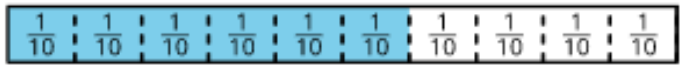
Course Title: Mathematics	Full Year	Required
Course Description: The mathematical work for Grade 4 is partitioned into 9 units: ● Understanding Factors and Multiples ● Fraction Equivalence and Comparison ● Extending Operations to Fractions ● From Hundredths to Hundred-thousands ● Multiplicative Comparison and Measurement ● Multiplying and Dividing Multi-digit Numbers ● Angles and Angle Measurement ● Properties of Two-dimensional Shapes ● Putting it All Together		
Additional Course Information: The big ideas in Grade 4 include: ● generalizing place value understanding for multi-digit whole numbers. ● using place value understanding and properties of operations to perform multi-digit arithmetic and solve problems. ● developing understanding and fluency with multi-digit multiplication ● developing understanding of dividing to find quotients involving multi-digit dividends ● building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers. ● developing an understanding of fraction equivalence and ordering, as well as addition and subtraction of fractions with like denominators ● multiplication of fractions by whole numbers ● understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. ● Required fluency: Add and subtract within 1,000,000 (4.NBT.B.4)	**Core Resources:** **Illustrative Mathematics** **Instructional Routines and Math Language Routines** **Grade 4 Glossary** **Grade 4 Unit 1 Glossary** **Required Materials** **IM en Español:** **Grade 4 en Español**) **Developing a Mathematical Community**	**Are there any attachments at the course level that teachers will need?** Grade 4 Scope and Sequence Pacing Guide and Dependency Diagrams K-5

Unit 2: Fraction Equivalence and Comparison	Duration: *19 days*
Unit Overview - FOCUS: In this unit, students extend their prior understanding of equivalent fractions and comparison of fractions. In grade 3, students partitioned shapes into parts with equal area and expressed the area of each part as a unit fraction. They learned that any unit fraction, 1/*b*, results from a whole partitioned into *b* equal parts. They used unit fractions to build non-unit fractions, including fractions greater than 1, and represent them on fraction strips and tape diagrams. The denominators of these fractions were limited to 2, 3, 4, 6, and 8. Students also worked with fractions on a number line, establishing the idea of fractions as numbers and equivalent fractions as the same point on the number line. Here, students follow a similar progression of representations. They use fraction strips, tape diagrams, and number lines to make sense of the size of fractions, generate equivalent fractions, and compare and order fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students generalize that a fraction a/b is equivalent to fraction $\frac{(n\times a)}{(n\times b)}$ because each unit fraction is being broken into *n* times as many equal parts, making the size of the part *n* times as small $\frac{1}{(n\times b)}$ and the number of parts in the whole *n* times as many (*n* x a) . For example, we can see 3/5 is equivalent to 6/10 because when each fifth is partitioned into 2 parts, there are 2 x 3, or 6 shaded parts, twice as many as before, and the size of each part is half as small, $\frac{1}{(2\times 5)}$ or $\frac{1}{10}$. 	**Topic Titles:** **How will you break up the unit into big ideas?** ● Section A: Size and Location of Fractions ○ Make sense of fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12 through physical representations and diagrams. ○ Reason about the location of fractions on the number line ● Section B: Equivalent Fractions ○ Generate equivalent fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100. ○ Use visual representations to reason about fraction equivalence, including using benchmarks such as ½ and 1. ● Section C: Fraction Comparison ○ Use visual representations or a numerical process to reason about fraction



As the unit progresses, students use equivalent fractions and benchmarks such as 1 to reason about the relative location of fractions on a number line, and to compare and order fractions.

comparison.

Coherence: How does this unit build on and connect to prior knowledge and learning?

In grade 3, students partitioned shapes into parts with equal area and expressed the area of each part as a unit fraction. They learned that any unit fraction, 1/*b*, results from a whole partitioned into *b* equal parts. They used unit fractions to build non-unit fractions, including fractions greater than 1, and represent them on fraction strips and tape diagrams. The denominators of these fractions were limited to 2, 3, 4, 6, and 8. Students also worked with fractions on a number line, establishing the idea of fractions as numbers and equivalent fractions as the same point on the number line.

Essential Questions:

1. How can we make sense of fractions?
2. How can we name or identify a fraction in different ways?
3. How do we compare fractions?

Enduring Understanding:

We can use physical representations, diagrams, and number lines to make sense of fractions. These tools can help us to compare size, parts, and location. Fractions can be built from unit fractions by applying and extending understandings of operations on whole numbers.

Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value. The same fractional amount can be represented by an infinite set of different but equivalent fractions. Equivalent fractions are found by using factors and multiples. Although represented differently, these equivalent fractions have the same location on the number line.

Physical representations, diagrams, and number lines can help us compare fractions. Benchmark fractions such as 1⁄2 are useful when comparing two fractions to each other. Fraction models such as fraction bars and number lines are useful when determining fraction equivalence.

What Students Will Know:	What students will do:	Unit Specific Vocabulary:
• A numerator is a part of a whole. A numerator is the top number in a fraction. • A denominator is how many pieces make a whole. A denominator is the bottom number in a fraction. • Unit fractions have a numerator of 1. • Fractions can be compared to benchmark fractions of ½ and 1 whole. • Fractions can be represented visually. • Fractions hold value. • Benchmark fractions can be easily used. Examples include: ½ and 1 whole • Fractions can be rewritten to have like denominators. This is useful when comparing fractions. • Fractions can be ordered and compared on a number line.	• Make sense of unit fractions with denominators of 2, 3, 4, 5, 6, 8, 10 and 12 using physical and visual representations • Make sense of non-unit fractions (including those greater than 1) that have denominators of 2, 3, 4, 5, 6, 8, 10, and 12 • Use the meaning of numerator and denominator to compare fractions with the same numerator or same denominator • Use visual representations to reason about the fractions that have the same size and locate them on a number line • Compare fractions to ½ and 1 whole • Compare fractions using common numerators or denominators or relationships such as ½ and 1 benchmarks • Compare two fractions by rewriting one or both of them as an equivalent fraction with like denominators • Compare fractions to solve word problems in and out of context • Order fractions using any strategy • Locate and compare fractions on a number line	Doing Math Math Community **Academic Vocabulary:** Unit Fraction Part Whole Numerator Denominator Partition Diagram Greater than Less than Equal to Equivalent Tick Marks Number Line Equivalent Partitioning Factor Legend Table Axis Factors Multiples Common Denominator
Entry Level Assessment and Connection to Unit: Grade 4 Unit 2 Entry-Level Assessment	**Unit Materials, Resources and Technology:** • **Illustrative Mathematics** • **Instructional Routines and Math Language Routines**: • **Grade 4 Glossary** • **Required Materials** • **IM en Español** • **Pacing Guide and Dependency Diagrams K-5**	

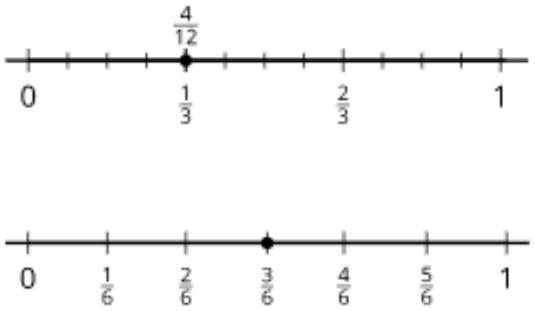
Opportunities for Interdisciplinary Connections: Baking can be a great way to showcase equivalent fractions. Students can investigate different equivalent fractions by using measuring cups of different sizes along with something that they can measure (for instance: rice). Through their investigation, students can discover that 1 cup is equivalent to 2 ½ cups, 3 ⅓ cups, and so on.	
Any links, attachments and resources: Instructional Routines Document Family Support Materials Unit 2	**Planning Ideas:** Components of a Typical IM Lesson What To Know About IM When Planning Gr 4 Where to Find the Mathematical Practices in the Units Assessing the Mathematical Practices

Topic # 1 (Section A)	Topic Name: Section A - *Size and Location of Fractions*	Duration: 6 days (6 lessons)

Topic Description:

In this section, students revisit ideas and representations of fractions from grade 3, working with denominators that now include 5, 10, and 12. They use physical fraction strips, diagrams of fraction strips, tape diagrams, and number lines to make sense of the size of fractions and fractional relationships.

Students reason about the relationship between fractions where one denominator is a multiple of the other denominator (such as 1/5 and 1/10 , or 1/6 and 1/12). They consider different ways to represent these relationships. Students also compare fractions to benchmarks such as 1/2 and 1.



The work in this section prepares students to reason about equivalence and comparison of fractions in the subsequent lessons.

Section Learning Goals

- Make sense of fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12 through physical representations and diagrams.
- Reason about the location of fractions on the number line.

Competencies Addressed:	Essential Question and Enduring Understanding Addressed in this Topic:
Understanding and Applying Number Systems 3. I can add and subtract whole numbers. **4.NBT.4** 5. I can apply my understanding of fractions for equivalence and comparing. **4.NF.A.1-2**	**Essential Question** How can we make sense of fractions?

	Enduring Understanding **We can use physical representations, diagrams, and number lines to make sense of fractions**. These tools can help us to compare size, parts, and location. Fractions can be built from unit fractions by applying and extending understandings of operations on whole numbers.
In this Topic, students will know: ● A numerator is a part of a whole. A numerator is the top number in a fraction. ● A denominator is how many pieces make a whole. A denominator is the bottom number in a fraction. ● Unit fractions have a numerator of 1. ● Fractions can be compared to benchmark fractions of ½ and 1 whole. ● Fractions can be represented visually. ● Equivalent fractions represent the same amount ● Equivalent fractions can be shown visually or on a number line ● Factors and multiples can be used to generate equivalent fractions	**Topic Vocabulary:** Doing Math Math Community **Academic Vocabulary:** Unit Fraction Part Whole Numerator Denominator Partition Diagram Greater than Less than Equal to Equivalent Tick Marks Number Line
In this Topic, students will be able to: ● Make sense of unit fractions with denominators of 2, 3, 4, 5, 6, 8, 10 and 12 using physical and visual representations ● Make sense of non-unit fractions (including those greater than 1) that have denominators of 2, 3, 4, 5, 6, 8, 10, and 12 ● Use the meaning of numerator and denominator to compare fractions with the same numerator or same denominator	**Plan for Student Reflection:** Student Journal Prompts and Reflection Practices **Plan for Teacher Reflection:** **Teacher Journal Reflecting Questions:**

● Use visual representations to reason about the fractions that have the same size and locate them on a number line ● Compare fractions to ½ and 1 whole	**Lesson 1:** What did you learn about each student and their foundational understanding of fractions based on their work today? **Lesson 2:** Who participated in math class today? What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class? **Lesson 3:** Most students may find it more intuitive to compare fractions with a common denominator than those with a common numerator. Did you see students who grasp both equally well? How did they conceptualize the latter? **Lesson 4:** This lesson is students' first experience with the number line in grade 4. What understandings or misunderstandings about the number line did you observe today as students worked? Did you see students relating the idea of partitioning a tape diagram to partitioning a number line? **Lesson 5:** In the next lesson, students will be comparing fractions to ½ and 1, applying what they know about equivalence and distance on a number line. How did today's work prepare them for that lesson? **Lesson 6:** What question asked today seemed to promote students' reasoning about benchmarks to compare fractions?

	Utilize additional strategies for Teacher Reflection: ● Reviewing formative assessments ● Developing scaffolds ● Collaborative scoring ● PLCs ● Planning for small groups

Topic 1 (Section A) Task Development

<table>
<tr><td>Task Title: Topic 1 - Size and Location of Fractions</td><td>Grade Level and Unit: Fourth Grade, Unit 2</td></tr>
<tr><td>Description of Task:
In this optional activity, students sort a set of fractions into groups based on whether they are less than, equal to, or greater than ½.</td><td>Purpose of Task:
Sorting enables students to estimate or to reason informally about the size of fractions relative to this benchmark before they go on to do so more precisely. As students discuss and justify their decisions, they share a mathematical claim and the thinking behind it (MP3).</td></tr>
<tr><td>Background of Students/Learning Progression:
In grade 3, students were introduced to fractions as numbers. They learned to name and represent fractions, to recognize simple equivalent fractions, and to compare fractions with like numerators and denominators (limited to 2, 3, 4, 6, and 8). They used fraction strips, area diagrams, tape diagrams, and number lines to support their reasoning with fractions.</td><td>Ensure all competencies are addressed in the task:

▢ Yes, all competencies are addressed
▢ No - Task needs modification</td></tr>
<tr><td colspan="2">Getting Started:
The purpose of this warm-up is to invite students to share what they know about the number ½ and elicit ways in which it can be represented. It gives the teacher the opportunity to hear students' understandings about and experiences with fractions, ½ in particular.

What do you know about ½?

Sample responses:
● It is a fraction.
● I shared half of my sandwich with my friend.
● It is what we get when we split something into two parts.
● We can "halve" something.
● Dividing by 2.
● It is halfway between 0 and 1 on a number line.
● It is less than 1.
● It is a number.

As a whole group, discuss the following: "What different ways can we represent ½?" (Cut an object, a rectangle, or another shape into two equal</td></tr>
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parts, mark the middle point between 0 and 1 on a number line.)

Learning Cycle Model:

Section A

IM Lesson	L1: Representations of Fractions Part 1	L2: Representations of Fractions Part 2	L3: Same Denominator or Numerator	L4: Same Sizes/ Related Sizes	L5: Fractions on Number Lines	L6: Relate Fractions to Benchmarks
Learning Cycle Model	**Make Meaning**	**Make Meaning**	**Investigating**	**Investigating**	**Investigating**	**Create & Produce**
Naugatuck Math Competency	**4.NS.5**	**4.NS.5**	**4.NS.3** **4.NS.5**	**4.NS.5**	**4.NS.5**	**4.NS.5**
Math Practice Standards	-	MP6	MP3, MP8	MP7	MP8	MP7
Lesson Purpose	The purpose of this lesson is for students to make sense of unit fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12, using physical and visual representations.	The purpose of this lesson is for students to make sense of non-unit fractions (including those greater than 1) that have denominators 2, 3, 4, 5, 6, 8, 10, and 12.	The purpose of this lesson is for students to use the meaning of numerator and denominator and to compare fractions with the same numerator or the same denominator.	The purpose of this lesson is for students to use visual representations to reason about the fractions that have the same size and to locate them on the number line.	The purpose of this lesson is for students to recognize that equivalent fractions describe the same point on the number line and to identify such fractions on the number line.	The purpose of this lesson is for students to locate fractions on the number line and compare their size to ½ and to 1.
Vocabulary Focus	Part Whole Numerator Denominator Partition Unit Fraction	Diagram	Greater than Less than	Equivalent Tick marks	Number line	Greater than Less than Equal to
Lesson Materials/ Resources	Lesson 1 Slides Teacher Presentation Materials Student Pages *For Activity 1:* ● Each group of 2 needs 4 strips of equal-size paper	Lesson 2 Slides Teacher Presentation Materials Student Pages *For Activity 1:* ● Give each student a straightedge	Lesson 3 Slides Teacher Presentation Materials Student Pages	Lesson 4 Slides Teacher Presentation Materials Student Pages *For Activity 1:* ● Give each student a straightedge	Lesson 5 Slides Teacher Presentation Materials Student Pages *For Activity 1:* ● Give students access to	Lesson 6 Slides Teacher Presentation Materials Student Pages *For Activity 2:* ● Create a set of fraction cards from the blackline master

	(cut lengthwise from letter-size or larger paper or use the provided blackline master) ● Give each group a straightedge *For Activity 2:* ● Give each student a straightedge Cool-Down: **What do the diagrams show?**	*For Activity 2:* ● Each student needs access to their fraction strips from the previous lesson (here is the blackline master) ● Give each student a straightedge **Cool-Down: What do the diagrams show?**	**Cool-Down: Sizing up Fractions**	**Cool-Down: Where on the number line?**	straightedges **Cool-Down: Two of the same**	for each group of students (Where Do They Belong) **Cool-Down: Greater than or Less than**
	Formative Assessment Strategies: observation, questioning, student discourse - Monitoring Sheet **See : Section A Checkpoint, Section A Checkpoint Teachers Guide**					
Assessment						**Section A Practice Problems**
Suggested Centers	Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing) Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)	Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing) Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)	Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing) Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)	Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing) Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Supporting)	Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing) Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Supporting)	Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing) Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Supporting)

Making Meaning:

Lessons 1 and 2 provide opportunities in which the teacher can activate students' prior knowledge of unit fractions and include fractions with new denominators 5, 6, 10, and 12. Students revisit the meaning of numerator and denominator, name unit fractions, create representations for them, and recall some strategies and tools for reasoning about fractions.

The idea of equivalence may naturally come up (and will help to prepare students for upcoming work), but it is not the focus of this lesson.

In Lesson 2, students are reminded of what they learned in grade 3: that a non-unit fraction can be understood as parts of a unit fraction , and that fractions with different numerators and denominators can be equivalent. Unlike in grade 3, the denominators they see here now include 5, 10, and 12.

In both lessons, rulers can be provided to help students draw, extend, or align partition lines, but should not be used to measure the location of a fraction on any diagram.

Lesson 1: Representations of Fractions (Part 1)

- The purpose of this lesson is for students to make sense of unit fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12, using physical and visual representations.
- Teacher Presentation Materials
- Lesson 1 Slides

Lesson 2: Representations of Fractions (Part 2)

- The purpose of this lesson is for students to make sense of non-unit fractions (including those greater than 1) that have denominators 2, 3, 4, 5, 6, 8, 10, and 12.
- Teacher Presentation Materials
- Lesson 2 Slides

Investigation:

This series of lessons provides students with opportunities to investigate different characteristics of fractions. In Lesson 3, students reason about the relative size of fractions based on the meaning of numerator and denominator, and use fraction strips to support their reasoning. Students first compare pairs of fractions with the same denominator. They recall that fractions with the same denominator are composed of the same unit fractions or have parts that are the same size, so the numerators can tell us how the fractions compare: the greater the numerator, the greater the fraction. Next, students compare fractions with the same numerator. They recognize that we cannot simply look at the denominators and see which is greater. Because the denominator tells us the number of parts in 1 whole, the greater that number, the smaller the fractional part.

In Lessons 4 and 5, students begin to revisit the idea of equivalence. Students examine fractions that have the same size but are expressed with different numerators and denominators. They use diagrams of fraction strips, now expanded to include fractions with denominator 10 and 12, and then transition to using number lines to support their reasoning. To determine whether two fractions are equivalent, students rely on their

understanding of fractions with related denominators (in which one denominator is a multiple of another). They practice thinking of certain fractions in terms of other fractions (for instance, thinking that they can split 1 third into 2 sixths, or 1 fifth into 2 tenths).

The relationships between fractions such as ¼ and ⅛, ⅕ and 1/10, and ⅙ and 1/12, in which one denominator is a multiple of the other, continue to be highlighted and offer many opportunities for students to look for and make use of structure (MP7).

As in earlier activities, rulers can be provided to help students draw, extend, or align partition lines, but should not be used to measure the location of a fraction.

Lesson 3: Same Denominator or Numerator

- The purpose of this lesson is for students to use the meaning of numerator and denominator and to compare fractions with the same numerator or the same denominator.
- Teacher Presentation Materials
- Lesson 3 Slides

Lesson 4: Same Size, Related Sizes

- The purpose of this lesson is for students to use visual representations to reason about the fractions that have the same size and to locate them on the number line.
- Teacher Presentation Materials
- Lesson 4 Slides

Lesson 5: Fractions on Number Lines

- The purpose of this lesson is for students to recognize that equivalent fractions describe the same point on the number line and to identify such fractions on the number line.
- Teacher Presentation Materials
- Lesson 5 Slides

Create and Produce:

In this lesson, students continue to identify fractions on the number line by reasoning about known distances or intervals. They also consider the size of fractions in relation to ½ and 1, by examining the position and distance of fractions from these benchmarks on the number line.

Although students consider the distance between a point on the number line and either ½ or 1, finding differences of two fractions is not the focus of this lesson. (That mathematical work will take place in a future unit.) What is important is for students to reason about the relative sizes

of fractions using number lines, their knowledge of equivalent fractions and familiar benchmarks, and the meaning of numerator and denominator. Activity 2 is optional and allows an opportunity for students to use the relationships between numerator and denominator in a fraction and between different denominators without requiring them to use the number line.

Lesson 6: Relate Fractions to Benchmarks

- The purpose of this lesson is for students to locate fractions on the number line and compare their size to ½ and to 1.
- Teacher Presentation Materials
- Lesson 6 Slides

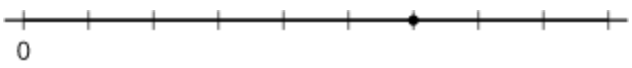
Communicate and Present:

- Invite groups to share how they sorted the fractions.
- "How did the numerator and denominator of each fraction tell you how a fraction relates to ½?" (Sample responses:
  - We already know fractions that are equivalent to ½, so we could compare any fraction to one of those equivalent fractions that has the same denominator.
  - A fraction that is equal to ½ has a denominator that is twice the numerator.
  - If a numerator is less than half of the denominator, the fraction is less than ½. If it is more than half of the denominator, it is more than ½.
  - If a numerator is 1 or is much less than the denominator, then the fraction is small and less than ½.
  - If a numerator is really close to the denominator, then the fraction is close to 1, which means it is more than ½.)
- Give students 2–3 minutes of quiet time to complete the sentence frames in the activity.

Reflection:

"Today we identified fractions on a number line and compared them to ½ and 1."

Display the number line from the warm-up (or ask students to refer to the diagram there).



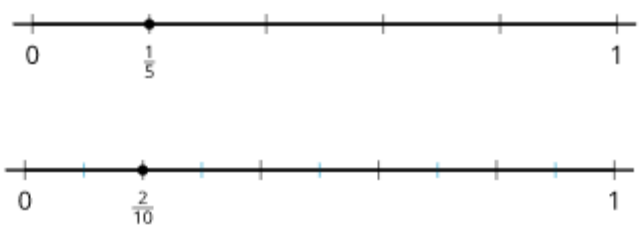
Label one of the tick marks (other than the one with the point) with "½". "Suppose a classmate is absent today, and you are asked to explain how to figure out the fraction that the point represents and how far away it is from ½. What would you say?" (I'd see how far away ½ is from 0 and then double that distance to know where 1 is, which would tell me the size of each space between tick marks. If ½ is 4 spaces away from 0, then 1 must be 8 spaces away, and each space must represent ⅛. I'd count the spaces from 0 to know the fraction. I'd count the spaces between the point and ½ to know its distance from ½.)

Notes: Complete all lessons in numerical order.

Complete File with Resources and Task:
Task-Based Learning Plan Format for Topic 1

Topic # 2 (Section B)	Topic Name: Section B - Equivalent Fractions	Duration: 5 days (5 lessons)

Topic Description:
In this section, students develop their ability to reason about and generate equivalent fractions. They begin by using number lines as a tool for finding equivalent fractions and verifying equivalence of two fractions.



Through repeated reasoning, students notice regularity in the visual representations and begin to make sense of a numerical way to determine equivalence and generate equivalent fractions (MP8). They generalize that fraction *a/b* is equivalent to fraction $\frac{n \times a}{n \times b}$.

Note that students do not need to describe this generalization in algebraic notation. Given their understanding of the size of fractions and relationship between fractions, however, they should be able to explain it with fractions that have denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

As they identify and generate equivalent fractions numerically, students apply their knowledge of factors and multiples from an earlier unit.

Section Learning Goals

- Generate equivalent fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Use visual representations to reason about fraction equivalence, including using benchmarks such as ½ and 1.

Competencies Addressed:	Essential Question and Enduring Understanding Addressed in this Topic:
Understanding and Applying Number Systems 5. I can apply my understanding of fractions for equivalence and comparing. **4.NF.A.1-2**	**Essential Question** How can we name or identify a fraction in

<table>
<tr><td></td><td>different ways?

Enduring Understanding
Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value. The same fractional amount can be represented by an infinite set of different but equivalent fractions. Equivalent fractions are found by using factors and multiples. Although represented differently, these equivalent fractions have the same location on the number line.</td></tr>
<tr><td>In this Topic, students will know:

● Equivalent fractions represent the same amount
● Equivalent fractions can be shown visually or on a number line
● Factors and multiples can be used to generate equivalent fractions</td><td>Topic Vocabulary
Doing Math
Math Community
Academic vocabulary
Equivalent
Partitioning
Factor</td></tr>
<tr><td rowspan="2">In this Topic, students will be able to:

● Generate equivalent fractions using a representation that makes sense to them
● Reason about and generate equivalent fractions on a number line
● Determine if two fractions are equivalent, and if they are explain why they are equivalent
● Identify and generate equivalent fractions by using multiples of the numerator and denominator
● Generate equivalent fractions using factors and multiples of the numerator and denominator</td><td>Plan for Student Reflection:

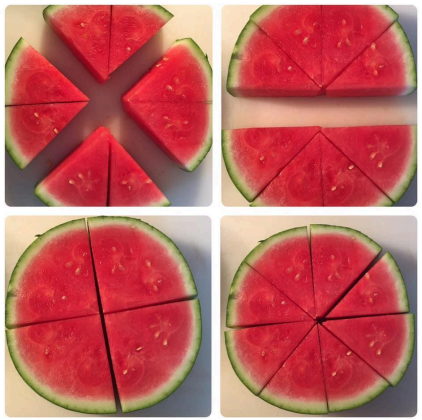
Student Journal Prompts and Reflection Practices</td></tr>
<tr><td>Teacher Journal Reflection Questions:

Lesson 7: Whose ideas and voices were heard, valued, and accepted today? How can you adjust the group structure tomorrow to ensure each student’s ideas are a part of the collective learning?

Lesson 8: In past lessons and in grade 3,</td></tr>
</table>

	students partitioned unit fractions such as 1/2, 1/3, and 1/4 (on fraction strips, tape diagrams, and number lines) into smaller fractional parts such as ⅛ and ⅛ . How readily did students transfer those insights to work with fractions with larger denominators on the number line? What was intuitive to them and what wasn't? **Lesson 9:** Lesson 9 centers on explanations for equivalence. What representations and strategies did most students rely on to justify equivalence? What aspects of the explanation was manageable for them? What was more challenging than anticipated? **Lesson 10:** To reason numerically we hope students begin to describe number relationships without visual representations. Did it seem that students were doing this in today's lesson? Which diagrams are they still holding on to? **Lesson 11:** What evidence did you see of students choosing a method strategically as they generated equivalent fractions? For students who chose a fixed way regardless of the given fractions, what questions could you ask them to prompt them to be more strategic? **Utilize additional strategies for Teacher Reflection:** ● Reviewing formative assessments ● Developing scaffolds ● Collaborative scoring ● PLCs ● Planning for small groups

Topic 2 (Section B) Task Development

Task Title: Topic 2 - Equivalent Fractions	Grade Level and Unit: Fourth Grade, Unit 2
Description of Task: Students generate equivalent fractions by applying the numerical strategies they learned. (Students might opt to use other strategies, but most of the given fractions have numbers that would make visual representation and reasoning inconvenient.) Depending on the given fractions, students need to decide whether it makes sense to multiply or divide the numerator and denominator by a common number.	**Purpose of Task:** The purpose of this task is for students to generate equivalent fractions numerically, by using factors and multiples of the numerator and denominator.
Background of Students/Learning Progression: In grade 3, students learned to recognize and generate simple equivalent fractions. In earlier lessons, they reasoned about the size of fractions and identified some equivalent fractions. Throughout those experiences, they used fraction strips, tape diagrams, number lines, and benchmark fractions to support their reasoning.	**Ensure all competencies are addressed in the task:** ▢ Yes, all competencies are addressed ▢ No - Task needs modification
Getting Started: Show the following image to students:  Ask students, “What equivalent fractions do you see represented in the picture?” As students are sharing their equivalent fractions, record them and have students explain how they know they are equivalent.	

Section B

IM Lesson	L7: Equivalent Fractions	L8: Equivalent Fractions on the Number Line	L9: Explain Equivalence	L10: Use Multiples to Find Equivalent Fractions	L11: Use Factors to Find Equivalent Fractions
Learning Cycle Model	**Making Meaning**	**Making Meaning**	**Making Meaning**	**Investigation**	**Create and Produce**
Naugatuck Math Competency	4.NS.5	4.NS.5	4.NS.5	4.NS.5	4.NS.5
Math Practice Standards	MP6	MP8	MP3 MP6	-	MP8
Lesson Purpose	The purpose of this lesson is for students to generate equivalent fractions using a representation that makes sense to them.	The purpose of this lesson is for students to reason about and generate equivalent fractions on the number line.	The purpose of this lesson is for students to determine if two fractions are equivalent, and if they are, explain why they are equivalent.	The purpose of this lesson is for students to make sense of a way to identify and generate equivalent fractions by using multiples of the numerator and denominator.	The purpose of this lesson is for students to generate equivalent fractions numerically, by using factors and multiples of the numerator and denominator.
Vocabulary Focus	Equivalent	Partitioning			Factor
Lesson Materials/ Resources	Lesson 7 Slides Teacher Presentation Materials Student Pages *For Activity 2:* ● Provide access to tools for creating a visual display to each group of 2 (tools for creating a visual display may include chart paper and markers, whiteboard space and markers, shared online drawing tool, and access to a document camera)	Lesson 8 Slides Teacher Presentation Materials Student Pages *For Activity 1:* ● Consider creating a human number line by placing a strip of masking tape or painter's tape, at least 25 feet long, on the floor of the classroom or a hallway.	Lesson 9 Slides Teacher Presentation Materials Student Pages *For Activity 1:* ● Give students access to rulers or straightedges *For Activity 2:* ● Each group needs 4 sticky notes ● Print How Do You Know. Cut papers on dotted line and place these posters around the room	Lesson 10 Slides Teacher Presentation Materials Student Pages	Lesson 11 Slides Teacher Presentation Materials Student Pages *For Activity 3:* ● Create a set of Fraction Galore cards from the blackline for each group of 3.

	Lesson 7 Cool-Down: Two Equivalent Fractions	Lesson 8 Cool-Down: In Search of Equivalence	Lesson 9 Cool-Down: To Be or Not to Be	Lesson 10 Cool-Down: Fractions of the Same Size	Lesson 11 Cool-Down: Find Three or More
	Formative Assessment Strategies: observation, questioning, student discourse - Monitoring Sheet **See : Section B Checkpoint, Section B Checkpoint Teachers Guide**				
Assessment					**Section B Practice Problems**
Suggested Centers	Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing) Mystery Number (1–4), Stage 4:' Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)	Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing) Mystery Number (1–4), Stage 4:' Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)	Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing) Mystery Number (1–4), Stage 4:' Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)	Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing) Mystery Number (1–4), Stage 4:' Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)	Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing) Mystery Number (1–4), Stage 4:' Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

Making Meaning

In Lesson 7, students continue to rely on different representations and reasoning strategies to generate equivalent fractions (including those with denominators 5, 10, and 12, and fractions greater than 1). They also hone their ability to communicate their reasoning clearly.

In Lesson 8, students take a closer look at the relationships between fractions with denominator 5, 10, and other multiples of 5. They begin to consider the meaning of fractions with denominator 100.

Lesson 9 enables students to consolidate the work so far and communicate their understanding conceptually, before they move on to reason about equivalent fractions numerically in the next lesson.

Students work with some fractions in the hundredths. Although students might try to partition a number line into 100 parts, they are not expected to do so. The idea is to motivate students to look for another way—one that is less tedious and more general—to generate equivalent fractions.

Lesson 7: Equivalent Fractions

- The purpose of this lesson is for students to generate equivalent fractions using a representation that makes sense to them.
- Teacher Presentation Materials
- Lesson 7 Slides

Lesson 8: Equivalent Fractions on the Number Line

- The purpose of this lesson is for students to reason about and generate equivalent fractions on the number line.
- Teacher Presentation Materials
- Lesson 8 Slides

Lesson 9: Explain Equivalence

- The purpose of this lesson is for students to determine if two fractions are equivalent, and if they are, explain why they are equivalent.
- Teacher Presentation Materials
- Lesson 9 Slides

Investigation:

Up until this point, students have used visual representations or other strategies to reason about and generate equivalent fractions. Along the way, they are likely to have noticed patterns in the numerator and denominator of equivalent fractions. While some students may have generalized and applied those observations intuitively, this is the first lesson in which students are prompted to reason numerically about the numbers in equivalent fractions.

Students notice that a fraction $\frac{a}{b}$ has the same location on the number line as a fraction $\frac{n \times a}{n \times b}$, so we can generate fractions that are equivalent to $\frac{a}{b}$ by multiplying both a and b by n. In other words, they can use multiples of a and b to generate fractions that are equivalent to $\frac{a}{b}$. Sample responses are shown in the form $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$ but students do not need to use this notation.

In earlier lessons, students saw that one way to generate equivalent fractions is by grouping unit fractions on a number line into larger units. For instance, 12 twelfths could be put in groups of 3 to make 4 equal parts, each part being a fourth. Or they could be put into groups of 2 to make 6 equal parts, each part being a sixth, which means that $\frac{12}{12} = \frac{4}{4} = \frac{6}{6}$. Some students may have related these observations to the fact that $12 \div 4 = 3$ and $12 \div 2 = 6$. These insights are formalized and generalized in this lesson.

Students have also generated equivalent fractions and verified equivalence by multiplying the numerator and denominator by the same number. In this lesson, they find equivalent fractions by dividing and by a factor that is common to both numbers.

Lesson 10: Use Multiples to Find Equivalent Fractions

- The purpose of this lesson is for students to make sense of a way to identify and generate equivalent fractions by using multiples of the numerator and denominator.
- Teacher Presentation Materials
- Lesson 10 Slides

Create and Produce:

Lesson 11: Use Factors to Find Equivalent Fractions

- The purpose of this lesson is for students to generate equivalent fractions numerically, by using factors and multiples of the numerator and denominator.
- Teacher Presentation Materials
- Lesson 11 Slides

Communicate and Present:

Students will work independently before engaging in partner discussions and sharing their work.

Students will work in small groups.

Reflection:

"What strategy did your group use to find equivalent fractions? How well did the strategy work? How efficient was it?" (We looked at the numerators and denominators to see if they were multiples or factors we recognized.)
"Did you notice any new patterns in the fractions that are equivalent?"

Notes: Complete all lessons in numerical order.

Complete File with Resources and Task:

Task-based Learning Plan Format for Topic 2

Topic # 3 (Section C)	Topic Name: Section C - Fraction Comparison	Duration: 6 days (6 lessons)

Topic Description:
By the time they reach this section, students have an expanded set of understandings and strategies for reasoning about the size of fractions. Here, they further develop these skills and work to compare fractions with different numerators and different denominators.

To make comparisons, students may use visual representations, equivalent fractions, and their understanding of the size of fractions (for instance, relative to benchmarks such as 1/2 and 1). They may rely on the meaning of the numerator and denominator, and choose a way to compare based on the numbers at hand. Students record the results of comparisons with symbols <, =, or > .

At the end of the section, students learn to write equivalent fractions with a particular denominator as a way to compare any fractions, another opportunity to apply the idea of factors and multiples. Having a numerical strategy notwithstanding, students are still encouraged to use flexible methods to reason about the relative size of fractions.

Section Learning Goals

- Use visual representations or a numerical process to reason about fraction comparison

Competencies Addressed:	Essential Question and Enduring Understanding Addressed in this Topic:
Understanding and Applying Number Systems 5. I can apply my understanding of fractions for equivalence and comparing.**4.NF.A.1-2**	**Essential Question** How do we compare fractions? **Enduring Understanding** **Physical representations, diagrams, and number lines can help us compare fractions.** Benchmark fractions such as 1⁄2 are useful when comparing two fractions to each other. Fraction models such as fraction bars and number lines are useful when determining fraction equivalence.

In this Topic, students will know: ● Fractions hold value. ● Benchmark fractions can be easily used. Examples include: ½ and 1 whole ● Fractions can be rewritten to have like denominators. This is useful when comparing fractions. ● Fractions can be ordered and compared on a number line.	**Topic Vocabulary:** Doing Math Math Community **Academic vocabulary** Legend Table Axis Factors Multiples Common Denominator
In this Topic, students will be able to: ● Compare fractions using common numerators or denominators or relationships such as ½ and 1 benchmarks ● Compare two fractions by rewriting one or both of them as an equivalent fraction with like denominators ● Compare fractions to solve word problems in and out of context ● Order fractions using any strategy ● Locate and compare fractions on a number line	**Plan for Student Reflection:** Student Journal Prompts and Reflection Practices **Teacher Journal Reflection Questions:** **Lesson 12:** Which questions did you ask today that were effective in prompting students to compare the size of fractions strategically or structurally? Which ones might have pushed them toward a particular method or process? **Lesson 13:** How readily did students grasp the idea of writing equivalent fractions with a common denominator as a way to compare fractions? What evidence did you see of students connecting it to the reasoning they did about equivalent fractions on number lines? How could the connections be made more explicit? **Lesson 14:** Were there students with unique approaches who didn't get air time? If so, what

	might be some possible reasons? How can their thinking be made visible in upcoming lessons? **Lesson 15:** How did students' earlier work on factors and multiples support their work in this lesson? What surprised you about the insights students brought forth to help them find common denominators? What challenges did you not anticipate seeing? **Lesson 16:** As you wrap up this unit, reflect on the norms that have supported your students in learning math. How have you seen each student grow as a learner? How have you seen yourself grow as a teacher? **Lesson 17:** Reflect on whose thinking was heard today. Reflect on whose thinking was not heard but could have enriched the conversations. What prompts or structures might better enable the latter to share their voices and reasoning? **Utilize additional strategies for Teacher Reflection:** ● Reviewing formative assessments ● Developing scaffolds ● Collaborative scoring ● PLCs ● Planning for small groups

Topic 3 (Section C) Task Development

Each Topic has its own Task that serves as a roadmap for instruction during the unit. The task follows the Learning Cycle Model that drives teaching and learning in Naugatuck Public Schools.

<table>
<tr><td>Task Title: Topic 3 - Fraction Comparison</td><td>Grade Level and Unit: Fourth Grade, Unit 2</td></tr>
<tr><td>Description of Task:
Students work in pairs to create a game based on these skills and concepts. They identify fractions on a number line, decide how numerical results will be interpreted to determine the winner, draft rules for their game, share their game with classmates, and revise rules based on feedback.</td><td>Purpose of Task:
When students make choices about their approach, analyze numerical information, interpret results, and describe mathematical procedures, they model with mathematics (MP4). When they label fractions on the number line and revise their procedures to better communicate the rules for the game, they are attending to precision (MP6).</td></tr>
<tr><td>Background of Students/Learning Progression:
Previously, students have investigated the relative sizes of fractions with the same numerator or denominator. They have also compared fractions to ½ and 1.</td><td>Ensure all competencies are addressed in the task:

▢ Yes, all competencies are addressed
▢ No - Task needs modification</td></tr>
<tr><td colspan="2">Getting Started:
Have students work with a partner. Display the number line. Ask students “What is an estimate that is too high? Too low? About right?” Have students discuss their thinking with their partner. As students are sharing, record their responses.

What is the value represented by the point on the number line?
 

Make an estimate that is:
<table><tr><th>too low</th><th>about right</th><th>too high</th></tr><tr><td></td><td></td><td></td></tr></table></td></tr>
</table>

After recording students' thinking, ask the following questions:

- How did you decide what fraction would be "about right"?
- Would writing the label "1" as "$\frac{10}{10}$" or as "$\frac{100}{100}$" help us make better estimates? Why or why not?

Learning Cycle Model
Section C

IM Lesson	L12: Ways to Compare Fractions	L13: Use Equivalent Fractions to Compare	L14: Fraction Comparison Problems	L15: Common Denominators to Compare	L16: Compare and Order Fractions	L17: Paper Clip Games (Optional)
Learning Cycle Model	**Making Meaning**	**Making Meaning**	**Investigation**	**Investigation**	**Investigation**	**Create & Produce**
Naugatuck Math Competency	**4.NS.5**	**4.NS.5**	**4.NS.5**	**4.NS.5**	**4.NS.5**	**4.NS.5**
Math Practice Standards	MP3	-	MP1	-	MP7	MP4
Lesson Purpose	The purpose of this lesson is for students to compare fractions in a way that makes sense to them, including by reasoning about the size of fractional parts, common numerators or denominators, or relationships to benchmarks such as ½ and 1.	The purpose of this lesson is for students to compare two fractions by rewriting one of them as an equivalent fraction with the same denominator as the other.	The purpose of this lesson is for students to compare fractions to solve problems in and out of context.	The purpose of this lesson is for students to compare two fractions with different denominators by rewriting both into an equivalent fraction with a common denominator.	The purpose of this lesson is for students to compare and order fractions using any strategy.	The mathematical purpose of this lesson is for students to create and play a game about locating and comparing fractions on a number line.
Vocabulary Focus	Legend Table Axis			Factors Multiples Common Denominator		
Lesson Materials/ Resources	Lesson 12 Slides Teacher Presentation Materials	Lesson 13 Slides Teacher Presentation Materials	Lesson 14 Slides Teacher Presentation Materials	Lesson 15 Slides Teacher Presentation Materials	Lesson 16 Slides Teacher Presentation Materials	Lesson 17 Slides Teacher Presentation Materials

	Student Pages *For Activity 2:* ● Each group of 2 needs 3 colored pencils (3 different colors) **Lesson 12 Cool-Down: Pick the Greater Fraction**	Student Pages **Lesson 13 Cool-Down: Make it True**	Student Pages *For Lesson Synthesis:* ● Provide access to tools for creating a visual display to each group of 2 (tools for creating a visual display may include chart paper and markers, whiteboard space and markers, shared online drawing tool, and access to a document camera) **Lesson 14 Cool-Down: Who Ran the Farthest?**	Student Pages **Lesson 15 Cool-Down: Which is greater?**	Student Pages *For Activity 1:* ● Create a set of cards from the blackline master for each group of 2-4 students (Fraction Cards Grade 4 and Compare Stage 3-8 Directions) **Lesson 16 Cool-Down: All in Order**	Student Pages *For Activity 1:* ● Each group of 2 needs 1-inch paper strips and 10-12 paper clips ● Each group will also need access to markers, paper, paper clips, and tape (painter's or masking) *For Activity 2:* ● Each group will need access to markers, paper, paper clips, and tape (painter's or masking)
	Formative Assessment Strategies: observation, questioning, student discourse - Monitoring Sheet **See : Section C Checkpoint, Section C Checkpoint Teacher Guide** **End of Unit Assessment, End of Unit Assessment Teacher Guide**					
Assessment						**Section C Practice Problems**

Centers Materials	Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting)	Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting)	Compare (1–5), Stage 5: Fractions (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting) How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)	Compare (1–5), Stage 5: Fractions (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting) How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)	Compare (1–5), Stage 5: Fractions (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting) How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)	Compare (1–5), Stage 5: Fractions (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting) How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)

Making Meaning

Previously, students have investigated the relative sizes of fractions with the same numerator or denominator. They have also compared fractions to ½ and 1. In Lesson 12, they apply those understandings to compare a wider range of fractions.

Some students may make comparisons by writing equivalent fractions, which shows they are applying learning from earlier in the unit. It is not necessary to highlight this approach at this point, however. In the next lesson, students will take a closer look at how equivalence can be used to compare fractions.

In Lesson 13, they focus on writing equivalent fractions as a way to compare fractions. Here the denominator of one fraction is a factor or a multiple of the denominator of the other fraction, making it likely for students to see one fraction in terms of the fractional part of the other. In a future lesson, students will compare fractions in which the denominators have no common factors.

Lesson 12: Ways to Compare Fractions

- The purpose of this lesson is for students to compare fractions in a way that makes sense to them, including by reasoning about the size of fractional parts, common numerators or denominators, or relationships to benchmarks such as ½ and 1.
- Teacher Presentation Materials
- Lesson 12 Slides

Lesson 13: Use Equivalent Fractions to Compare

- The purpose of this lesson is for students to compare two fractions by rewriting one of them as an equivalent fraction with the same denominator as the other.
- Teacher Presentation Materials
- Lesson 13 Slides

Investigation:

In the previous lesson, students wrote equivalent fractions to help them compare pairs of fractions with different denominators. In Lesson 14, they include this newly developed strategy in their toolkit for comparing fractions.

In the first activity of Lesson 14, students compare sets of fractions with like and unlike denominators. They do so by using benchmarks, writing equivalent fractions, or reasoning about the numerators and denominators. In the second activity, students interpret and solve problems involving fractional measurements in context. Both activities present a new setup, structure, or context, requiring students to make sense of the given information and the problems, and to persevere in solving them (MP1).

In Lesson 15, students encounter pairs of fractions with different denominators, in which neither denominator is a factor or multiple of the other, and for which other means of comparison are not feasible or intuitive. These fractions motivate students to find another way to compare: by rewriting both fractions into equivalent fractions with a shared denominator.

In Lesson 16, students consolidate their understanding and skills and use them to solve new fraction comparison problems strategically and with flexibility.

Lesson 14: Fraction Comparison Problems

- The purpose of this lesson is for students to compare fractions to solve problems in and out of context.
- Teacher Presentation Materials
- Lesson 14 Slides

Lesson 15: Common Denominators to Compare

- The purpose of this lesson is for students to compare two fractions with different denominators by rewriting both into an equivalent fraction with a common denominator.
- Teacher Presentation Materials

- Lesson 15 Slides

Lesson 16: Compare and Order Fractions

- The purpose of this lesson is for students to compare and order fractions using any strategy.
- Teacher Presentation Materials
- Lesson 16 Slides

Create and Produce:

This lesson offers students an opportunity to model with mathematics to create a game.

In previous lessons, students located and compared fractions on number lines using benchmark fractions and numerical relationships. Here, students work in pairs to create a game based on these skills and concepts. They identify fractions on a number line, decide how numerical results will be interpreted to determine the winner, draft rules for their game, share their game with classmates, and revise rules based on feedback.

When students make choices about their approach, analyze numerical information, interpret results, and describe mathematical procedures, they model with mathematics (MP4). When they label fractions on the number line and revise their procedures to better communicate the rules for the game, they are attending to precision (MP6).

This lesson may take more than 60 minutes, as students may need additional time to create, play, and revise their games. Consider modifying the activities or expanding the lesson across 2 days to meet students' needs and any time constraints.

Lesson 17: Paper Clip Games (Optional)

- The mathematical purpose of this lesson is for students to create and play a game about locating and comparing fractions on a number line.
- Teacher Presentation Materials
- Lesson 17 Slides

Communicate and Present:	**Reflection:**
Lesson 17: Paper Clip Games (Activity 3)	"Today, we made up a game involving labeling fractions."

• The purpose of this activity is for students to share their games with their classmates. This provides pairs the opportunity to articulate their rules and check to see if they are clear to their audience. Based on feedback, they will be able to revise their directions. • Teacher Presentation Materials • Lesson 17 Slides	"What was your favorite part of the game making experience? What was challenging about the experience?"
Notes:	**Complete File with Resources and Task:** Task-based Learning Plan Format for Topic 3