

Henri Poincaré and the Aesthetics of Mathematics

Henri Poincaré, a giant in the French mathematical scene during the late 19th and early 20th centuries, is well known for his widespread contributions across the breadth of his field. He is most often spoken about in conjunction with his close connection to and crucial input on the emerging field of relativity¹, his conceptualization of algebraic topology, and formative involvement regarding the creation of both theoretical and utilitarian mathematical conceptions – including but not limited to the Poincaré Conjecture and the Three-Body Problem. He was also a member, and later president, of l'Académie des sciences during some of its most prolific years and briefly served as the director of l'Académie française. His influence reverberates through to this day which was well evidenced during the surveys I carried out during the initial conceptualization of my senior thesis; it was difficult to come across anyone, history or mathematics aligned, who had not heard of or directly contended with his body of work.²

However, while much history of science has been conducted on the mechanics of Poincaré's mathematical impact, his philosophical writing, particularly as it relates to the aesthetics of equations and proof, is not equally established. There exists this idea of beauty for mathematicians which seems to go deeper than just equations that translate to gracefully curving graphs. Moreover, there is a propensity in the field to ascribe characteristically visual terms like 'elegant' and 'ugly' as they relate to a heuristic idea of appeal. It is an interesting phenomenon and one that Poincaré wrote about extensively, including, as part of two long-form books titled *The Value of Science*³ and *Science and Method*⁴ respectively. In them, he delineates different

¹ Peter Galison, *Einstein's Clocks, Poincaré's Maps: Empires of Time* (New York: WW Norton & Company, 2004).

² Here I am especially grateful to Professor James Myer (CUNY Graduate Center), Professor Matthew Stanley (NYU Gallatin), Professors Carl Wennerlind and Mark Mazower (Columbia University), as well as an incredibly talented group of individuals from the Emergent Ventures grant program (running under George Mason University's Mercatus Center) for inputs on both the viability of the concept and recommendations on scholarship.

³ Henri Poincaré, *The Value of Science*, ed. George Bruce Halsted (New York: Dover Publications, 1958).

⁴ Henri Poincaré, *Science and Method*, ed. Francis Maitland (Mineola, N.Y.: Dover Publications, 1914).

types of propositions one might find pleasing and attempts to verbalize philosophical reasonings for each of them in a way that many scholars find firmly Neo-Kantian.⁵ Despite leaving much nuance on the table, his position, based on consultation with secondary sources, could briefly be summarized by saying that for him, aesthetic judgments have “intersubjective validity” due to man’s shared cognitive faculties, even though they do not correspond to objective features of reality.⁶

There is much more to be gleaned here, and while it can be hypothesized that the relative scarcity of scholarship on this topic is largely due to how niche the field of mathematical aesthetics is, I argue that the conception of beauty should not be treated as separate to the logical work when it comes to Poincaré. This belief is supported by both the background against which he worked, with the idea of *esthétisme fantastique* reaching an apex during his lifetime, and his implicit epistemic emphasis that conflated the truth value of a proposition with its potential for beauty.⁷ The aim of my senior thesis, therefore, is to understand how Poincaré’s philosophy of aesthetics influenced his mathematical work. I strongly anticipate that this line of inquiry will also shed light on the way he both explicitly, through his published corpus of essays, and indirectly helped create the tradition of a “standard of beauty” in mathematics.⁸

It is this secondary extrapolation that I will make while consulting sources on the more straightforward primary question that will serve as the distinguishing feature of my examination

⁵ Terry F. Godlove, “Poincaré, Kant, and the Scope of Mathematical Intuition,” *The Review of Metaphysics* 62, no. 4 (2009): 779–801, <http://www.jstor.org/stable/40387765>.

⁶ Although there is much debate between structuralist and instrumentalist interpretations, this paper summarizes and pragmatically reconciles some of them: Milena Ivanova, “Poincaré’s Aesthetics of Science,” *Synthese* 194, no. 7 (2017): 2581–94, <https://www.jstor.org/stable/48726139>.

⁷ An idea supported particularly strongly in the last three subsections of the first chapter of the following (the book also contains an extensive bibliography of primary and secondary sources making it a good resource in general too): Jeremy Gray, *Henri Poincaré: A Scientific Biography* (Princeton: Princeton University Press, 2013), <http://www.jstor.org/stable/j.ctt1r2fwt>, chapter 1.

⁸ With a very strong link to the tradition he creates regarding the “beauty standard” implied in the following, much better known work: G. H. Hardy, *A Mathematician’s Apology* (London: Cambridge University Press, 1967).

of Poincaré. To do so, my research will rely on two parallel tracks of inquiry where I simultaneously concern myself with the primary mathematical work he produced alongside the abundant and indiscriminate correspondence he wrote during his academic career. The goal is that upon cross-referencing his ideas, the way he spoke about them to his contemporaries, and employing the body of secondary scholarship that exists on mathematical aesthetics, I will be able to construct a novel and more holistic picture – shedding light on the way his philosophical ideas influenced the technical work him and his contemporaries produced.

It is prudent that I consider the problem of scope and, therefore, plan to work systematically in terms of research methodology to overcome it. My archival readings will be constrained to the final decade of his life, one marked by some of his more overt philosophical opinions on beauty. This is a particularly important time in his life, one cut short by his untimely death, during which he reached an inflection point in both his academic and political careers – not only did he publish prolifically on epistemic and aesthetic matters, but he also produced some of his most seminal mathematical work and had an audience to witness it all due to his prominence within l'Académie des sciences.⁹ This provides a concrete direction for archival work and the evidence it can lend regarding his personal influences and impact. Specifically, I will be consulting his correspondence during this time of political and academic prominence along with the minutes from the meetings of l'Académie. These sources will be read in conjunction with the philosophical interpretations that arose from them in both modern and contemporary scholarship. This time frame also helps narrow down the technical work necessary, and I will restrict myself to understanding why the work he most concerned himself with during

⁹ Jeremy Gray, *Henri Poincaré: A Scientific Biography* (Princeton: Princeton University Press, 2013), <http://www.jstor.org/stable/j.ctt1r2fwt>, chapter 2.

this time was philosophically special – focusing on Poincaré’s Conjecture regarding the topology of three-dimensional spaces.^{10 11 12}

Before moving to the specifics of the plan surrounding my archival research, it is worth contending with the concern of depth throughout the thesis given the mathematical complexity of the topic. However, I am confident in my ability to carry out the above plan in part due to my double major in Neuroscience where previous coursework has helped with some of the necessary foundational knowledge. Crucially, the early conceptualization of this thesis idea provided me with ample time to anticipate the technical challenge that the history of mathematics raises. In preparation, I have spent the past two months independently deep-diving into algebra, geometry, and the basics of topology to better grapple with the complexity of Poincaré’s Conjecture.¹³

Other prerequisites to consider, given that all of the research takes place in France, are language proficiency and visa requirements due to my status as an international student. With regards to those, I have previous French immersion experience and have been studying the language at Barnard for two semesters now.¹⁴ Paperwork for the visa has also been filed and all going well, I should have a fairly flexible right to travel for the duration of the summer.

My current plan is to be in the Schengen region, flying to Paris on May 16th with research efforts beginning from the 1st of June. The first stop is going to be in Nancy at the

Archives Henri-Poincaré Philosophie et Recherches sur les Sciences et les Technologies at the

¹⁰ It is interesting to note here that while it was his aesthetic corpus that led me to these specifics, more than one person I consulted has independently used topology as an example when trying to make concrete their schema of mathematical beauty.

¹¹ While the conjecture has its own set of scholarship it is less researched and spoken about as compared to Poincaré’s work on early Relativity. My understanding is that this discrepancy exists because Einstein’s own epistemic concerns offered a good counter to Poincaré. Equations around Relativity are also well researched because of the absurdity of their independent arrival at the same conclusion via very disparate philosophical routes, despite never directly corresponding, that too within months of each-other.

¹² Donal O’Shea, *The Poincaré Conjecture: In Search of the Shape of the Universe* (New York: Bloomsbury Publishing USA, 2009).

¹³ One of the foundational texts I used includes: John Stillwell, *Elements of Mathematics: From Euclid to Gödel* (Princeton: Princeton University Press, 2016), <https://doi.org/10.2307/j.ctvc77h7p>.

¹⁴ With further specifics provided in the application form.

Université de Lorraine. They are an extensive and well-organized collection of his correspondence and academic bibliography. Here, I am particularly interested in the Klein letters of 1881, and later letters on topology to Gaston Darboux, David Hilbert, and Tuvio Levi-Civita, all of whom interacted with Poincaré over his conjecture or closely allied concepts. While about half of their archives are digitized, the value of being physically present comes from interactions with the working group and the archivists who will help inform non-obvious connections that are limited by the modality of online search inquiries. It is also worth noting that many of the documents relating to Poincaré's conversations with people of initial interest to my posed question are not available on the internet. Consulting 10-12 documents a day, I plan to be in Nancy for ten days which provides sufficient time to fill in the gaps inaccessible via the online repository.

I then return to Paris for the remainder of my trip, approximately five weeks that conclude shortly after the meeting at Reid Hall, where I focus my attention on *les archives de l'Académie des sciences*. For the time period of Poincaré's life, the minutes of the meetings of the academy are digitized. However, the files of the individual sessions, or the documents mentioned in the minutes themselves, are not accessible – therefore, via digital resources on their own, one knows the broad motions but not the context or the specifics in which they are undertaken. Furthermore, the archives contain in-depth bibliographical files that help investigate the life of a person beyond their academic work and will let me understand how his personal experiences and relationships may have shaped his philosophy of aesthetics.

My last, large, archival source will be at the *Centre Historique des Archives Nationales*, a subset of the *Bibliothèque Nationale Française*, with the aim of comprehending the general milieu of the selected time period. Some particular resources identified can be searched via the

list number 510.92 (23e éd.), and under publications of Académie des sciences (France) section de géométrie, le Congrès national des sociétés savantes, and l'Institut général psychologique. I hope that these documents, on the general communication of his work, provide missing insight into how the widely circulated primary corpus was received by the scientific community as a whole. Lastly, additional points of reference, although not currently thought to be necessary, could be accessed by petitioning for select documents at the archives of the *Institut Henri Poincaré* at Sorbonne Université. For the entirety of the proposed research and travels, reasonable efforts have been made to become aware of any potential closures and restrictions and most archives have been contacted to learn of any authorizations required to conduct research on premises.¹⁵

To conclude, by methodically relying on French archival sources, I will be able to create a complete picture of Henri Poincaré. One that will help illuminate the intrinsic connection between mechanics and aesthetics and make clearer his larger influence on the way we perceive mathematical beauty.

¹⁵ With many thanks to Professor Pauline Guedj (Columbia University) for introducing me to best practices while navigating the French archives.

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