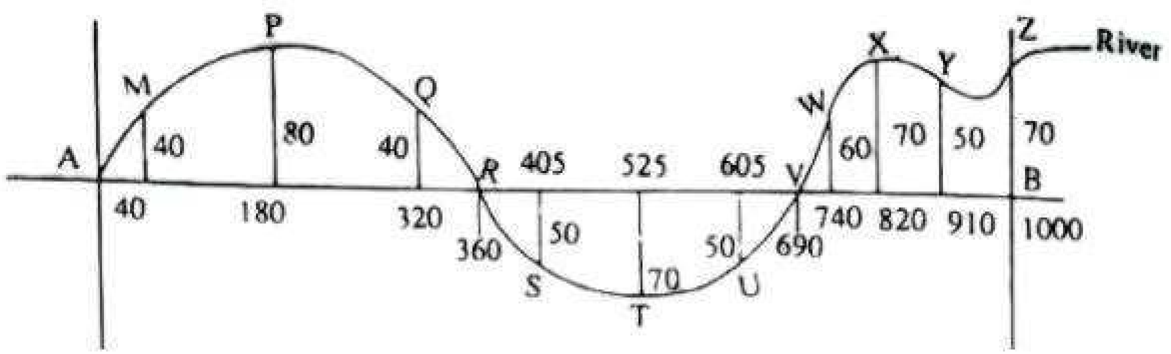
	Y	
	240	**180 to N**
To R90	**180**	
	120	**60 to M**
	X	

Assignment

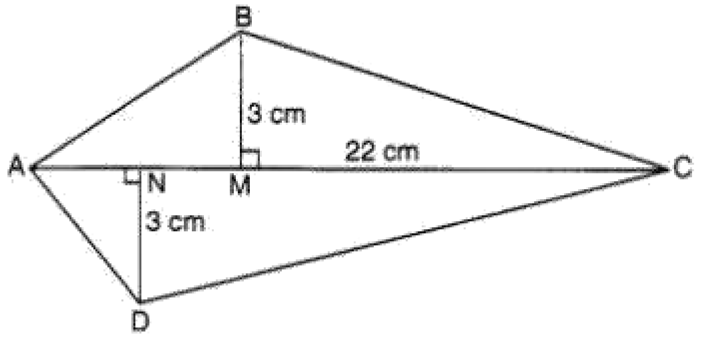
(1). A survey of a small island was made by using a triangle PQRSTU as shown in the sketch of figure below. Record the measurements shown in the figure in a tabular form in a field book.



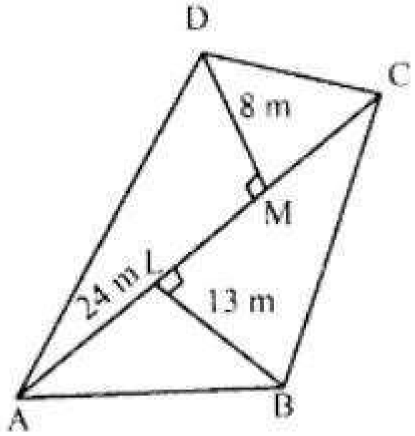
(2). A river can also be surveyed and its map made using base line AB as shown in the sketch of figure below. Record the measurements from the sketch in a field book.



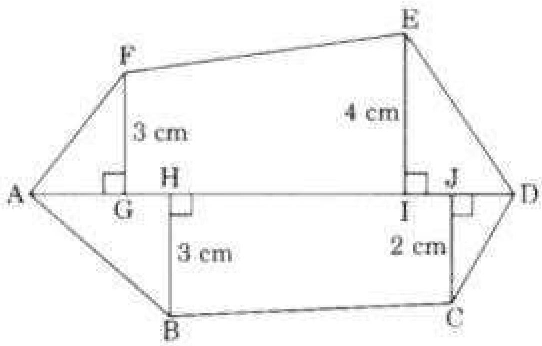
(3). A plot of land is draw as shown below. Record the measurements from the plot in a field book given that AN = NM = 2 cm. Use a scale of 1cm rep 25 m.



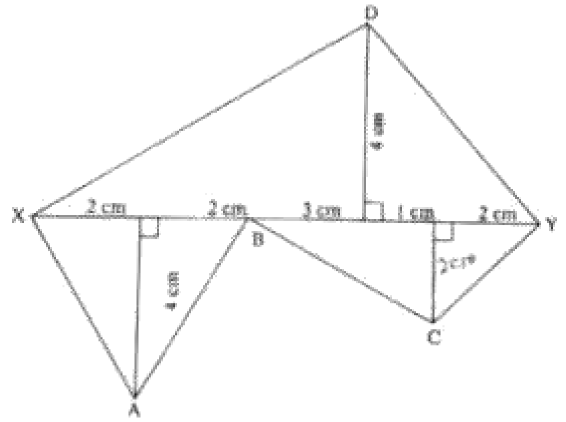
(4). A piece of land is drawn with a base line AC as shown below. Record the measurements from the plot in a field book given that LM = 12 m MC = 20 m.



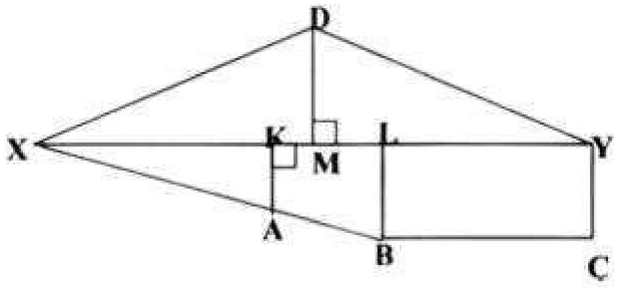
(5). A survey of a small island was drawn as shown below. Record the measurements shown in the figure in a tabular form in a field book given that baseline AD = 10 cm and AG = JD = 2.5 cm. Use a scale of 1cm rep 45 m.



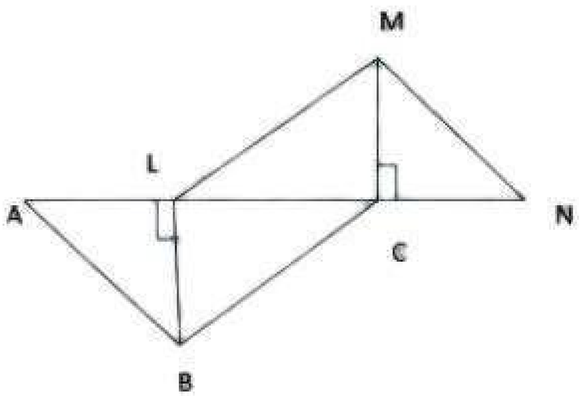
(6). Record the measurements shown in the figure in a tabular form in a field book for a survey of a small piece of land. Use a scale of 1cm rep 60m.



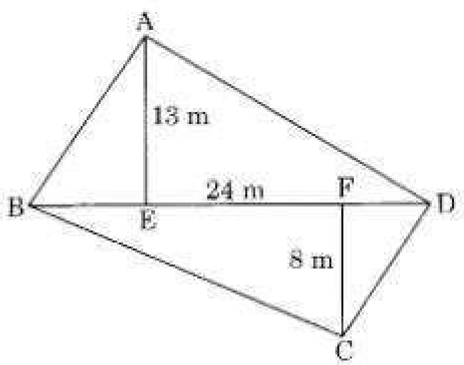
(7). The figure below represents a surveyor's sketch of a plot of land. Record the measurements from the sketch in a field book given that XY = 50m, XK = 20m, XM = 25m, XL = 35m, KA = 40m, MD = 38m and LB = YC = 60m.



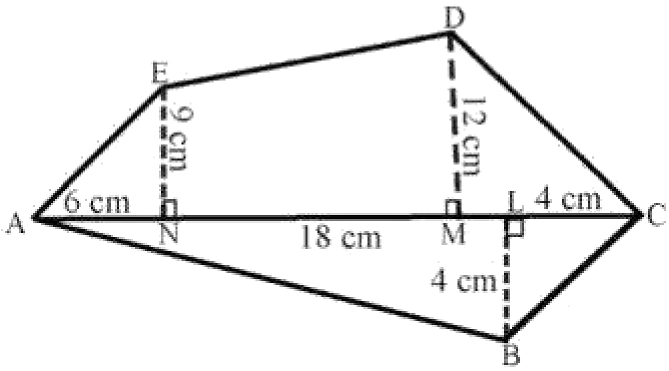
(8). A surveyor's sketch of a plot of land is as shown below. Record the measurements from the sketch in a field book given that AN = 10 cm, AL = 2 cm, LC = 2.5 cm, CN = 3.5 cm, LB = 4 cm, CM = 3.8 cm. use a scale of 1 cm rep 50 m.



(9). The figure below represents a surveyor's plot of land. Record the measurements from the plot in a field book.



(10). Using a scale of 1cm rep 20 m, record the measurements shown in the figure below in a field book for a survey of a small piece of land.



Survey B

<u>**Survey**</u>

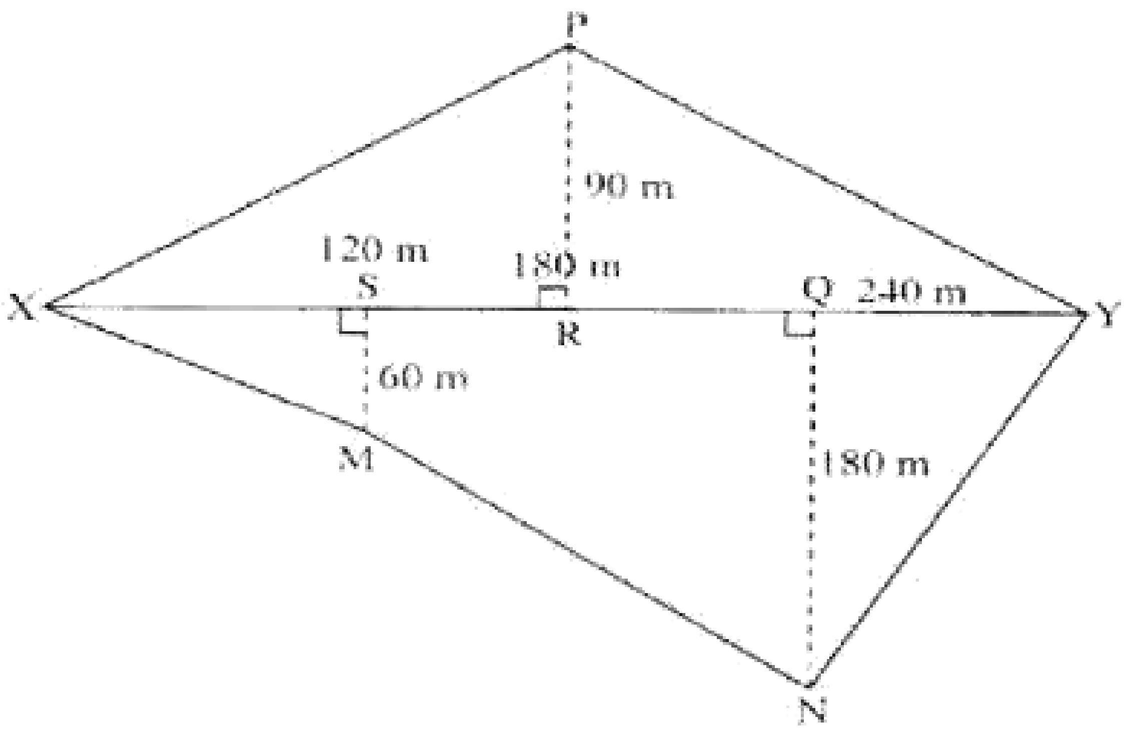
- Areas of pieces of land which have irregular shapes can be obtained by subdividing them into convenient geometrical shapes, e.g., triangles, rectangles or trapezia. This is done by the use of baseline and offsets of the area required.

Example.

(1). Find the area in hectares of a coffee field whose measurements are entered in a field book as follows. (Take XY = 360 m as the base line)

	Y	
	240	180 to N
To R90	180	
	120	60 to M
	X	

Solution



Area of:

- *Triangle XPR is $^1/_2$ x 180 x 90 m² = 8 100 m²*
- *Triangle PRY is $^1/_2$ x 180 x 90 m² = 8 100 m²*
- *Triangle XSM is $^1/_2$ x 120 x 60 m² = 3 600 m²*
- *Triangle QNY is $^1/_2$ x 120 x 180 m² = 10 800 m²*
- *Trapezium SQNM = $^1/_2$ (QN+SM) x SQ m²*
- *$^1/_2$ (180+60) x 120 m² = 14 400 m²*
- *Total area = 45 000 m²*
- *Therefore, the area of the field is 4.5 ha.*

<u>Assignment</u>

(1). Find the area in hectares of a coffee field whose measurements are entered in a field book as follows. (Take XY = 400 m as the base line).

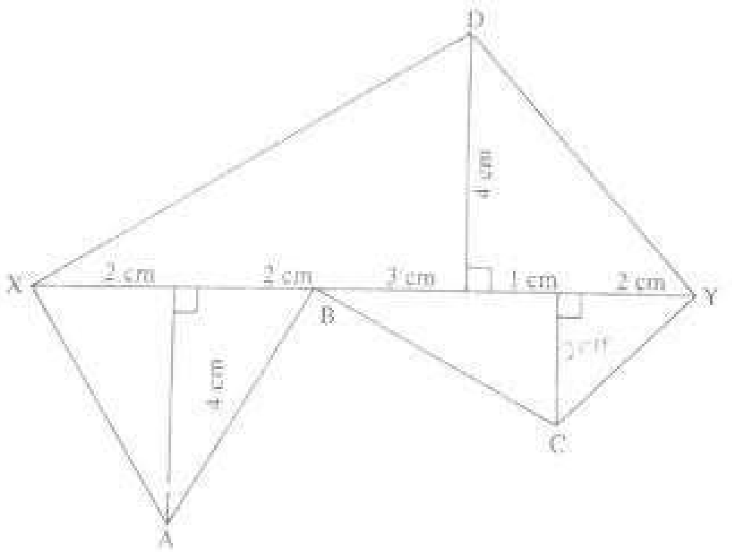
	Y	
	360	80 to Q
To R80	280	
To S160	200	
	80	200 to P
	X	

(2). A farm of land of measurements are shown in field book below. XY = 500 m. Find the area of the farm in hectares.

	Y	
To B100	450	
	300	20 to C
	120	70 to D
To A40	60	
	X	

(3). Use a scale of 1 cm to 40 m to draw the map of the coffee field.

(a). Find the area in hectares of a maize farm XABCYD in figure below which is drawn to a scale of 1 cm to 50 m.



(b). Taking XY as the base line and that the survey is from X to Y, enter the actual measurements of the farm in a field book.

(4). Measurements of a maize field using a base line XY were recorded as shown below. (Measurements are in metres)

Use a suitable scale to draw the map of the maize field hence find the area of the field in hectares.

	240	
60	190	
	180	75 TO Q
	150	50 TO P
TO S 100	120	
	100	100 TO N
30	50	
	20	20 TO M

(5). Find the area in hectares of farms whose measurements are shown in field book as in the tables below.

AB = 600 m.

	B	
	550	120 to L
To K150	450	
	250	90 to M
To J60	40	
	A	

(6). A surveyor recorded the measurements of a small field in a field book using base lines AB = 75 cm, as shown below, draw the map of the field.

	B		C		A
R9	55	W5	80	Z17	70
Q7	42	V6	70	Y5	50
P15	30	U7	60	X6	25
A	A	S10	20		C
			B		

(7). Use a suitable scale to draw the map of the tea field hence find the area of the field in hectares. If a surveyor recorded the measurements of the tea field in a field book using base lines BC = 100 cm, as shown below.

	B		C		A
R9	55	W5	80	Z17	70
Q7	42	V6	70	Y5	50
P15	30	U7	60	X6	25
A	A	S10	20		C
			B		

(8). The table below gives a field book showing the results of a survey of a section of a piece of land between A and E. All measurements are in metres.

(a) Draw a sketch of the land.

	E	
D33	95	
	90	F 36
C21	70	
B 42	30	G 25
	25	H 40
	A	

(9). A field was surveyed and its measurements recorded in a field book as shown below.

(a) Using a scale of 1cm to represent 10m, draw a map of the field.

	F	
	100	
E 40	80	
	60	D 50
C 40	40	
	20	B 30
	A	

(b) Calculate the area of the field in hectares.

(10). A tea farm in Kakamega forest was surveyed and the results were recorded in the surveyors note book as shown below. The measurements are in meters. Using a scale of 1: 25, draw the map of the plot and hence calculate the area of the plot in Hectares.

	250	**Y**
	240	D70
C80	170	
	70	B60
A60	50	
X	**0**	

Similarity and Enlargement
Introduction

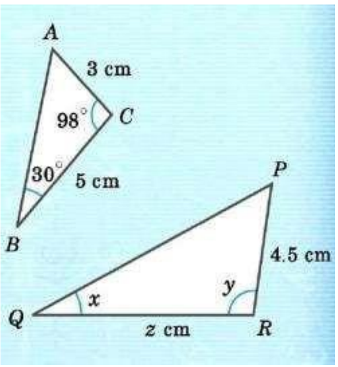
Similar Figures

- Two or more figures are said to be similar if:
- The ratio of the corresponding sides is

constant. • The corresponding angle are similar

Example 1

In the figures below, given that △ ABC ~ △ PQR, find the unknowns x, y and z.



Solution

BA corresponds to QP each of them has opposite angle y and 980 . Hence y is equal to 980 BC

corresponds to QR and AC corresponds to PR.

BA/QR = BC/QR = AC/PR

AC/PR = BC/QR

3 /4.5 = 5 /z

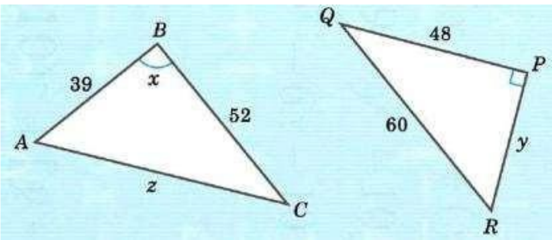
z = 7.5 cm

Note:

- Two figures can have the ratio of corresponding sides equal but fail to be similar if the corresponding angles are not the same.
- Two triangles are similar if either their all their corresponding angles are equal or the ratio of their corresponding sides is constant

Example:

In the figure, △ ABC is similar to △ RPQ. Find the values of the unknowns.



Solution

Since △ ABC~ △ RPQ,

∠B=∠P

44. x = 90°

Also,

AB/RP =

BC/PQ 39/y =

52/48 (48 × 39)

52

23. y = 36

Also,

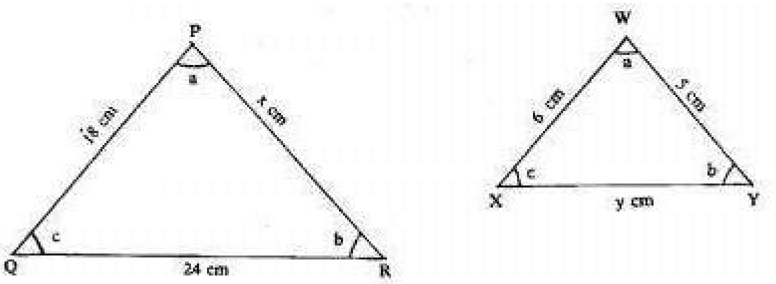
AC/RQ = BC/PQ

z /60 = 52/48

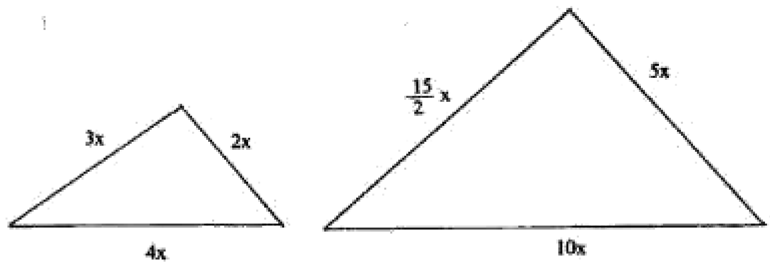
38. z = 65

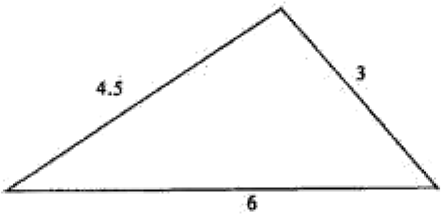
<u>QUESTIONS</u>

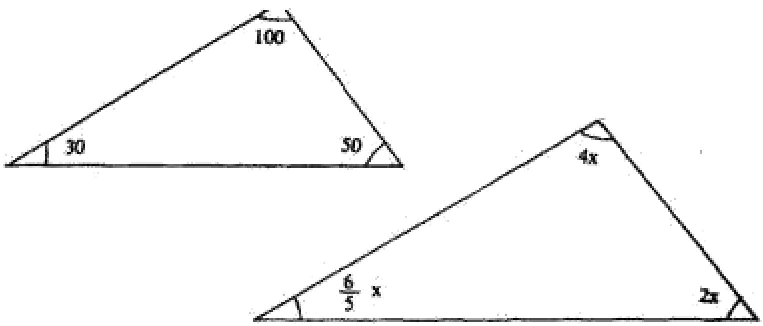
(1). In figure 7.2, the triangle PQR and WXY are similar. calculate the length of PR and XY.

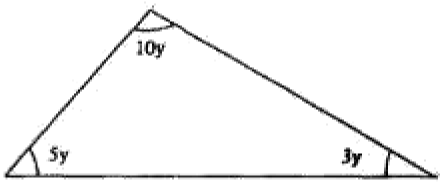


Identify the similar triangles.





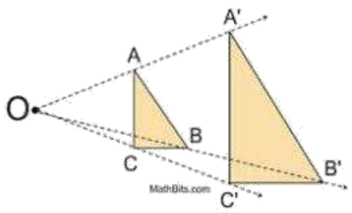




Enlargement

Enlargement

- Enlargement, sometimes called **scaling**, is a kind of transformation that changes the size of an object.
- The image created is **similar** to the object. Despite the name enlargement, it includes making objects smaller.
- For every enlargement, a **scale factor** must be specified. The scale factor is how many times larger than the object the image is.
- *Length of side in image = length of side in object × scale factor*
- For any enlargement, there must be a point called the **center of enlargement**.
- *Distance from center of enlargement to point on image = Distance from Centre of enlargement to point on object X scale factor*
- The Centre of enlargement can be anywhere, but it has to exist.



- This process of obtaining triangle A' B 'C' from triangle A B C is called enlargement.
- Triangle ABC is the object and triangles A' B 'C 'Its image under enlargement scale factor 2.

Hence;

OA'/OA = OB'/OB = OC'/OC = 2.

The ratio is called **scale factor of enlargement**. The scale factor is called **linear scale factor**

By measurement OA =1 .5 cm, OB = 3 cm and OC = 2.9 cm. To get A', the image of A, we proceed

as follows

OA =1 .5 cm

OA'/OA = 2 (scale factor 2)

OA'=1.5×2

= 3 cm

Also OB'/OB = 2

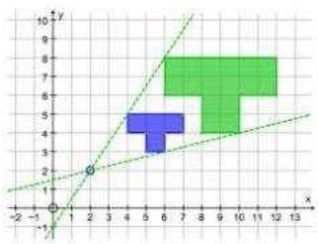
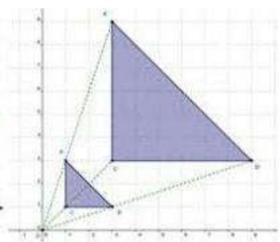
39. 3×2

40. 6 cm

Note: Lines joining object points to their corresponding image points meet at the Centre of enlargement.

Center of Enlargement

- To find center of enlargement join object points to their corresponding image points and extend the lines, where they meet gives you the Centre of enlargement. Or Draw straight lines from each point on the image, through its corresponding point on the object, and continuing for a little further.
- The point where all the lines cross is the **Centre of enlargement.**

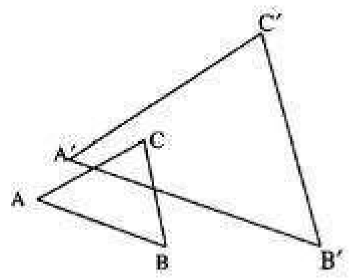
QUESTIONS

(1). Construct any triangle ABC. Take a point O outside the triangle. With O as the centre of enlargement and scale factor of 3. construct the image of ABC under the enlargement.

(2). Triangle A'B'C' is the image of triangle ABC under an enlargement

a. Locate the centre of the enlargement

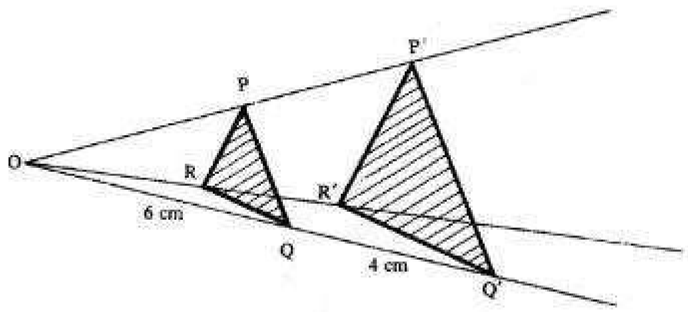
b. Find the scale factor of the enlargement.



(3). In figure 7.18,ΔP'Q'R' under an enlargement, center O.

a. If OQ = 6 cm and QQ' = 4 cm, find the scale factor of the enlargement.

b. If PQ = 4 cm, calculate the length of P'Q'



(4). In figure 7.19, rectangle A'B'C'D' is the image of rectangle ABCD under an enlargement with centre at O. OA = 12 cm, OA' = 4 cm, AB = 6cm and A'D' = 3cm

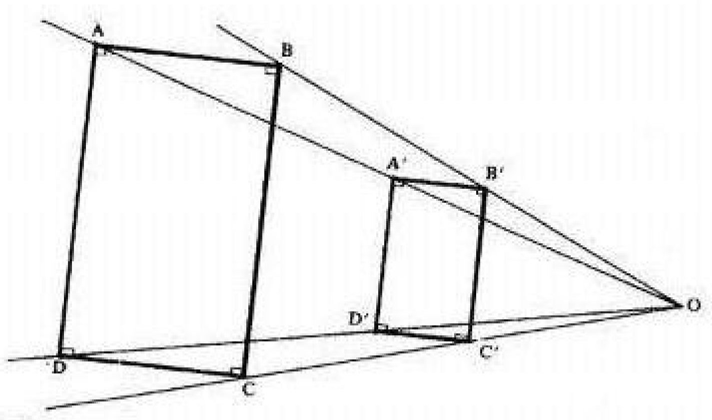


Fig. 7.19

Calculate:

a. the linear scale factor

b. the length of A'B'

c. the length of BC

Given that p(3. 4), Q (4, 4), R (6, $), S97, 1) and TS (5,0) are the vertices of an object, Find the vertices of the image after an enlargement with the centre at (0, 0) and scale factor

55. 1/2

56. 3

Negative Scale Factor

Negative Scale Factor

In figure 7.25, P'Q'R' and P'' Q'' R'' are the images of PQR under an enlargement, center O. Both images are twice as large as PQR.

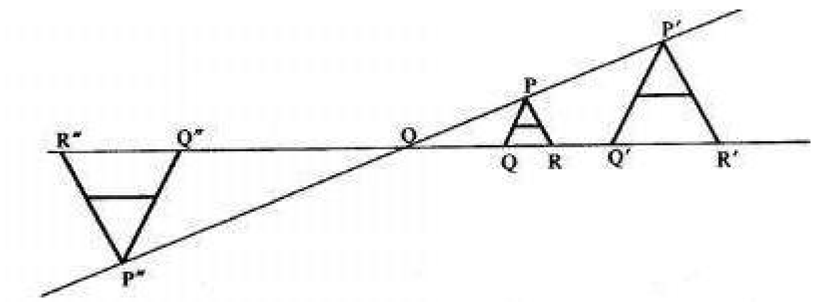


Fig. 7. 25

P'Q'R' is the image of PQR under an enlargement centre O, scale factor 2. How can we describe P'Q'R', the image of PQR under an enlargement with O as the centre?

Note:

i. PQR and its image P'Q'R' are on opposite sides of the centre of enlargement.

57. The image is inverted.

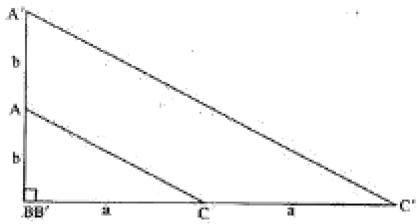
When this happens, the scale factor is said to be negative. In this case, the scale factor is -2.

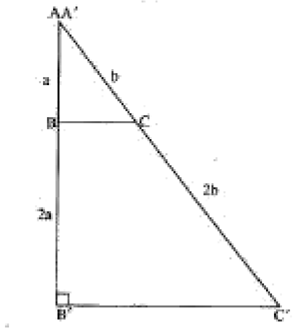
Note;

- To locate the image of the object under an enlargement with a negative scale factor. the same procedure as for the positive scale factor is followed. However, the object and the image fall on opposite sides of the centre of enlargement.

Questions

(1). Scale the centre of enlargement and the linear scale factor in each of the following figures if Δ A'B'C' is the image of ΔABC.





(2). Points A(4, 2), B(9, 2), C(7, -2) and D(2, -2) are the vertices of a parallelogram. Taking the origin as the centre of enlargement, find the image when the scale factor is;

58. -4
59. -2
60. $-^1/_4$

Area Scale Factor

Area scale factor

- Is a ratio in the form e: f or e/f. This ratio describes how many times to enlarge. Or reduce the area of two dimensional figure. Area scale factor can be calculated using

$$= \frac{\text{New Area}}{\text{Original Area}}$$

- Area scale factor = (linear scale factor)^2

Example;

A triangle whose area is 12 cm^2 is given an enlargement with linear scale factor.3

Find the area of the image.

Solution

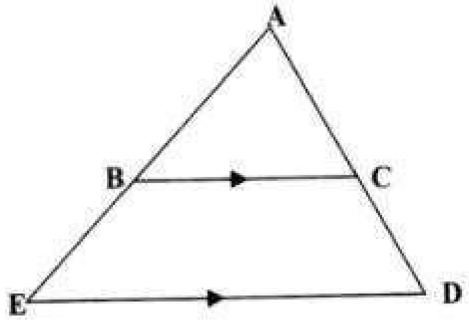
Linear scale factor is 3.
Area scale factor is 32 = 9

$$\frac{\text{Area of image}}{12} = 9$$

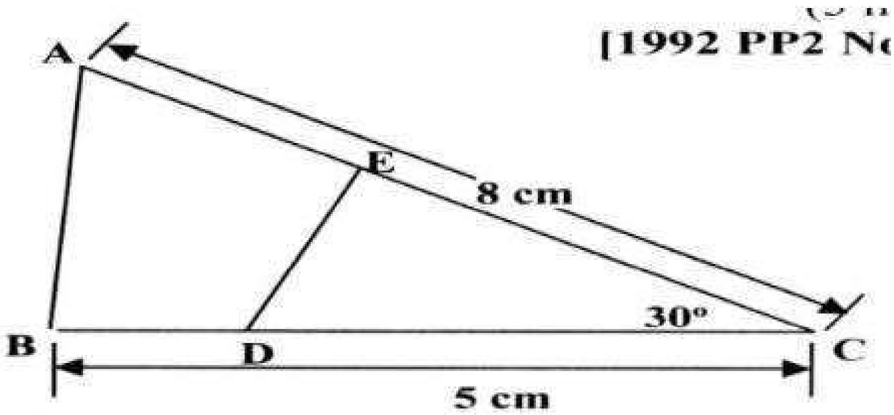
Therefore, area if image is 12 × 9 = 108cm^2

Questions

(1). In the figure above, triangle ABC is similar to triangle AED and BC // ED. Given that the ratio AB: AE = 2:5, find the ratio of the area of triangle ABC to that of the trapezium BCDE.

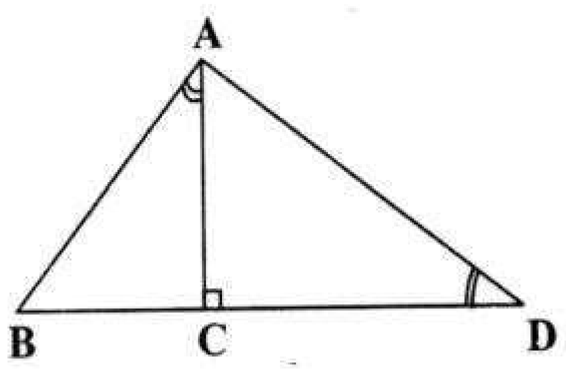


(2). In the triangle ABC below AC = 8 cm, BC = 5 cm and angle BCA = 30°. point D divides BC in the ratio 1:4 and point E divides AC in the ratio 2:3. Find the area of the quadrilateral ABDE.



(3). In the figure below, angles BAC and ADC are equal. Angle ACD is a right angle.

The ratio of the sides AC: BC = 4:3



Given that the area of triangle ABC is 24 cm^2, find the area of triangle ACD.

Volume Scale Factor

Volume scale factor

- Is the ratio that describes how many times to enlarge or reduce the volume of a three dimensional figure.

  Volume scale factor can be calculated using.

New Volume / Original Volume

Volume scale factor = (linear scale factor)3

Example

The base radii of two similar cones are 6 cm and 8 cm. If the volume of the smaller cone is $324cm^3$, Find the volume of the larger one.

Solution

The ratio of the radii is 8 cm/6 cm = 4/3

linear factor is 4/3.

Therefore, the volume scale factor is $\left(\frac{4}{3}\right)^3$ = 64/27.

Thus, volume of the larger cone 64/27 x 324 = 768 cm^3

<u>QUESTIONS</u>

(1). A football tube in the form of a sphere is inflated so that its radius increases in the ratio of 4:3. Find the ratio in which the volume is increased

(2). A container of height 30cm has a capacity of 1.5 litres. What is the height of a similar container of capacity 3.0 m^3?

(3). The ratio of the lengths of the corresponding sides of two similar rectangular water tanks is 3:5. The volume of the smaller tank is 8.1 m^3. Calculate the volume of the larger tank.

(4). Pieces of soap are packed in a cuboid container measuring 36 cm by 24 cm by 18 cm. Each piece of soap is similar to the container. If the linear scale factor between the container and the soap is 1/6, find the volume of each piece of soap

(5). The volumes of two similar solid cylinders are 4752 cm^3 and 1408 cm^3. If the area of the curved surface of the smaller cylinder is 352 cm^2, find the
area of the curved surface of the larger cylinder

TRIGONOMETRY

Tangent of an Acute Angle

<u>Introduction</u>

<u>Tangent of Acute Angle</u>

- The constant ratio between the vertical distance/horizontal distance is called the tangent. It's abbreviated as **tan**

QUESTIONS

(1). When the angle of elevation of the sun is 580, a vertical pole casts a shadow of length 5m on a horizontal ground. Find the height of the pole.

(2). The angle of elevation of the top of a cliff from point P is 450. From a point Q which is 10m from P towards the foot of the cliff, the angle of elevation is 480.

Calculate the height of the cliff.

(3). A flag 10m long is fixed on top of a tower. From a point on horizontal ground, the angles of elevation of the top and bottom of the flag post are 400 and 330 respectively.

Calculate

a. The height of the tower (6mks)

b. The shortest distance from the point on the ground to the top of the flag post (2mks).

Table of Tangents

- Scale drawing is one method of obtaining the tangent of an angle.
- Special tables have been prepared and can be used to obtain tangents of acute angles (see tables of natural tangents in your mathematical tables). the technique of reading tables of tangents is similar to that of reading tables of logarithms or square roots.

i. In the tables of tangents, the angles are expressed as decimals and degrees. or in degrees and minutes.

ii. One degree is equal to 60' (60 minutes). Thus. 30' = 0.50° , 54'' = 0.9° and 6' = 0.1 .

Example;

Find the tangent of each of the following angles from the tables:

45. 42°

46. 42.75°

47. 42° 47'

Solution;

From the tables:

a. tan 42° = 0.9004

b. Using decimal tables, tan 42.4 = 0.9228. from the difference column under 5, we read 0.0016.

Therefore, tan 42.47° = 0.9228 + 0.0016

= 0.9244

c. Using degrees and minutes tables, tan 42° 42' = 0.9228 and from difference column under 5, we read 0.0027.

Therefore, tan 42° 47' = 0.9228 + 0.0027

= 0.9255

<u>Questions</u>

(1). Express each of the following in degrees and minutes:

22. 15.3°
23. 25.75°
24. 30 $\frac{1}{2}$°
25. 34 $^3/_4$°

(2). Read from the tables the tangent of:

41. 70.53°

42. 18.73°

43. 55° 53'

d. 63° 12'

e. 81° 08'

f. 10° 30'

g. 75° 58'

h. 89° 54'

i. 29° 34'

j. 87° 50'

k. 78° 08°

l. 48° 42'

44. 84°

45. 43° 51'

46. 57.17°

(3). Find from the tables the angle whose tangent is:

61. 0.3317

62. 0.6255

63. 1.6391

64. 0.44444

65. 0.0122

66. 0.8799

67. 0.1867

68. 0.5903

69. 5.1006

70. 1.0000

71. 0.2839

72. 2.0011

73. 3.6703

74. 0.7400

75. 40.92

Sine of an Acute Angle

Sine of an Angle

- The ratio of the side of angle x to the hypotenuse side is called the sine.

$$\text{Sin}\varnothing = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Example

In figure 9.17, AB = 5 cm, CB = 12 cm and < ABC = 90° . Calculate:

a. sin x

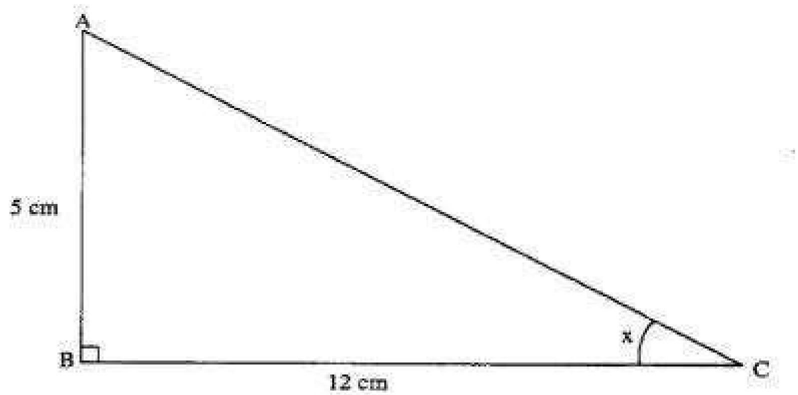


Fig. 9.17

Solution

a. sin x = $\frac{\text{opp}}{\text{hyp}} = \frac{AB}{AC} = \frac{5}{AC}$

But $AC^2 = 12^2 + 5^2$

=144 + 25

= 169

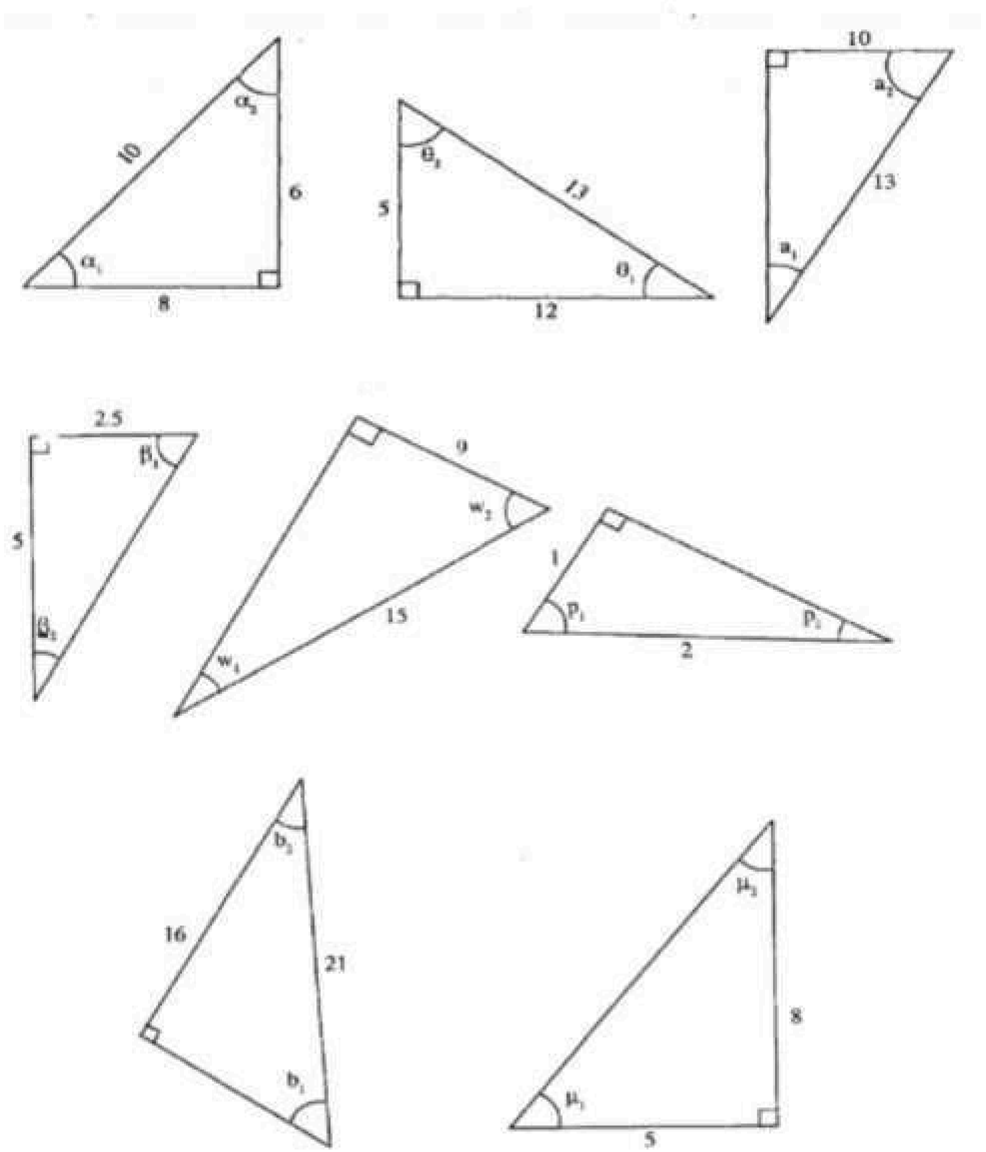
Therefore, AC = 13 cm

76. sin x = $^5/_{13}$

= 0.3846

Questions

(1). Find the cosine and sine of each in the following marked angle. 9 units are in centimeters.

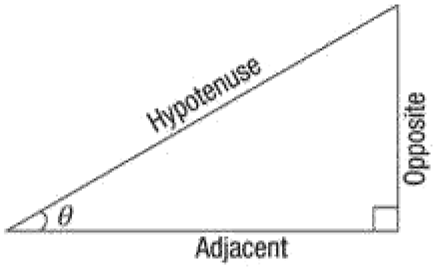


Cosine of an Acute Angle

Cosine of an Angle

- The ratio of the side adjacent to the angle and hypotenuse

$$\text{Cosine}\varnothing = \frac{\text{Adjacent}}{\text{hypotenuse}}$$



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

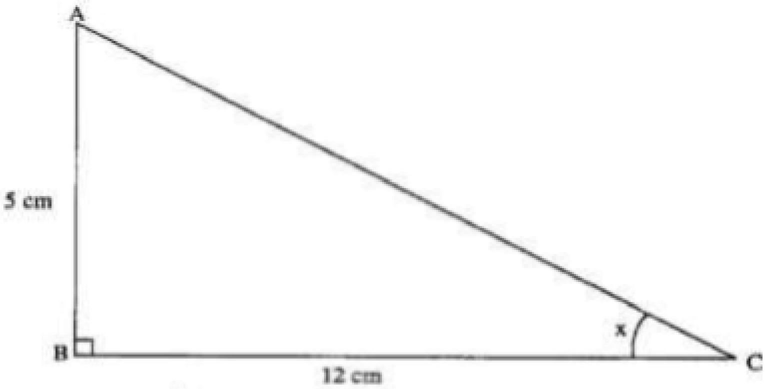
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Example

In figure 9.17, AB = 5 cm, CB = 12 and < ABC = 90°. **Calculate:**

cos x.



$$= \frac{\text{adj}}{\text{hyp}}$$

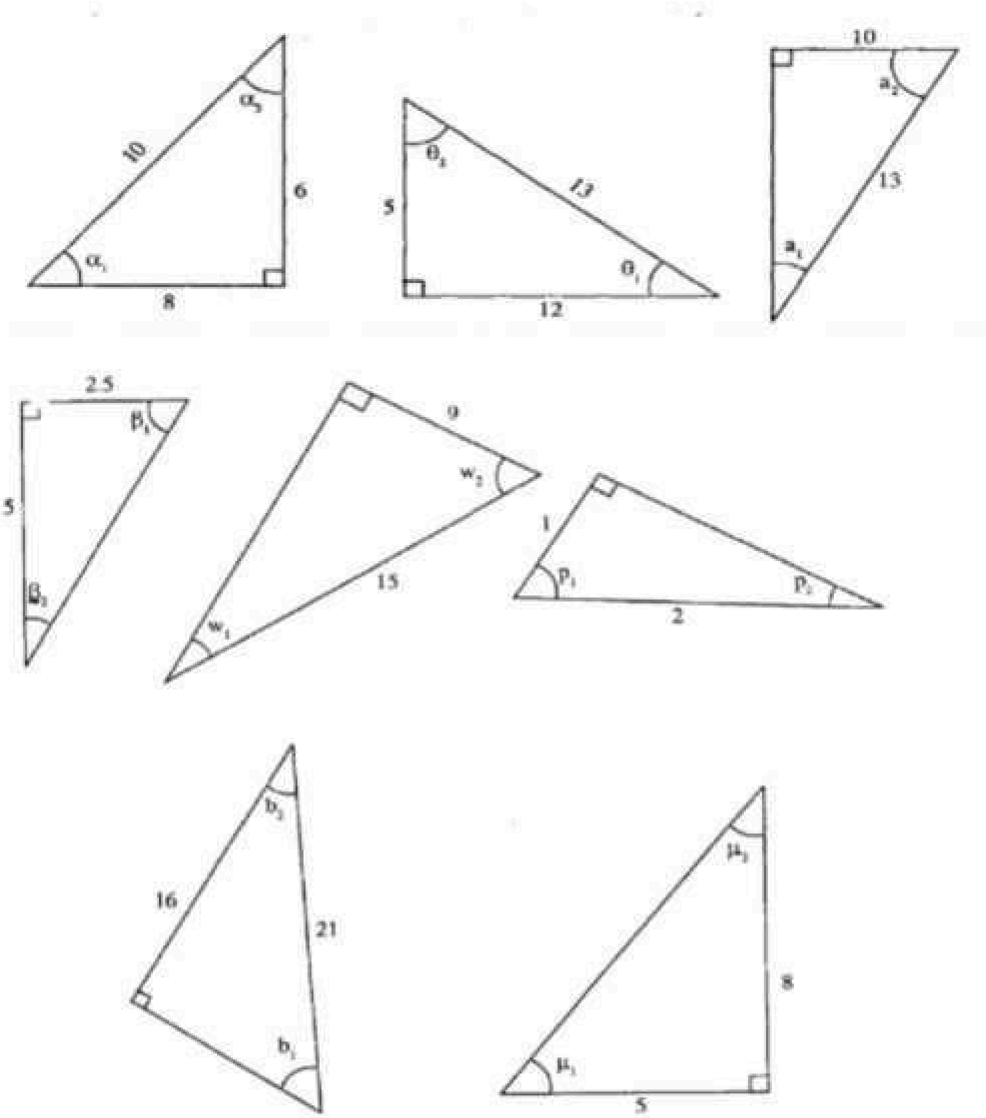
$$= \frac{CB}{AC}$$

$$= \frac{12}{13}$$

b. $\cos x = 0.9231$

<u>Questions</u>

(1). Find the cosine and sine of each in the following marked angle. (units are in centimeters)



Tables of Sine

- The sine and cosine tables are read and used in the same way as the tangent tables. As the angles increase from 0° to 9°

i. The values of their sines increase from 0 to 1

ii. The values of their cosines decrease from 1 to 0.

- Therefore, the values in the difference columns of cosine tables have to be subtracted and those in the difference columns of the sine tables have to be added.

Example;

Read the sine and cosine values of the following angles from the tables:

32. 47.3°

33. 69.55°

Solution

a.

sin 47.3° = 0.7349

cos 47.3° = 0.6782

b.

sin 69.55° = 0.9370

cos 69.55° = 0.3494

<u>Questions</u>

(1). Find from the tables the angle whose sine is:

(xiii) 0.3367

(xiv) 0.5871

(xv) 0.0523

(xvi) 0.8500

(xvii) 0.1822

(xviii) 0.9834

(xix) 0.5012

(xx) 0.2518

(2). Read from the tables the sine of:

14. 31.46°

15. 77° 34'

16. 52° 9'

17. 66° 31'

18. 6.76°

19. 40.13°

20. 26° 47'

21. 13.07°

Tables of Cosine

- The sine and cosine tables are read and used in the same way as the tangent tables. As the angles increase from 0° to 90°

i. the values of their sines increase from 0° to 90° .

ii. the values of their cosines decrease from 1 to 0.

- Therefore, the values in the difference columns of cosines tables have to be subtracted and those in the difference columns of the sine tables have to be added.

Example;

Read the sine and cosine values of the following angles from the tables:

7. 47.3°

8. 69.55°

Solution

a.

sin 47.3° = 0.7349

cos 47.3°= 0.6782

b.

sin 69.55° = 0.9370

cos 69.55° = 0.3494

Questions

(1). Find from the tables the angle whose cosine is:

73. 0.1643

74. 0.7196

75. 0.9970

76. 0.8660

77. 0.4009

78. 0.9481

79. 0

80. 0.7371

(2). Read from the tables the cosine of:

a. 79° 42'

11.24.23°

12. 5° 37'

13. 60°

14. 88°.59'

15. 55.97°

16. 33.33°

17. 17° 52'

Sine and Cosine of Complementary Angles

Sine and Cosines of Complementary Angles

- For any two complementary angles x and y, sin x = cos y; cos x = sin y e.g. sin 600 = cos 300 , Sin 300 = cos 600 , sin 700 = cos 200 ,

Example

Find acute angles α and β if Sin α = cos 330

Solution

sin α = cos 33

Therefore α + 33 = 90

α=57

Find acute angles ∝ and 3β if:

17. sin ∝ = cos 33°

18. cos β = sin3 β

Solution

114. sin ∝ = cos 33

Therefore, ∝ + 33 = 90

∝ = 90° - 33°

= 57°

17. $\cos \beta = \sin 3\beta$

Therefore, $\beta + 3\beta = 90°$

$4\beta = 90°$ $\beta = 22\,{}^{1}/_{2}°$

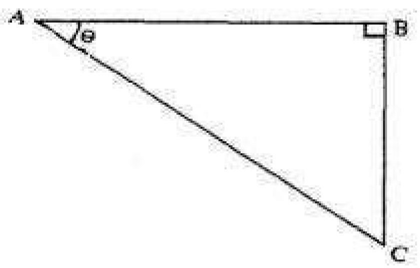
Questions

(1). If A and B are complementary angles and sin A = $^{4}/_{5}$, find cos B

(2). In figure 9.32, AB = 16 cm and AC = 20cm. Find:

a sin θ

b cos θ



(3). X and B are complementary angles. If sign B = 0.9975. Find X

(4). A and B are complementary angles. If A = $\frac{1}{2}$B Find:

a. sin A.

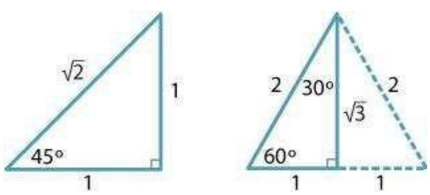
b. cos A

(5). Find the acute angle X given that cos x° = sin 2x°

Tangent, Cosine and Sine of 45 degrees

Trigonometric Ratios of Special Angles: 30° , 45° , 60° .

- These trigonometric ratios can be deducted by the use of isosceles right – angled triangle and equilateral triangles as follows.



Tangent, Cosine and Sine of 45°

- The triangle should have a base and a height of one unit each, giving hypotenuse of $\sqrt{2}$.

Cos 45° = $1/\sqrt{2}$

Sin 45° = $1/\sqrt{2}$

Tan 45° = 1

QUESTIONS

(1). Simplify the following without using tables (use trigonometric ratios):

Note: $\sqrt{a} \times \sqrt{a} = a$

a. sin 30° cos 30°

b. 4 cos 45° sin 60°

c. 3 cos 30° + cos 60°

d. tan 45° + cos 45° sin 45°

e. sin 60° cos 30° + sin 30° cos 30°

f. $\cos^2 60° + \sin^2 60°$ **(sin θ × sin θ is written as $\sin^2$ θ)**

(2). Given sin (90 – a) = ½ , find without using trigonometric tables the value of cos a (2mks)

(3). If θ = 24/25, find without using tables or calculator, the

value of tan θ – cos θ

cos θ + sin θ

(4). At point A, David observed the top of a tall building at an angle of 30°. After walking for 100 meters towards the foot of the building he stopped at point B where he observed it again at an angle of 60°. Find the height of the building

(5). Find the value of θ, given that ½ sinθ = 0.35 for 0o ≤ θ ≤ 360°

(6). A man walks from point A towards the foot of a tall building 240 m away. After covering 180m, he observes that the angle of elevation of the top of the building is 45°. Determine the angle of elevation of the top of the building from A

(7). Solve for x in 2 Cos 2 x 0 = 0.6000; 00≤ x ≤ 3600 .

(8). Wangechi whose eye level is 182cm tall observed the angle of elevation to the top of her house to be 32° from her eye level at point A. She walks 20 m towards the house on a straight line to a point B at which point she observes the angle of elevation to the top of the building to the 40°.

Calculate, correct to 2 decimal places the ;

a. Distance of A from the house

b. The height of the house

(9). Given that $\cos A = 5/13$ and angle A is acute, find the value

of:- $2 \tan A + 3 \sin A$

(10). Given that $\tan 5° = 3 + \sqrt{5}$, without using tables or a calculator, determine tan 25°, leaving your answer in the form $a + b\sqrt{c}$

(11). Given that $\tan x = 5/12$, find the value of the following without using mathematical tables or calculator:

a. Cos x

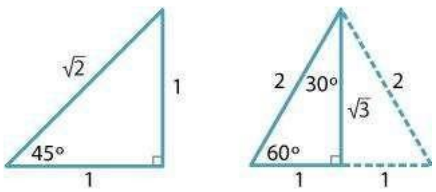
b. Sin2 (90–x)

(12). If $\tan \theta = 8/15$, find the value of Sinθ – Cosθ without using a calculator or table Cosθ + Sinθ

Tangent , Cosine and Sine of 30 and 60 Degrees

Trigonometric Ratios of Special Angles: 30°, 45°, 60°.

- These trigonometric ratios can be deducted by the use of isosceles right – angled triangle and equilateral triangles as follows.



Tangent, Cosine and Sine of 30° and 60° .

- The equilateral triangle has a sides of 2 units each

$\sin 30° = 1/2$

$\cos 30° = \sqrt{3}/2$

$\tan 30° = 1/\sqrt{3}$

$\sin 60° = \sqrt{3}/2$

$\cos 60° = 1/2$

$\tan 60° = \sqrt{3}/1 = \sqrt{3}$

QUESTIONS

(1). Simplify the following without using tablets (use trigonometric ratios):

Note: $\sqrt{a} \times \sqrt{a} = a$

a. sin 30° cos 30°

b. $4 \cos 45° \sin 60°$

c. $3 \cos 30° + \cos 60°$

d. $\tan 45° + \cos 45° \sin 45°$

e. $\sin 60° \cos 30° + \sin 30° \cos 60°$

f. $\cos^2 60° + \sin^2 60°$ ($\sin \theta \times \sin \theta$ is written as $\sin^2 \theta$)

(2). Find the height of an equilateral triangle of side x cm. Use the triangle to show that $\sin^2 60° + \cos^2 60° = 1$ (without using tables).

(3). Given $\sin (90 - a) = \frac{1}{2}$, find without using trigonometric tables the value of cos a (2mks)

(4). If $\theta = 24/25$, find without using tables or calculator, the

value of $\tan \theta - \cos \theta$

$\cos \theta + \sin \theta$

(5). At point A, David observed the top of a tall building at an angle of 30°. After walking for 100 meters towards the foot of the building he stopped at point B where he observed it again at an angle of 60°. Find the height of the building.

(6). Find the value of θ, given that $\frac{1}{2} \sin\theta = 0.35$ for $0o \leq \theta \leq 360°$

(7). A man walks from point A towards the foot of a tall building 240 m away. After covering 180 m, he observes that the angle of elevation of the top of the building is 45°. Determine the angle of

elevation of the top of the building from A

(8). Solve for x in $2Cos\ 2 \times 0 = 0.6000;\ 00 \leq x \leq 3600$.

(9). Wangechi whose eye level is 182 cm tall observed the angle of elevation to the top of her house to be 32° from her eye level at point A. She walks 20 m towards the house on a straight line to a point B at which point she observes the angle of elevation to the top of the building to the 40°.

Calculate, correct to 2 decimal places the ;

a. Distance of A from the house

b. The height of the house

(8). Given that $\cos A = 5/13$ and angle A is acute, find the

value of:- $2 \tan A + 3 \sin A$

(9). Given that $\tan 5° = 3 + \sqrt{5}$, without using tables or a calculator, determine tan 25°, leaving your answer in the form $a + b\sqrt{c}$

(10). Given that $\tan x = 5/12$, find the value of the following without using mathematical tables or calculator:

a. Cos x

b. Sin2 (90–x)

(11). If $\tan \theta = 8/15$, find the value of $\sin\theta - \cos\theta$ without using a calculator or table $\cos\theta + \sin\theta$

Logarithm of Tangent, Cosine and Sine

Logarithms of Tangents, Sines and Cosines

- In this section, we extend the use of logarithms in computations involving trigonometry.

For example,

To evaluate 234 sin 36°, we can use the following method: from tables, sin 36° = 0.5878

Therefore, 234 sin 36° = 234 × 0.5878

No	Log
234	2.3692
0.5878	$\bar{1}.7692$ +
1.375×10^2	2.1384

Therefore; 234 sin 36 = 137.5

Alternatively, log sin 36° can be read directly from the tables of logarithms of sines.

No	Log
234	2.3692
sin 36^0	$\bar{1}.7692$ +
1.375×10^2	2.1384

Therefore, 234 sin 36° = 137.5.

Similarly, values of logo(cos x) and log(tan x) can be read from their respective tables.

Questions

(1). Evaluate;

a. 8.52 tan 42.2°

b. 7.9 sin 79°

c. $\dfrac{7\cos 50.2^0}{9.5 \sin 60^0}$

d. $\dfrac{7\cos 50.2^0}{9.5 \sin 60^0}$

(2). In a triangle PQR, <PQR = <PQR = 58° and QR = 5.2 cm. Calculate the length of PQ.

(3). In a triangle PQR, OR = 5.2 cm and PQ = PR = 8.2 cm. Calculate:

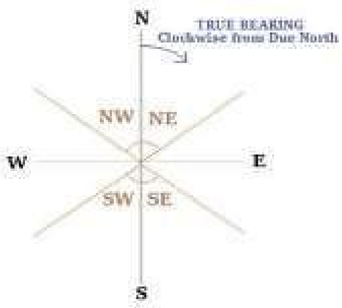
a. <PQR and <QPR

b. The area of ΔPQR.

Bearing

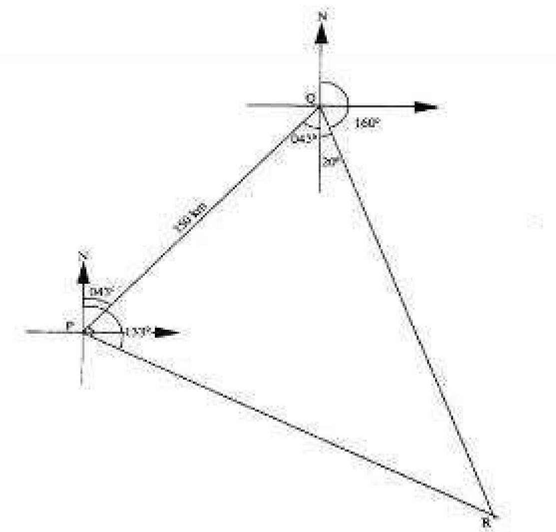
Bearings

- Bearings are used in navigation. A bearing is an angle measurement used to describe precisely the direction of one location from a given reference point.
- Three-figure bearings, also called true bearings, use angles from 000°to 360°to show the amount of turning measured clockwise from north 000°. Note that the angles are always written with three digits.



Example;

Three towns P, Q and R are such that Q is 150 km from P on a bearing of 043° (see figure 9.36). The bearing of R from P is 133° and the bearing of R from Q is 160°. Calculate the distance of R from P, Q from R and the bearing of P from R.



Solution;

In the figure, ΔPQR is right-angled at P <PQR = 63°,

Hence, tan 63° = PR/150

150 tan 63° = PR

No	Log
150	2.1761
tan 63^0	0.2928 +
2.944×10^2	2.4689

Therefore, PR = 294.4 km

In the right-angled ΔPQR, QR is the hypotenuse;

cos 63° = 150/QR

QR = 150/cos 63°

No	Log
150	2.1761
cos 63^0	$\bar{1}.6570$ –
3.305×10^2	2.5191

Therefore, QR = 330.5 km

<PRQ = 180° - (90 + 63)°

= 27°

<PRT = 90° - (20 + 27)°

= 43°

Therefore, the bearing of p from R = 270° + 43° = 313°

QUESTIONS

(1). A man walks directly from point A towards the foot of a tall building 240m away. After covering 180m, he observes that the angle of the top of the building is 450. Determine the angle of elevation of the top of the building from A.

(2). There are two signposts A and B on the edge of the road. A is 400 m to the west of b. A tree is on a bearing of 0600 from A and a bearing of 3300 from B Calculate the shortest distance of the tree from the edge of the road.

(3). A point A is directly below a window. Another point B is 15 m from A and at the same horizontal level. From B angle of elevation of the top of the

bottom of the window is 300 and the angle of elevation of the top of the window is 350

Calculate the vertical distance.

a. From A to the top of the window

b. From A to the bottom of the window

c. From the bottom to top of the window

(4). An electric pylon is 30m high. A point S on top of the pylon is vertically above another point on the ground . Points A and B are on the same horizontal ground as R. Point A is due south of the pylon and the angle of elevation of S from A is

260. Point B is due west of the pylon and the angle of elevation of S from B is 320.

a. Distance from A and B