Properties of Matrix Operations

When working in algebra, there are a number of properties that we regularly apply when problem-solving:

Commutative Property of Addition: $a + b = b + a$

Commutative Property of Multiplication: $a \times b = b \times a$

Associative Property of Addition: $(a + b) + c = a + (b + c)$

Associative Property of Multiplication: $(a \times b) \times c = a \times (b \times c)$

Distributive Property: $a \times (b + c) = a \times b + a \times c$

Additive inverse: $a + (-a) = 0$

Multiplicative inverse: $a \times a^{-1} = 1$ for any non-zero real number a

If $a \times b = 0$, then either $a = 0$ or $b = 0$ (or both a and b equal 0)

Investigate which of these properties work for matrices.

- choose your own matrices A, B and C
  (note: some properties may only work for square matrices—be sure to investigate this too!)
- note that a zero matrix is one that has all elements equal to 0
- note that for the multiplicative inverse, you'll need to investigate $A \times A^{-1} = I$ where I is the identity matrix and a is an invertible matrix

Extensions

i) Show that $(kA)^{-1} = k^{-1} A^{-1}$ where k is a scalar and $k \neq 0$ and A is an invertible matrix.

ii) Show that $(AB)^{-1} = B^{-1}A^{-1}$ where both A and B are invertible matrices.

iii) Given an m x n matrix A, the transpose of the matrix, A^T, is an n x m matrix where the ij^{th} entry of matrix A equals the ji^{th} entry of matrix A^T.
For instance, if $A = (1\ 2\ 3\ 4\ 5\ 6\ )$ then its transpose will be $A^T = (1\ 4\ 2\ 5\ 3\ 6\ )$.
Show the following properties:

a) $(A^T)^T = A$
b) $(A + B)^T = A^T + B^T$
c) $(AB)^T = B^T A^T$
d) $(A^T)^{-1} = (A^{-1})^T$ where both A and its transpose are invertible matrices