Worked examples; $F = Gm_1m_2/r^2$

Worked examples – Newton's Law of universal gravitation – teacher's sheet

Note: Remember that a lot of students have difficulties in using standard form correctly on a scientific calculator. You may need to tutor them in the use of the EXP button.

Data required: $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, mass of the Earth = 6.0×10^{24} kg, radius of the Earth = 6.4×10^6 m, mass of the Sun = 2.0×10^{30} kg, average distance from the Earth to the Sun = 1.5×10^{11} m.

1) Communications satellites orbit the Earth at a height of 36 000 km. How far is this from the centre of the Earth? If such a satellite has a mass of 250 kg, what is the force of attraction on it from the Earth?

It is (3.6×10^7 m + 6.4×10^6 m) = 4.24×10^7 m from the centre of the Earth. (They should really give this as 4.2×10^7 m – this may be an opportunity to reinforce the role of significant figures in physical calculations).

The force is $F = Gm_1m_2/r^2 = (6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 250)/ (4.24 \times 10^7)^2$. This gives an answer of about 56 N, which for information is about less than the weight as a one year old toddler.

2) What is the force of attraction from the Earth on you? What do we call this force? What is the force of attraction on the Earth from you?

They will need to estimate their own mass in kg. If they need to convert, 1 stone is 6.4kg (and there are 14 pounds in a stone). They then use $F=Gm_1m_2/r^2$ where r is the radius of the Earth.

This force is usually called their weight.

The force on the Earth from the student is exactly the same as their first answer, but in the opposite direction.

3) What is the force of attraction from the Sun on you? How many times smaller is this than the force of attraction from the Earth on you?

Again, they will need to use their own mass, and the equation $F=Gm_1m_2/r^2$, but this time r is the average distance from the Sun to the Earth. This force should be about 1650 times less than their weight, of the order of 0.3-0.5 N. Small, but not negligible.

4) The average force of attraction on the Moon from the Sun is 4.4×10^{20} N. Taking the distance from the Sun to the Moon to be about the same as that from the Sun to the Earth, what value of mass does this give for the Moon?

$m_2 = Fr^2/Gm_1 = (4.4 \times 10^{20} \times (1.5 \times 10^{11})^2)/(6.67 \times 10^{-11} \times 2.0 \times 10^{30}) = 7.4 \times 10^{22}$ kg

5) Using the mass of the Moon you calculated in question 4, what is the pull of the Earth on the Moon, if the Moon is 380 000 km away? How does this compare with the pull of the Sun on the Moon?

$F = Gm_1m_2/r^2 = (6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 7.4 \times 10^{22})/ (3.8 \times 10^8)^2 = 2.1 \times 10^{20}$ N

This is actually smaller than the pull of the Sun on the Moon. You could discuss whether that means the Moon is orbiting the Sun rather than the Earth. In fact, it depends on the most useful frame of reference in a particular situation – from the Sun's point of view, the Moon and the Earth orbit the Sun, in a way that is affected by the presence of the other; from the Moon's point of view, both the Sun and the Earth orbit the Moon, in a way that is affected by the presence of the other, etc.

6. What is the force of attraction between two people, one of mass 80 kg and the other 100 kg if they are 0.5m apart?

$$F = Gm_1m_2/r^2$$

$$F = G \times 100 \times 80 / 0.5^2 = 2.14 \times 10^{-6} \text{ N.}$$

This is a very small force but it does increase as the people get closer together!

Actually this example is not accurate because Newton's law really only applies to spherical objects, or at least objects so far apart that they can be effectively considered as spherical.

7. What is the force of attraction between the Earth and the Sun?

Mass of the Sun = 2×10^{30} kg, mass of the Earth = 6×10^{24} kg, distance from the Earth to the Sun = 1.5×10^{11} m

$$F = Gm_1m_2/r^2$$

$$F = G \times 2 \times 10^{30} \times 6 \times 10^{24}/ [1.5 \times 10^{11}]^2 = 6.7 \times 10^{11} \text{ N} \qquad \text{an enormous force!}$$

External reference

Questions 6 and 7 taken from Resourceful Physics