Big Idea: Multivariable thinking and assessing variation due to random assignment

- **Two group comparison of quantitative response**
- **Three group comparison of quantitative response**

Example 1: Tai Chi with 2 groups

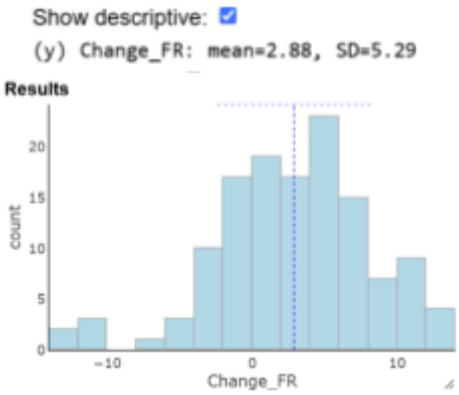
The *New England Journal of Medicine* (Feb 2012) published a study investigating the effects of tai chi on postural stability in Parkinson's patients. Parkinson's disease causes impaired balance, which leads to diminished functional ability and increased risk of falling. The disease has five stages, with stage 1 being the most mild and stage 5 the most severe.

Two different exercise programs were compared: (1) tai chi and (2) stretching. A group of 130 people with Parkinson's (stages 1-4) ages 40 to 85 with stable medication use and medical clearance. Half (n = 65) were randomly assigned to each exercise program for a period of 12-weeks. The weekly frequency and duration of exercise sessions were kept the same for all participants. One of the outcomes measured was *change in functional reach* (measured in centimeters, cm). Functional reach is the distance one can reach out away from the body to retrieve an item without losing balance and falling over.

1. What are the response variable and explanatory variables in the study? What type of variable is each?

2. What type of study is this, an experiment or an observational study? Will a cause-and-effect conclusion be possible for this study? Explain.

3. Brainstorm sources of variation in the changes in functional reach. Provide at least 3 extraneous sources of variation in change in functional reach.

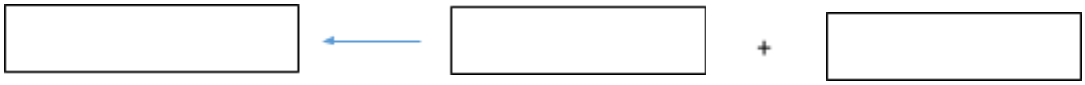


4. What would the histogram above look like if there was no variation in the change in functional reach?

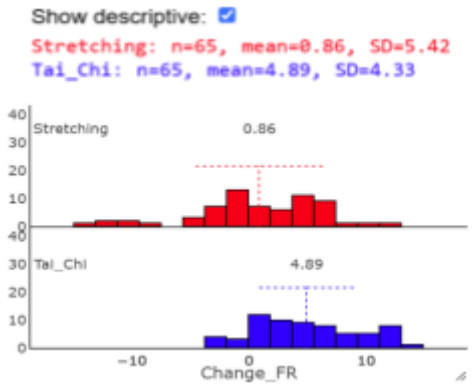
5. Fill in the Sources of Variation diagram. Include at least one inclusion criterion, at least one source of variability that was (or should have been) controlled by design, and at least 3 sources of unexplained variability.

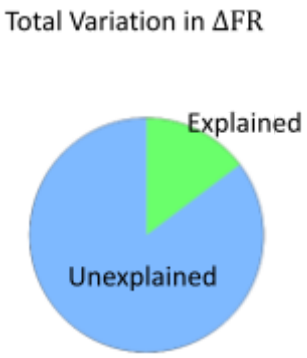
Observed Variation in:	Sources of explained variation	Sources of unexplained variation
Inclusion criteria: *Constant by Design:*		

Word Model:



Fitted Model:





6. Has exercise program explained variation in the change in functional reach? Explain.

Statistical Inference:

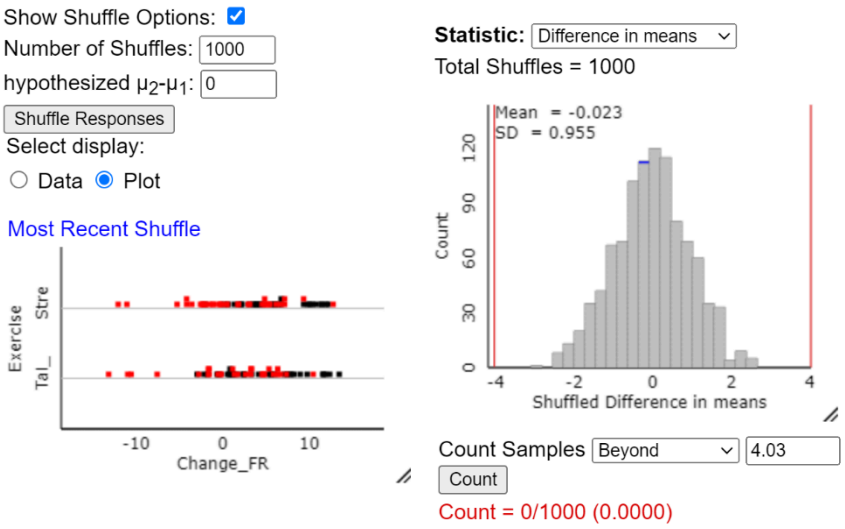
7. Theory-based Two-sample t-test: t-statistic = standardized ratio of $\frac{explained}{unexplained}$

Unpooled: $t = \frac{\bar{y}_{TC} - \bar{y}_S}{\sqrt{\frac{s_{TC}^2}{n_{TC}} + \frac{s_S^2}{n_S}}} = 4.69$ vs. Pooled: $t = \frac{\bar{y}_{TC} - \bar{y}_S}{s_{pooled}\sqrt{\frac{1}{n_{TC}} + \frac{1}{n_S}}} = 4.69$

Where $s^2_{pooled} = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

8. Simulation-based Inference: Using the variation from the random assignment process of 130 response outcomes into two groups of 65 each

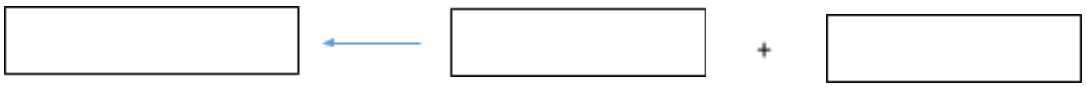
What statistic can we use to measure how different the two group means are?



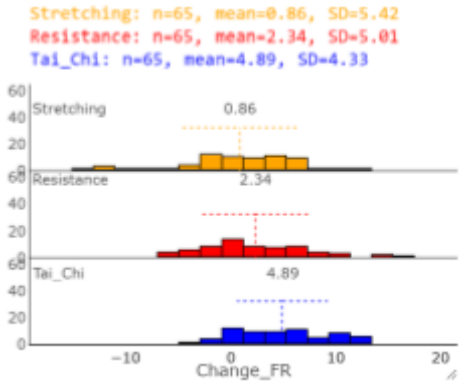
Example 2: Tai Chi with 3 groups

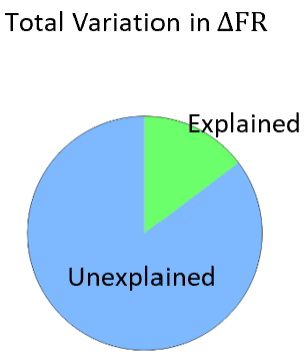
In the actual study the researchers investigated three different exercise programs: (1) tai chi (2) resistance training and (3) stretching. A group of 195 people with Parkinson's (stages 1-4) were enrolled in the study. One-third of the participants (n = 65) were randomly assigned to each exercise program for a period of 12-weeks.

Word Model:



Fitted Model:

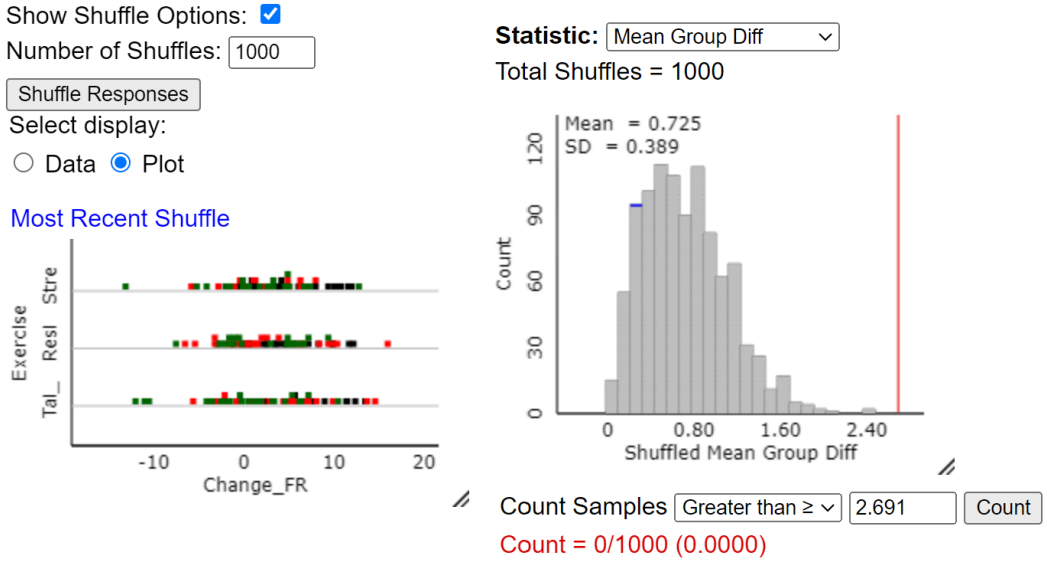




1. Has exercise program explained variation in the change in functional reach? Explain.
Statistical Inference:

Simulation-based Inference: Using the variation from the random assignment process of 130 response outcomes into 3 groups of 65 each

2. How could we measure, with one statistic, the 'difference' in the 3 group means?



3. Theory-based F-statistic

F-statistic = standardized ratio of $\frac{explained}{unexplained} = \frac{\frac{SSModel}{df\ Model}}{\frac{SSError}{df\ Error}}$

SSTotal = 5231.378

ANOVA Table:

Source	df	SS	MS	F-stat	p-value
Exercise	2	542.07	271.03	11.10	< 0.0001
Error	192	4689.31	24.42		
Total	194	5231.38