

Geometry

Third Time's A Cosine

1. What can you say about the tangent of a k -degree angle, when k is a number between 90 and 180? Explain your response, then check with your calculator.
2. Draw a right triangle that has an 18-cm hypotenuse and a 70-degree angle. To within 0.1 cm, measure the leg adjacent to the 70-degree angle, and express your answer as a percentage of the hypotenuse. Compare your answer with the value obtained from your calculator when you enter $\cos 70$ in degree mode. This is an example of the cosine ratio.
3. The sides of a triangle are 12 cm, 35 cm, and 37 cm long.
 - (a) Show that this is a right triangle. *Easy ways and hard(er) ways to do that.*
 - (b) Show that $\tan^{-1}$, $\sin^{-1}$, and $\cos^{-1}$ can all be used to find the size of the smallest angle of this triangle.

Once you are at this point let me know. Once the entire class is here we'll debrief a bit.

4. Show that the area of a square is half the product of its diagonals.
5. Ask your calculator for the sine of a 56-degree angle, and for the cosine of a 34-degree angle. Ask your calculator for the sine of a 23-degree angle, and for the cosine of a 67-degree angle. The word cosine abbreviates "sine of the complement." Explain the terminology. Do you really need the cosine button?
6. To the nearest tenth of a degree, find the angles of the triangle with vertices (0, 0), (6, 3), and (1, 8). Use your protractor to check your calculations, and explain your method.
7. In a right triangle, the 58-cm hypotenuse makes a 51-degree angle with one of the legs. To the nearest tenth of a cm, how long is that leg? Once you have the answer, find two ways to calculate the length of the other leg. They should both give the same answer.
8. Make an accurate drawing of a regular hexagon ABCDEF. Be prepared to report on the method you used to draw this figure. Measure the length of diagonal AC and the length of side AB. Form the ratio of the diagonal measurement to the side measurement. When you compare answers with your classmates, on which of these three numbers do you expect to find agreement?

For a bit more of a challenge:

9. The sides of a square are parallel to the coordinate axes. Its vertices lie on the circle of radius 5 whose center is at the origin. Find coordinates for the four vertices of this square.

10. Suppose that triangle ABC has a 30-degree angle at A and a 60-degree angle at B. Let O be the midpoint of AB. Draw the circle centered at O that goes through A. Explain why this circle also goes through B and C. Angle BOC is called a central angle of the circle because its vertex is at the center. The minor arc BC is called a 60-degree arc because it subtends a 60-degree angle at the center. What is the angular size of minor arc AC? of major arc AC? How does the actual length of minor arc AC compare to the length of minor arc BC?

11. If two chords of a circle have the same length, then their minor arcs have the same length too. True or false? Explain. What about the converse statement? Is it true? Why?

12. An equilateral triangle is inscribed in the circle of radius 1 centered at the origin (the unit circle). If one of the vertices is $(1, 0)$, what are the coordinates of the other two? The three points divide the circle into three arcs; what are the angular sizes of these arcs?