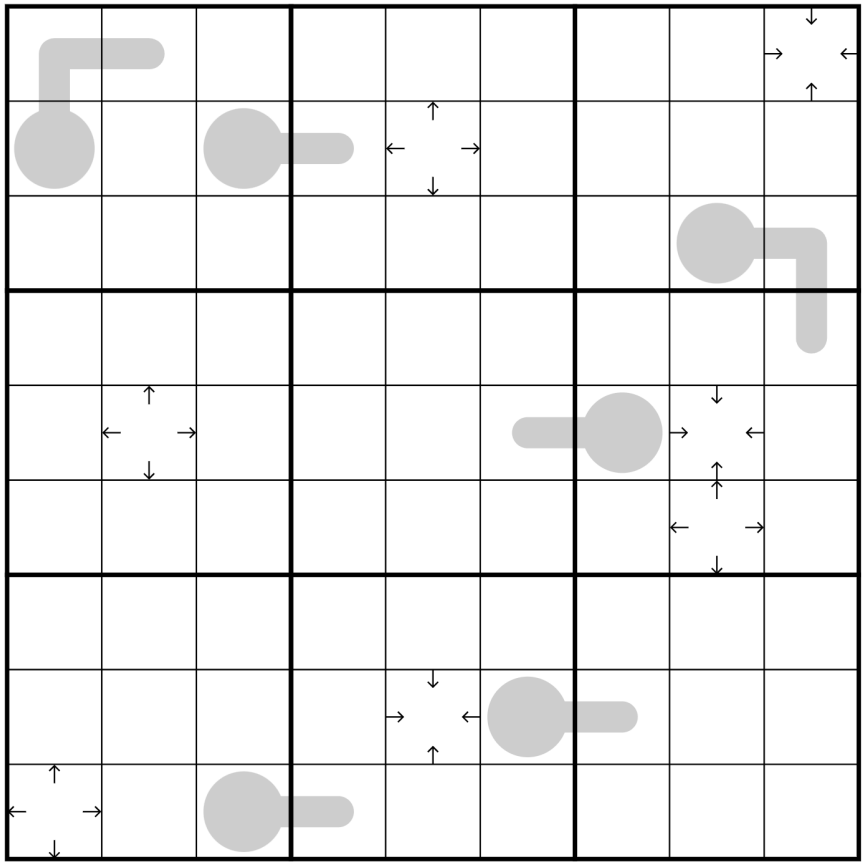
Reticulating Splines (2/5/22) (F-Puzzles, CtC)



Rules:
Normal sudoku rules apply.

Thermometers: Along thermometers, digits increase from the bulb to the end.

Peaks: A cell with arrows pointing outward is a peak, which is greater than its orthogonally adjacent cells. A peak sees all digits less than it to the north, south, east, and west, except for those hidden by digits larger than the peak. A peak of value *x* sees a total of *x* digits.

Valleys: A cell with arrows pointing inward is a valley, which is less than its orthogonally adjacent cells. A valley sees all digits greater than it to the north, south, east, and west, except for those hidden behind larger digits. A valley of value *x* sees a total of *x* digits.

Example:
Suppose a row contains the digits 5 1 2 7 6 9 4 8 3.

If 7 is a peak, it sees all three cells to the west (2, 1, and 5) and one cell to the east (6). It doesn't see 8 or 9 because they are larger than 7, and the 9 hides the 4 and the 3.

If 1 is a valley, it sees one cell to the west (5) and three cells to the east (2, 7, and 9). The 7 hides the 6 and the 9 hides the 4, 8, and 3.

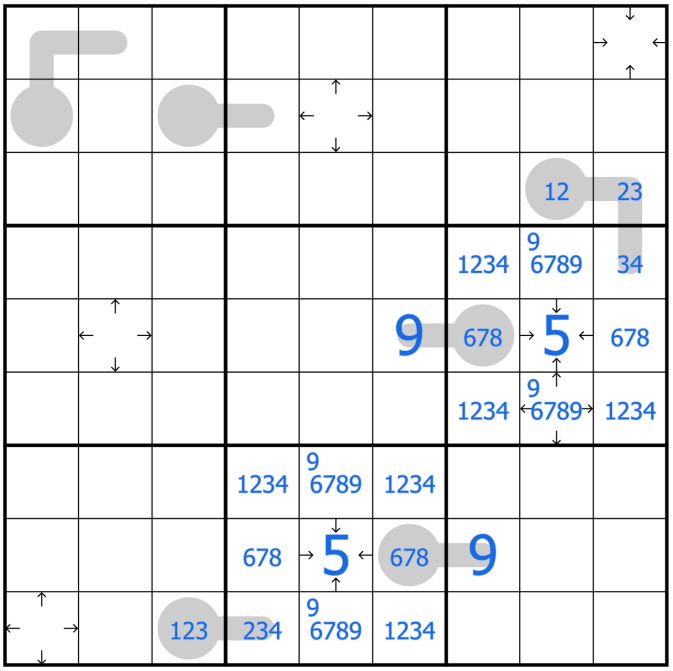
WALKTHROUGH OF SOLUTION:

A. Central Valleys

Let's start by considering the valley in R8C5. By definition, all orthogonally adjacent cells are greater than it. Those adjacent cells at most can be a 6/7/8/9 quadruple, which means R8C5 can be at most 5. The valley definitely sees its orthogonally adjacent cells, so it sees at least four cells. Therefore R8C5 is 4 or 5. The thermometer tells us R8C7 is greater than R8C6, so if the valley sees R8C6, then it also sees R8C7, resulting in now seeing five cells. Therefore R8C5 is 5, and so its orthogonally adjacent digits form a 6/7/8/9 quadruple. Because of the thermometer, R8C6 is 6, 7, or 8, and so R8C7 is 7, 8, or 9.

If the 9 in Column 5 were in one of the first six cells, then the valley in R8C5 would see it, resulting in a sixth cell seen. Therefore the 9 in Column 5 is in Box 8 in R7C5 or R9C5. Therefore R8C4 can't be 9 anymore. If the 9 in Row 8 were in one of the first three cells or the last two cells, then the valley in R8C5 would see it, resulting in a sixth cell seen. Therefore the 9 in Row 8 must be in R8C7.

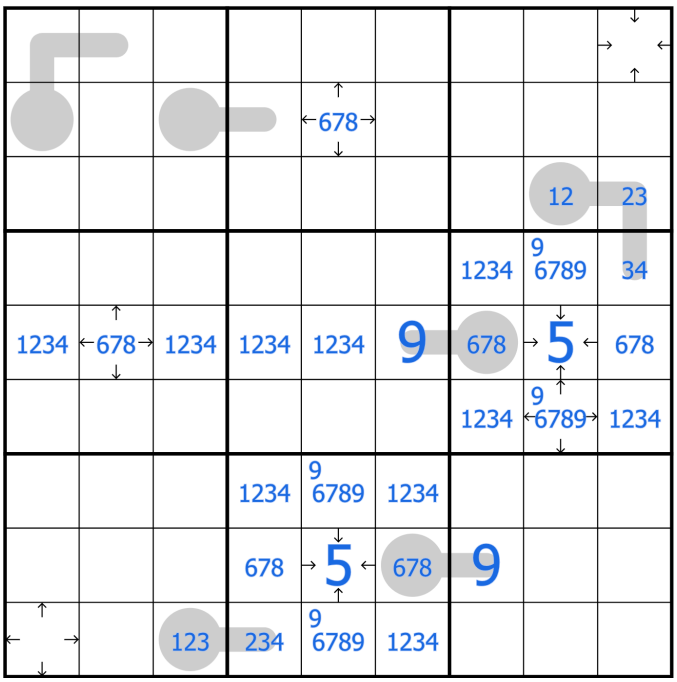
We can place a 1/2/3/4 quadruple in the corner cells of Box 8. Because of the thermometer, R9C4 must be from 2, 3, and 4, and R9C3 must be from 1, 2, and 3.



The same logic above applies to the valley in R5C8 and the thermometer with its bulb in R5C7. Therefore R5C8 is 5 and its orthogonally adjacent digits form a 6/7/8/9 quadruple. Because of the thermometer, R5C7 is 6, 7, or 8, and so R5C6 is 7, 8, or 9.

If the 9 in Column 8 were in one of the first three cells or the last three cells, then the valley in R5C8 would see it, resulting in a sixth cell seen. Therefore the 9 in Column 8 is in Box 6 in R4C8 or R6C8. Therefore R5C9 can't be 9 anymore. If the 9 in Row 5 were in one of the first five cells, then the valley in R5C8 would see it, resulting in a sixth cell seen. Therefore the 9 in Row 5 must be in R5C6.

We can place a 1/2/3/4 quadruple in the corner cells of Box 6. Because of the thermometer, R4C9 must be from 3 and 4, R3C9 must be from 2 and 3, and R3C8 must be from 1 and 2.



B. Central Peaks

Now let's consider the peak in R5C2. By definition, all orthogonally adjacent cells are less than it. Those adjacent cells at least can be a 1/2/3/4 quadruple, which means R5C2 is at least 5. There is already a 5 and 9 in Row 5, so R5C2 is 6, 7, or 8. The remaining cells in the row—R5C1, R5C3, R5C4, and R5C5—form a 1/2/3/4 quadruple.

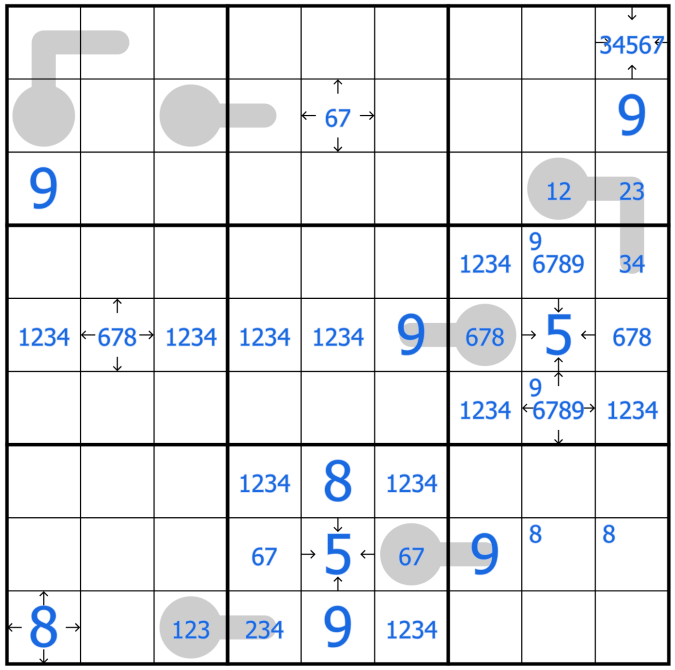
The peak in R2C5 is at least 5 for the same reasons, and

the 5 and 9 in Column 5 are already accounted for, so R2C5 is also 6, 7, or 8.

C. The Corners

The valley in R1C9 has two orthogonally adjacent cells which at most are an 8/9 pair, so R1C9 is at most 7. Therefore the only place for 9 in Column 9 is R2C9. The valley definitely sees R2C9 (which is a 9), R1C8 (which is not a 9), *and* wherever the 9 in Row 1 is. Therefore the valley is at least 3.

The peak in R9C1 has two orthogonally adjacent cells which at least are a 1/2 pair, so R9C1 is at least 3. A peak can never be 9, because then it would be greater than all eight other cells in the column and all eight other cells in the row, resulting in seeing sixteen cells. R9C2 must be less than R9C1, so the only place for 9 in Row 9 is R9C5.

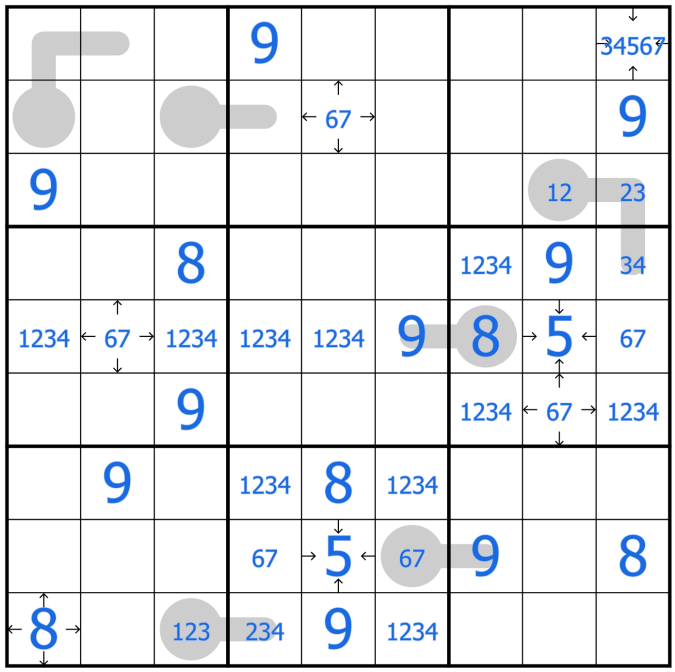


Now let's go back to the valley in R8C5. If R7C5 were 6 or 7, there would be an 8 in one of the first six cells of Column 5, resulting in the valley seeing a sixth cell. Therefore R7C5 is 8. Since R8C4 is now 6 or 7, the 8 in Row 8 can't be in the first three cells anymore, because then the valley would see at least six cells again. Therefore the 8 in Row 8 is in R8C8 or R8C9. Now where does the 8 in Row 9 go? It can't go in R9C2 because R9C1 has to be greater and we said R9C1 can't be 9. Therefore R9C1 is 8.

The peak in R9C1 sees R9C2 because it is adjacent, and R9C3 and R9C4 are seen because they are definitely less than 8. R9C5 is 9, so it sees exactly three cells to the east. Therefore to see a total of eight cells, the peak must see five cells to the north. The digit 8 as a peak sees everything except 9, so the 9 in Column 1 must go in R3C1 in order for the peak to see exactly five cells to the north.

D. 9's and 8's

We can place the remaining 9's. The only place for 9 in Box 2 is R1C4. In Column 2, there can't be a 9 in the first three cells nor the last two cells. R4C2 and R6C2 can't be 9 because those cells must be less than the peak in R5C2. Therefore The 9 in Column 2 must be in R7C2.



R6C8 is a peak, and therefore cannot be a 9 because we previously established peaks can never be 9. So the 9 in Box 6 is in R4C8. The final 9 in the grid goes in R6C3.

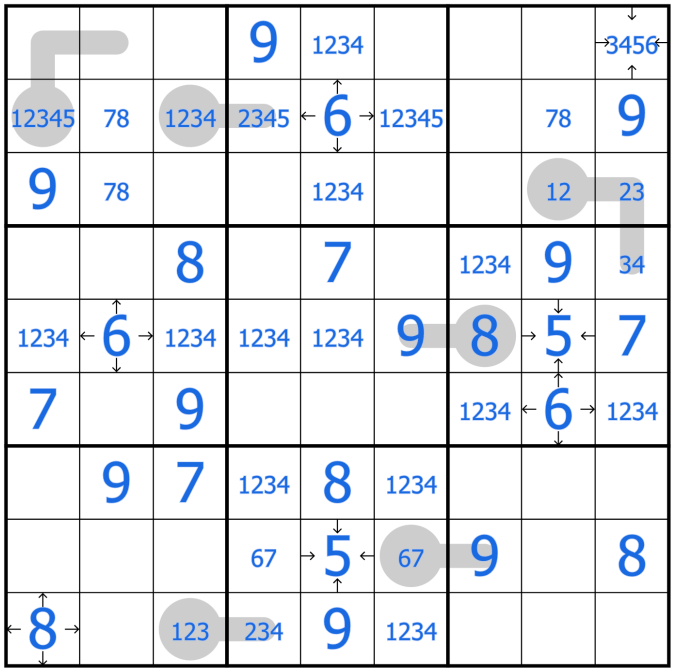
Now where does the 8 go in Row 8? If it went in R8C8, then R6C8 would be 6 or 7 and the valley in R5C8 would see R8C8, resulting in it seeing at least six cells. Therefore R8C9 is 8.

Since the 9's are all placed, it's easier to determine if a peak can be 8. If R5C2 were 8, it would see all four cells

to the north, one cell to the west, three cells to the east, and one cell to the south, resulting in seeing nine cells instead of 8. Therefore R5C2 is not 8 and so the 8 in Row 5 is in R5C7. Since R5C2 is now 6 or 7, none of its orthogonally adjacent cells can be 8. The only place for 8 in Box 4 is R4C3.

E. Back to Central Peaks

Let's go back to the peak in R2C5, which is now a 6 or 7. It sees all four orthogonally adjacent cells. Because of the thermometer, R2C3 is less than R2C4, so R2C5 sees R2C3 as well, seeing a total of five cells now. Could the peak be 7? If so, R4C5, R5C5, and R6C5 would all be less than it, resulting in a total of at least eight cells seen. Therefore R2C5 is 6, so this peak sees one more cell. It if saw R4C5, then it'd also see R5C5 which is 1, 2, 3, or 4. Therefore it doesn't see R4C5, so R4C5 must be greater than 6 and the only option is 7. The bulb in R2C1 must be between 1 and 5 in order for the thermometer to work. Therefore if the peak were to see R2C2, then it'd also see R2C1. Again we see too many cells. Therefore R2C2 is 7 or 8. The final cell the peak sees must be R2C7, so R2C8 is also 7 or 8.

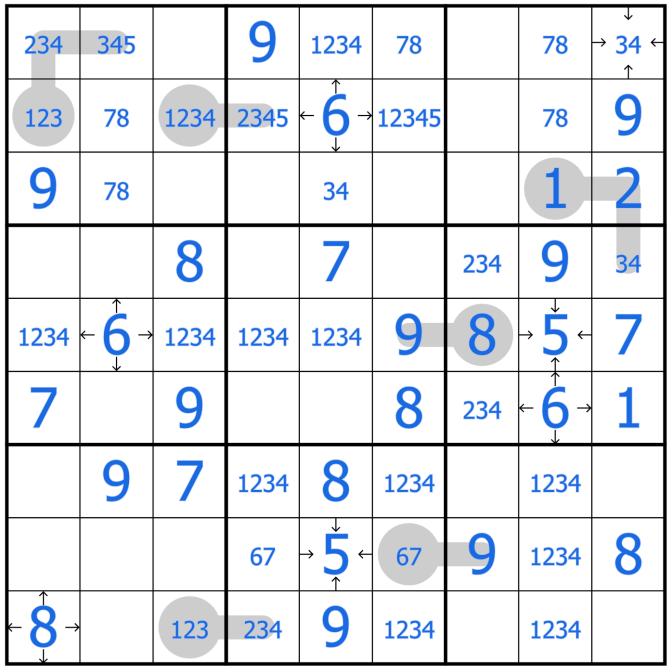


Note that R5C2 and R5C9 are a 6/7 pair, and R5C9 and R6C8 are also a 6/7 pair, meaning that R5C2 and R6C8 are the same digit. If they were both 7, then both R2C2 and R2C8 would be 8. Therefore R5C2 and R6C8 are 6, and R5C9 is 7.

The peak in R5C2 sees all four orthogonally adjacent cells, as well as R5C4 and R5C5, which are from 1, 2, 3, and 4. That's already all six cells the peak can see. Thus R3C2 must be greater than 6, so it is 7 or 8. Now the only place for 7 in Box 4 is R6C1. In Box 7, the 7 must go in R7C3.

F. R6C8 and R1C9

The peak in R6C8 sees its four orthogonally adjacent cells. Note that R8C8 cannot be 7, 8, or 9, so the peak sees R8C8, totalling to five cells. Which cell is the final one that the peak sees? If it were R6C6, the peak would also see R6C5, which is 1, 2, 3, or 4. Therefore the peak does not see R6C6, so R6C6 is greater than 6 and the only option is 8. The final cell the peak sees must be R9C8. That means the last three cells of Column 8 are from 1, 2, 3, and 4 while R1C8 and R2C8 form a 7/8 pair.



Now let's consider the valley in R1C9. First, we know that in Box 2, the 7 and 8 must go in two of the three cells: R1C6, R3C4, and R3C6. One of the digits 7 or 8 must be in R1C6, otherwise we'd have three cells from 7 and 8 in Row 3 (R3C2, R3C4, R3C6). The valley in R1C9, sees its two orthogonally adjacent cells, and the 9 in R1C4. It doesn't see R1C5 nor R1C7 which will be less than—therefore hidden by—7/8. The valley may see R1C6. Therefore the valley is 3 or 4.

We now have a 3/4 pair in Column 9, so R3C9 is 2, R3C8 is 1, and R6C9 is 1.

Looking at the top left thermometer, the end at R1C2 can be a maximum of 5. Therefore R2C1 is 1, 2, or 3; R1C1 is 2, 3, or 4; and R1C2 is 3, 4, or 5.

G. Finishing Up With Some Sudoku

The only place for 6 in Box 7 is R7C1. R7C9 is a naked single, a 5, and that puts a 6 in R9C9. The only place for 7 in Box 9 is R9C7. R9C2 must be 5.

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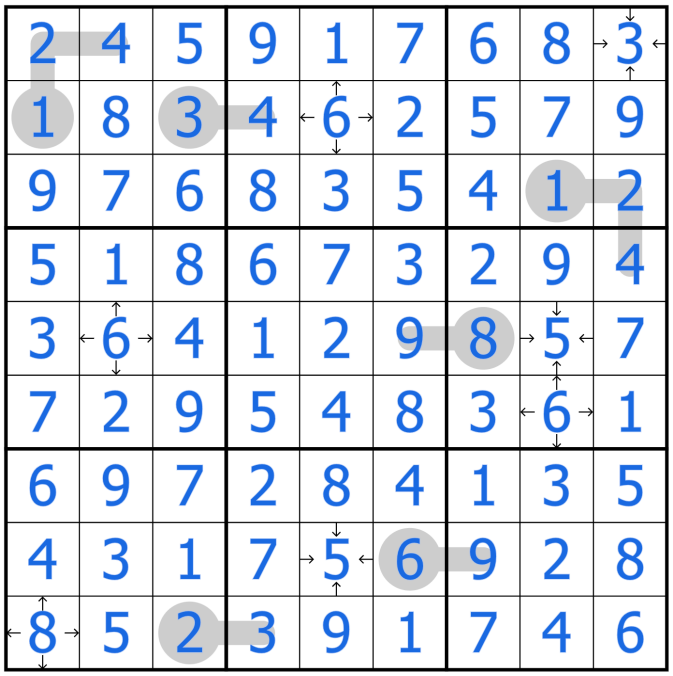
Now we have a 3/4 pair in Row 1. R1C1 must be 2, so R2C1 is 1. The only place for 1 in Row 1 is R1C5 and the only place for 2 in Row 2 is R2C6 (a 2 in R2C4 would force the bulb in R2C3 to be 1, putting a second 1 in the row).

In Row 6, the only place for 5 is R6C4. Going back to Row 2, the thermometer end in R2C4 is at most 4, and the bulb in R2C3 is at least 3. Therefore R2C3 is 3 and R2C4 is 4. R1C2 must be 4, so R1C9 is 3, R4C9 is 4, and R3C7 is 4



R2C7 must now be 5, and R1C7 is 6. R1C3 is 5, so R3C3 is 6. R3C6 and R4C1 are the remaining 5's. R3C5 is a naked single, a 3.

R7C7 must be 1 and R9C6 must be 1. R9C3 is 2, R9C4 is 3, and R9C8 is 4, so R7C4 is 2, R7C6 is 4, R7C8 is 3, and R8C8 is 2.

R1C6 is 7, so R1C8 is 8, R2C8 is 7, R2C2 is 8, R3C2 is 7, and R3C4 is 8.

R8C6 is 6 and R8C4 is 7. R4C6 is 3, so R4C7 is 2, R6C7 is 3, R6C2 is 2, R6C5 is 4, and R5C5 is 2.

R4C2 is 1, R4C4 is 6, R5C4 is 1, R5C3 is 4, and R5C1 is 3

Lastly, R8C1 is 4, R8C2 is 3, and R8C3 is 1.