

$$y = \sqrt{x^3+1}$$

$$\mathcal{D}: x^3+1 \geq 0 \quad x^3 \geq -1 \quad x \geq -1$$

segue $y > 0 \quad \forall x \in \mathcal{D}$

LIMITI AI BORDI DEL DOMINIO

$$\lim_{x \rightarrow +\infty} \sqrt{x^3+1} = +\infty$$

$$f(-1) = 0 \quad f(0) = 1$$

fr. né pari né d.

MAX MIN FL

$$y' = \frac{3x^2}{2\sqrt{x^3+1}}$$

$$y' = 0 \quad x = 0$$

$$y' > 0 \quad \forall x \in \mathcal{D}$$

dunque $x=0$ flesso
ascendente

fr. sempre
crescente

dominio y' : lo stesso di y
tranne il punto -1

studio la derivata in un intorno di -1

$$\lim_{x \rightarrow -1^+} \frac{3x^2}{2\sqrt{x^3+1}} = \frac{3}{0} = \infty \quad x = -1 \text{ punto a tg verticale}$$

$$y'' = \frac{6x \cdot 2\sqrt{x^3+1} - 3x^2 \cdot \frac{1}{2\sqrt{x^3+1}} \cdot 3x^2}{4(x^3+1)} = \frac{12x(x^3+1) - 9x^4}{4(x^3+1)} =$$

$$= \frac{12x^4 + 12x - 9x^4}{4(x^3+1)\sqrt{x^3+1}} = \frac{3x^4 + 12x}{4(x^3+1)\sqrt{x^3+1}}$$

$$y'' = 0 \quad 3x^4 + 12x = 0 \quad 3x(x^3+4) = 0 \quad \begin{matrix} x=0 \\ x = -\sqrt[3]{4} \end{matrix}$$

$$y'' > 0 \quad \begin{matrix} 3x > 0 & x > 0 \\ x^3+4 > 0 & x > -\sqrt[3]{4} \end{matrix} \quad \text{dunque } \forall x \in \mathcal{D} \text{ poich\'e } -\sqrt[3]{4} < -1.$$

$$4(x^3+1)\sqrt{x^3+1} > 0 \quad \forall x \in \mathcal{D}$$

