

## 2. Statistics II

|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |   |    |    |    |    |   |   |   |    |   |   |   |   |   |    |   |   |    |    |   |   |                 |   |   |    |    |    |    |                                                                                     |                                                                 |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----|----|----|----|---|---|---|----|---|---|---|---|---|----|---|---|----|----|---|---|-----------------|---|---|----|----|----|----|-------------------------------------------------------------------------------------|-----------------------------------------------------------------|
| 1               | <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>f</td><td>20</td><td>8</td><td>6</td><td>4</td><td>1</td><td>1</td></tr><tr><td>fx</td><td>0</td><td>8</td><td>12</td><td>12</td><td>4</td><td>5</td></tr><tr><td>fx<sup>2</sup></td><td>0</td><td>8</td><td>24</td><td>36</td><td>16</td><td>25</td></tr></table> <p><math>\sum f = 40 \quad \sum fx = 41 \quad \sum fx^2 = 109</math></p> $s.d = \sqrt{\frac{109}{40} - \left(\frac{41}{40}\right)^2}$ $= \sqrt{2.725 - 1.050625}$ $= \sqrt{1.674375}$ $= 1.294$ | X | 0  | 1  | 2  | 3  | 4 | 5 | f | 20 | 8 | 6 | 4 | 1 | 1 | fx | 0 | 8 | 12 | 12 | 4 | 5 | fx <sup>2</sup> | 0 | 8 | 24 | 36 | 16 | 25 | <p>M<sub>1</sub></p> <p>M<sub>1</sub></p> <p>M<sub>1</sub></p> <p>A<sub>1</sub></p> | <p>fx</p> <p>fx<sup>2</sup></p> <p>Allow</p> <p>1.293976429</p> |
| X               | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 1 | 2  | 3  | 4  | 5  |   |   |   |    |   |   |   |   |   |    |   |   |    |    |   |   |                 |   |   |    |    |    |    |                                                                                     |                                                                 |
| f               | 20                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 8 | 6  | 4  | 1  | 1  |   |   |   |    |   |   |   |   |   |    |   |   |    |    |   |   |                 |   |   |    |    |    |    |                                                                                     |                                                                 |
| fx              | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 8 | 12 | 12 | 4  | 5  |   |   |   |    |   |   |   |   |   |    |   |   |    |    |   |   |                 |   |   |    |    |    |    |                                                                                     |                                                                 |
| fx <sup>2</sup> | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 8 | 24 | 36 | 16 | 25 |   |   |   |    |   |   |   |   |   |    |   |   |    |    |   |   |                 |   |   |    |    |    |    |                                                                                     |                                                                 |
|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 4 |    |    |    |    |   |   |   |    |   |   |   |   |   |    |   |   |    |    |   |   |                 |   |   |    |    |    |    |                                                                                     |                                                                 |

1.

| Mass<br>kg | Mid<br>term<br>x | F                 | d = x A | fd                   | d <sup>2</sup> | fd <sup>2</sup>       |
|------------|------------------|-------------------|---------|----------------------|----------------|-----------------------|
| 50 - 54    | 52               | 19                | -15     | -285                 | 225            | 4275                  |
| 55 - 59    | 57               | 23                | -10     | -230                 | 100            | 2300                  |
| 60 - 64    | 62               | 40                | -5      | -200                 | 25             | 1000                  |
| 65 - 69    | 67               | 28                | 0       | 0                    | 0              | 0                     |
| 70 - 74    | 72               | 17                | 5       | 85                   | 25             | 425                   |
| 75 - 79    | 77               | 9                 | 10      | 90                   | 100            | 900                   |
| 80 - 84    | 82               | 4                 | 15      | 60                   | 225            | 900                   |
|            |                  | $\sum f =$<br>140 |         | $\sum fd =$<br>- 480 |                | $\sum fd^2 =$<br>9800 |

Marks awarded for ✓table as follows:-

$\sum f = 140$       B1

Column for d      B1

Column for fd      B1

$\sum fd = - 480$       B1

✓Column for d<sup>2</sup> = 9800 B<sub>1</sub>

$\sum fd^2 = 9800$  B<sub>1</sub>

$x = A + \frac{\sum fd}{\sum f}$

$= 67.0 + \frac{- 480}{140}$

$= 67.0 - 3.43 = 63.57$  ..... M1

$= 63.6$  kg ..... A1

Standard deviation      =       $\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2$

$$= \sqrt{\frac{9800}{140} - (3.43)^2}$$

$$= \sqrt{58.24} = 7.631$$

$$= 7.6$$

$$2. = \frac{8}{150} + \frac{6}{150} + \frac{9}{300} + \frac{3}{300}$$

$$= \frac{40}{300} = \frac{2}{15}$$

a) Construction of AB B1

Construction of BC B1

Construction of AC B1

b) Construction of bisect of AC B1

Construction of bisect BC B1

Radius 3.6 cm B1

c) Construction of bisect  $\angle CAB$  B1 OC B1

Construction of AD B1 AD = 12.8cm B1

3. a)

| Class   | f  | x    | d = A - x | fd                | d <sup>2</sup> | fd <sup>2</sup>        |
|---------|----|------|-----------|-------------------|----------------|------------------------|
| 41 - 50 | 20 | 45.5 | 15        | 300               | 225            | 4500                   |
| 51 - 55 | 60 | 53   | 7.5       | 450               | 56.25          | 3375                   |
| 56 - 65 | 60 | 60.5 | 0         | 0                 | 0              | 0                      |
| 66 - 70 | 50 | 68   | -7.5      | -375              | 56.25          | 2812.50                |
| 71 - 85 | 15 | 73   | -12.5     | 187.5             | 156.25         | 2343.75                |
|         |    |      |           | $\Sigma fd$ 562.5 |                | $\Sigma fd^2$ 13031.25 |

$$b) S = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left( \frac{\Sigma fd}{\Sigma f} \right)^2}$$

$$S = \sqrt{\frac{13031.25}{205} - \left( \frac{562.5}{205} \right)^2}$$

$$= \sqrt{63.567 - 7.529}$$

$$= \sqrt{56.038}$$

$$= 7.486$$

$$4. 15 (ax)^4 \left( \frac{2}{x} \right)^2 = 4860$$

$$60a^4 = 4860$$

$$a^4 = 81$$

$$a = 3$$

5.

| Marks(x) | Freq.(f)      | fx               | d=x-x  | d <sup>2</sup> | Fd <sup>2</sup>      |
|----------|---------------|------------------|--------|----------------|----------------------|
| 5.5      | 1             | 5.5              | -40.45 | 1636           | 1636                 |
| 15.5     | 6             | 99               | -30.45 | 927.2          | 5563                 |
| 25.5     | 10            | 255              | -20.45 | 418.2          | 4182                 |
| 35.5     | 20            | 710              | -10.45 | 109.2          | 2184                 |
| 45.5     | 15            | 682.5            | -0.45  | 0.2025         | 3038                 |
| 55.5     | 5             | 277.5            | 9.55   | 91.20          | 456                  |
| 65.6     | 14            | 917              | 19.55  | 382.2          | 535                  |
| 75.5     | 5             | 377.5            | 29.55  | 873.2          | 4366                 |
| 85.5     | 3             | 256.5            | 39.55  | 1564           | 4692                 |
| 95.5     | 1             | 95.5             | 49.55  | 2455           | 2455                 |
|          | $\Sigma f=80$ | $\Sigma fx=3676$ |        |                | $\Sigma fd^2=33,923$ |

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{3676}{80}$$

$$= 45.95$$

$$(b) Q1 = 30.5 + \frac{3}{14} \times 10$$

$$= 62.64$$

$$S.I.R = \frac{1}{2} (62.64 - 32)$$

$$= 15.32$$

(c) Standard deviation

$$= \sqrt{\frac{\Sigma fd^2}{\Sigma f}} = \frac{33923}{80}$$

$$= 20.59$$

$$6. \quad a) x = 90 - (2 + 13 + 51 + 27 + 14 + 1)$$

$$= 90 - 84 = 6$$

$$b) 15 - 19$$

c) i)

| Class | x  | f  | D= x-A | fd   | D <sup>2</sup> | Fd <sup>2</sup> |
|-------|----|----|--------|------|----------------|-----------------|
| 5-9   | 7  | 2  | -15    | -30  | 225            | 450             |
| 10-14 | 12 | 13 | -10    | -130 | 100            | 1300            |
| 15-19 | 17 | 31 | -5     | -155 | 25             | 775             |
| 20-24 | 22 | 23 | 0      | 0    | 0              | 0               |
| 25-29 | 27 | 14 | 5      | 70   | 350            | 4900            |
| 30-34 | 32 | 6  | 10     | 60   | 600            | 3600            |
| 35-39 | 37 | 1  | 15     | 15   | 225            | 225             |

$$Ef = 90 \quad Efd = 170 \quad Efd^2 = 11250$$

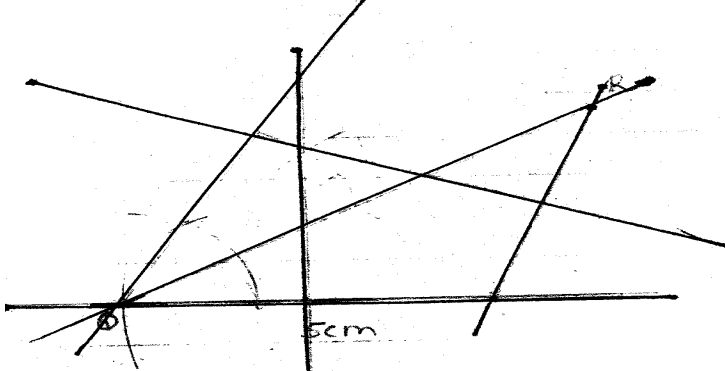
$$\text{Mean} = \frac{E + d}{Ef} + A$$

$$= \frac{-170}{90} + 22$$

$$= 22 - 1.888 = 20.11$$

$$\begin{aligned}
 \text{ii) } S.d &= \sqrt{\frac{Efd}{Ef} - \left[\frac{Efd}{Ef}\right]^2} \\
 &= \sqrt{122 - (-1.888)^2} \\
 &= \sqrt{125 - 3.566} = \sqrt{121.4} \\
 &= 11.02
 \end{aligned}$$

7.



$$\begin{aligned}
 RQ &= 7.5 \pm 0.1 \\
 < PRQ &40^\circ \pm 1 \\
 &\text{BI circle through } P, Q \text{ and } R
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } r &= 4.1 \pm 0 \\
 A &= \pi r^2 \\
 &= \frac{22}{7} \times 4.1 \times 4.1 = 52.83
 \end{aligned}$$

8.

| Class limits | $f$ | $cf$ |
|--------------|-----|------|
| -0.5 - 19.5  | 7   | 7    |
| 19.5 - 39.5  | 21  | 28   |
| 39.5 - 59.5  | 38  | 66   |
| 59.5 - 79.5  | 27  | 93   |
| 79.5 - 99.5  | 7   | 100  |

- i) from the curve – median = 52. MI AI  
 (ii) Inter quartile range = 66-38 = 28.  
 (iii) 7th  $\frac{7}{10}$  = 62.46marks  
 (iv) 60th percentile – 56.34

$$\begin{aligned}
 9. \quad &25^2 + 24^2 + 22^2 + 23^2 + x^2 + 26^2 + 21^2 + 23^2 + 22^2 + 27^2 = 5154 \\
 &5.625 + 576 + 2(484) + 2(529) + 676 + 441 + 729 + x^2 = 5154 \\
 &x^2 = 81 \\
 &x = 9
 \end{aligned}$$

$$\text{(ii) } X = \frac{222}{10} = 22.2$$

$$\begin{aligned}
 \Sigma(X-x)^2 &= 2.8^2 + 1.8^2 + 0.22 + 0.8^2 \\
 &13.2^2 + 3.8^2 + 1.22 + 0.8^2 + 0.2^2 + 4.8^2 \\
 (x-x)^2 &= 7.84 + 3.24 + 2(0.04) + 2(0.64) \\
 &+ 174.24 + 14.44 + 1.44 + 23.04 \\
 &= \frac{225.6}{10}
 \end{aligned}$$

$$s.d \ 22.56$$

$$= 4.75$$

$$(b) \ (i) \ \text{New mean} = 22.2 + 3$$

$$= 25.2$$

$$(ii) \ s.d = 4.75$$

$$10. \quad a) \ i) \ x = A + \frac{\sum fd}{\sum f}$$

$$= 45.6 + \frac{(-74)}{40}$$

$$= 43.75$$

| Class    | Mis-pt x | d = (x - A) | Frequency f | fd    | Fd <sup>2</sup> |
|----------|----------|-------------|-------------|-------|-----------------|
| 1 - 10   | 5.5      | -40.1       | 1           | -40.1 | 1608.01         |
| 11 - 20  | 15.5     | -30.1       | 3           | -90.3 | 8154.05         |
| 21 - 30  | 25.5     | -20.1       | 4           | -80.4 | 6464.16         |
| 31 - 40  | 35.5     | -10.1       | 7           | -70.7 | 4998.49         |
| 41 - 50  | 45.5     | -0.1        | 12          | -1.2  | 1.44            |
| 51 - 60  | 55.5     | 9.9         | 9           | 89.1  | 7938.81         |
| 61 - 70  | 65.5     | 19.9        | 2           | 39.8  | 1584.04         |
| 71 - 80  | 75.5     | 29.9        | 1           | 29.9  | 894.01          |
| 81 - 90  | 85.5     | 39.9        | 0           | 0     | 0               |
| 91 - 100 | 95.5     | 49.9        | 1           | 49.9  | 2410.01         |

i) Standard Deviation

$$D = \sqrt{\frac{\sum fd^2}{\sum f} - \left[ \frac{\sum fd}{\sum f} \right]^2}$$

$$= \sqrt{\frac{34135.11}{40} - \left[ \frac{-74}{40} \right]^2}$$

$$= \sqrt{853.37775 - 3.49}$$

$$= \sqrt{849.88775}$$

$$= 29.1531$$

$$b) \ 30^{\text{th}} \text{ student} = 10^{\text{th}} \text{ from bottom}$$

$$30.5 + \left[ \frac{30 - 10}{10} \right] 10$$

$$= 30.5 + 2.9 = 33.4 \text{ marks.}$$

$$11. \quad a) \text{ Mean } 45.5 + \frac{530}{60}$$

$$= 54.33$$

$$(b) \text{ Median } = 50.5 + \left[ \quad \right] \cdot 10$$

$$= 55.86$$

$$(c) \text{ Standard deviation } = \sqrt{\left[ \quad \right]^2 \frac{530}{60}}$$

$$= 17.52$$

(d) Modal class 51 – 60

12.

| $x$  | $f$ | $d$ | $d^2$ | $fd$ | $fd^2$ |
|------|-----|-----|-------|------|--------|
| 24.5 | 4   | -30 | 900   | -120 | 3600   |
| 34.5 | 26  | -20 | 400   | -520 | 10400  |
| 44.5 | 72  | -10 | 100   | -720 | 7200   |
| 54.5 | 53  | 0   | 0     | 0    | 0      |
| 64.5 | 25  | 10  | 100   | 250  | 2500   |
| 74.5 | 9   | 20  | 400   | 180  | 3600   |
| 84.5 | 11  | 30  | 900   | 330  | 9900   |
|      | 200 |     |       | -600 | 37200  |

$$(a) (i) \text{ Mean } = A + \frac{\sum fd}{\sum f}$$

$$= 54.5 - \frac{600}{200}$$

$$= 51.5$$

$$(ii) \text{ Standard deviation}$$

$$= \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

$$= \sqrt{\frac{37200}{200} - (-3)^2}$$

$$= \sqrt{186 - 9}$$

$$= 13.30$$

$$(b) Q_1 = 39.5 + \frac{50 - 30}{72} \times 10$$

$$= 42.28$$

$$Q_3 = 49.5 + \frac{150 - 102}{53} \times 10$$

$$= 58.56$$

$$Q_3 - Q_1 = 58.56 - 42.28$$

$$= 16.28$$