

Assignment 8

Due: Tue 7/31/2018 by 11:59pm

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Practice Problems

Prepare for graded problems by completing the following problems which have solutions in the textbook. Some of these problems may be covered in Discussion Exercises.

- Section 9.1: 3a,b,c,d, 5, 7a,c,e,g, 13, 31, 33, 55
- Section 9.2: 11, 17, 29
- Section 9.3: 3, 7, 13, 19, 21, 23, 31, 35
- Section 9.4: 1, 3, 19, 21, 25a,b
- Section 9.5: 3b,c,d,e, 9, 13, 19, 21, 29, 41, 43, 59
- Section 10.1: 15, 21, 25, 29, 33
- Section 10.2: 1, 5, 7, 15, 21, 23, 27, 33, 35
- Section 10.3: 1, 3, 5, 7, 11, 17, 19, 37, 47, 63
- Section 10.4: 1, 5, 7, 11, 21, 31, 39

Graded Problems

Complete the problems on the following pages. *Electronic submission is preferred* : save a copy of this document, fill in answers, and convert to a PDF to upload it.

If electronic work proves difficult, you may print these pages, write your answers on them, scan/photograph them, convert to a PDF, and then upload as your assignment.

Problem 1 (15 points)

Let the set S be

$$S = \{1,2,3,4,5,6\}.$$

Let the relation R on set S be

$$R = \{(x,y) \mid y \bmod x = 0\}$$

That is, R contains pairs (x,y) where x evenly divides y .

(A) Give a listing of the specific pairs in R .

(B) Show a Matrix (table) representation of the set as was demonstrated in class.

(C) Show a directed graph representation of the relation. If working online, it is useful to employ an appropriate graph drawing tool for this such as this one: [Graph Editor from CS Academy](#).

An alternative online editor is via [GraphViz Online](#) which involves editing text to produce the graph and taking a screenshot but this version allows self-loops.

Problem 2 (15 points)

Determine if each of the following relations on the real numbers is

- Reflexive: (x,x) is in the relation for all x
- Symmetric: (x,y) in the relation implies (y,x) is also in the relation
- Anti-symmetric: (x,y) in the relation implies (y,x) is NOT in the relation
- Transitive: (x,y) and (y,z) in the relation implies (x,z) is in the relation

If the property does not hold, give a counter example. If property does hold, give a brief explanation of why.

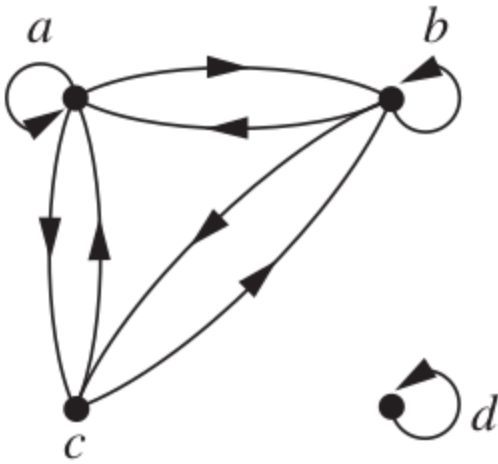
(A) $R = \{(x,y) \mid x+y = 0\}$

(B) $R = \{(x,y) \mid x = |y|\}$, where $|y|$ is the absolute value of y .

(C) $R = \{(x,y) \mid x \cdot y \geq 0\}$

Problem 3 (10 points)

Examine the directed graph below which represents a relation R on the set $S = \{a,b,c,d\}$



(A) Show the list of ordered pairs for the associated relation R

(B) Determine if the relation has the following properties justifying why or why not for each of them.

- Reflexive
- Symmetric
- Transitive
- Is an Equivalence Relation

Problem 4 (20 points)

5 points for part (A), 15 points for part (B)

Consider the following relation on the set $S = \{a,b,c,d,e\}$

$$R = \{(b, c), (b, e), (c, e), (d, a), (e, b), (e, c)\}$$

(A) Fill in the table below to show the matrix representation of the directed graph version of the relation

	a	b	c	d	e
a	-	-	-	-	-
b	-	-	-	-	-
c	-	-	-	-	-
d	-	-	-	-	-
e	-	-	-	-	-

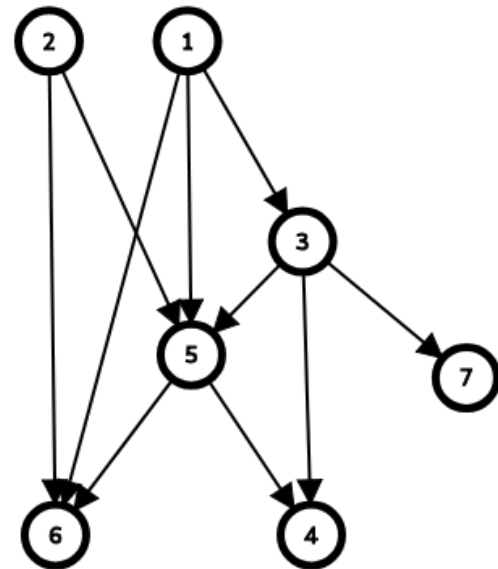
(B) Use Warshall's algorithm to determine the transitive closure of the algorithm. Use the following version and show the state of the matrix Rel at the position indicated in the code.

```
1 bool[][] warshall_tc(bool Rel[][]) {
2   assert(Rel is a square matrix);
3   int n = rows(Rel);
4   bool[][] Rel = copy(Rel);
5   for(int v=0; v<n; v++){
6     for(int i=0; i<n; i++){
7       for(int j=0; j<n; j++){
8         bool b = Rel[i][v] AND Rel[v][j];
9         Rel[i][j] = Rel[i][j] OR b;
10      }
11    }
12    // SHOW Rel HERE for v=0,1,2,...
13  }
14  // Rel is now its transitive closure
15 }
```

Problem 5 (20 points)

Consider the directed acyclic graph shown. Use the following algorithm covered in class to determine a valid topological sorting of the vertices of the DAG.

```
1 vertex[] topo_sort(graph G){
2   int n = number_of_vertices(G);
3   vertex order[] = new vertex[n];
4   for(i=n-1; i>=0; i--){
5     vmin = find a "minimal" vertex in G;
6     order[i] = vmin;
7     G = remove vmin from G;
8     // SHOW GRAPH HERE
9   }
10  return order;
11 }
```



Show the graph and order variable each time line 8 is reached.

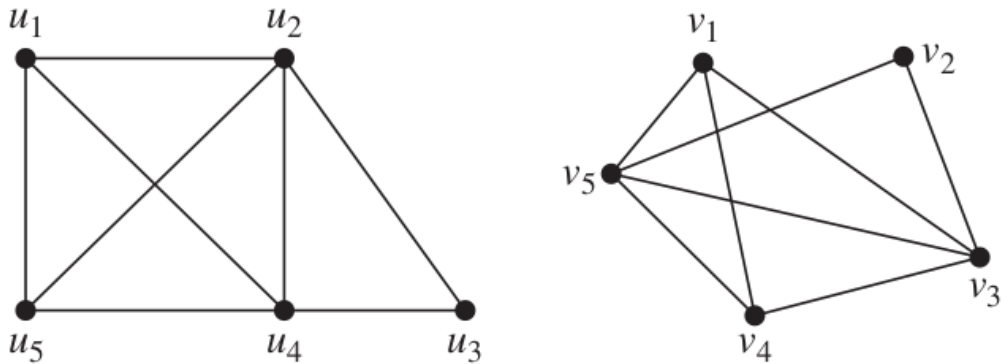
If working online, you may use the a graph editor such as as [Graph Editor from CS Academy](#).

Problem 6 (20 points)

Determine if the following pairs of graphs are isomorphic.

- If they are not isomorphic, describe a property that one graph has that the other does not.
- If they are isomorphic, give a valid mapping of the vertex set for the left graph to the right graph.

(A)



(B)

