ESCUELA POLITÉCNICA NACIONAL

INGENIERÍA EN SISTEMAS INFORMÁTICOS Y DE COMPUTACIÓN

Calculo Vectorial

Nombre: Parra Ordoñez Miguel Ángel

Tema: Integrales dobles

Fecha de entrega: 19/07/2016

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1. Evalúe la integral iterada

$$\int_0^1 \int_{x^2}^{x} (1+2y)\, dy\, dx = \int_0^1 (y+y^2)\Big|_{x^2}^{x} dx = \int_0^1 (x + \cancel{x^2} - \cancel{x^2} - x^4)\, dx$$

$$= \int_0^1 (x - x^4)\, dx = \frac{x^2}{2} - \frac{x^5}{5}\Big|_0^1 = \left(\frac{1}{2} - \frac{1}{5}\right) = 0{,}3$$

2. $$\int_0^{\pi/2} \int_0^{\cos\theta} e^{\operatorname{sen}\theta}\, dr\, d\theta = \int_0^{\pi/2} r\, e^{\operatorname{sen}\theta}\Big|_0^{\cos\theta} d\theta$$

$$= \int_0^{\pi/2} \cos\theta\, e^{\operatorname{sen}\theta}\, d\theta \Rightarrow \int_0^1 e^u\, du$$

Sea $u = \operatorname{sen}\theta$

$du = \cos\theta \cdot d\theta$

$$= e^u\Big|_0^1 \Rightarrow e^1 - e^0 \Rightarrow e - 1$$

3. Evalúe la integral doble

$$\iint_D y^2\, dA, \quad D = \{(x,y) \mid -1 \le y \le 1,\ -y-2 \le x \le y\}$$

$$\int_{-1}^{1} \int_{-y-2}^{y} y^2\, dx\, dy = \int_{-1}^{1} x y^2 \Big|_{-y-2}^{y} dy = \int_{-1}^{1} y^2 (y - (-y-2))\, dy$$

$$= \int_{-1}^{1} y^2 (2y+2)\, dy = \int_{-1}^{1} 2y^3 + 2y^2\, dy = \frac{2y^4}{4} + \frac{2y^3}{3}\Big|_{-1}^{1}$$

$$= \cancel{\frac{2}{4}} + \frac{2}{3} - \left(\cancel{\frac{2}{4}} + \frac{2}{3}\right)$$

$$= \frac{4}{3}$$

4.) Evalue la integral doble

$$\iint_D x\cos y\, dA \quad , \quad D \text{ está acotada por } y=0,\ y=x^2,\ x=1.$$

$$\int_0^1\int_0^{x^2} x\cos y \; dy\,dx = \int_0^1 x\,\mathrm{sen}\,y\Big|_0^{x^2} dx = \int_0^1 x\,(\mathrm{sen}\,x^2)\,dx$$

$u = x^2$

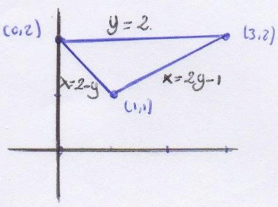
$du = 2x\,dx$

$$= \frac{1}{2}\int_0^1 \mathrm{sen}\,u\; du = -\frac{1}{2}\cos u\Big|_0^1 = -\frac{1}{2}\left(\cos(1) - \cos(0)\right)$$

$$= -\frac{1}{2}\cos(1) + \frac{1}{2} \Rightarrow \frac{1}{2}\left(1-\cos(1)\right)$$

5. Evalue la integral doble

$$\iint_D y^3\, dA \quad , \quad D \text{ es la región triangular con vértices } (0,2),(1,1),(3,2).$$



(1) $\dfrac{x-1}{3-1} = \dfrac{y-1}{2-1} \Rightarrow \dfrac{x-1}{2} = \dfrac{y-1}{1} \Rightarrow x-1 = 2y-2$

$x = 2y-1$

(2) $\dfrac{x-1}{0-1} = \dfrac{y-1}{2-1} \Rightarrow \dfrac{x-1}{-1} = \dfrac{y-1}{+1} \Rightarrow x-1 = -y+1$

$x = 2-y$

(3) $\dfrac{x-0}{3-0} = \dfrac{y-2}{2-2} \Rightarrow \dfrac{x}{3} = \dfrac{y-2}{0} \Rightarrow 3y-6=0$

$y = 2$

$$\int_1^2\int_{x=2-y}^{x=2y-1} y^3\,dx\,dy = \int_1^2 xy^3\Big|_{x=2-y}^{x=2y-1} dy = \int_1^2 \left((2y-1)-(2-y)\right)y^3\,dy$$

$$= \int_1^2 y^3(3y-3)\,dy = \int_1^2 (3y^4-3y^3)\,dy = \frac{3y^5}{5} - \frac{3y^4}{4}\Big|_1^2$$

$$= \frac{3(2)^5}{5} - \frac{3(2)^4}{4} - \left(\frac{3}{5}-\frac{3}{4}\right)$$

$$= \frac{147}{20} = 7{,}35$$