Solution of Quiz 2

Q1)

Solution: According to the definition of circular convolution, we have

LENGTH OF SEQUENCE = 4

TOTAL LENGTH = $(2)^2 \times 2 = 8$

$\therefore N = 8 - 1 = 07$

$$y(m) = \sum_{n=0}^{N-1} x_1(n) \cdot x_2((m-n))_N \qquad \text{...(i)}$$

Here, given sequences are x(n) and h(n). The length of sequence is 4 that means N = 7. Thus, equation (i) becomes,

$$y(m) = \sum_{n=0}^{N-1} x(n)h((m-n))_4 \qquad \text{...(ii)}$$

(i) We draw x(n) and h(n) as shown in figure 2.19(a) and (b).

It may be noted that x(n) and h(n) are plotted in anti-clockwise direction.

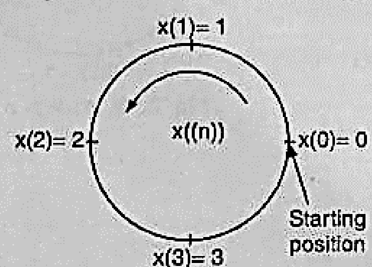


Fig. 2.19. (a) x(n) = {0, 1, 2, 3}.

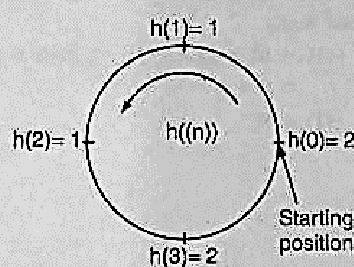


Fig. 2.19. (b) h(n) = {2, 1, 1, 2}.

Now, let us calculate different values of y(m) by putting m = 0 to m = 3 in equation (ii).

(ii) Calculation of y(0)

Substituting m = 0 in equation (ii), we get

$$y(0) = \sum_{n=0}^{3} x(n)h((-n))_4 \quad ...(iii)$$

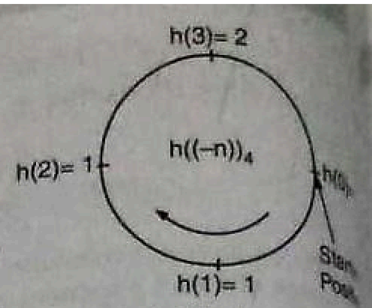


Fig. 2.19. (c) h(n) is plotted in clockwise direction

Equation (iii) shows that we have to obtain the product of x(n) and h((− n)), and then we have to take the summation of product elements. Using graphical method, this calculation is done as follows.

The sequence $h((-n))_4$ indicates circular folding of h(n). This sequence is obtained by plotting h(n) in a clockwise direction as shown in figure 2.19(c).

To do the calculations, we plot x(n) and h((− n)) on two concentric circles as shown in figure 2.19(d). Also, x(n) is plotted on the inner circle and h((− n)) is plotted on the outer circle.

Now, according to equation (ii), individual values of product x(n) and h((− n)) are obtained by multiplying two sequences point-by-point. Then, y(0) is obtained by adding all product terms.

Therefore, we have

$$y(0) = (0 \times 2) + (1 \times 2) + (1 \times 2) + (3 \times 1) = 0 + 2 + 2 + 3$$

or $$y(0) = 7$$

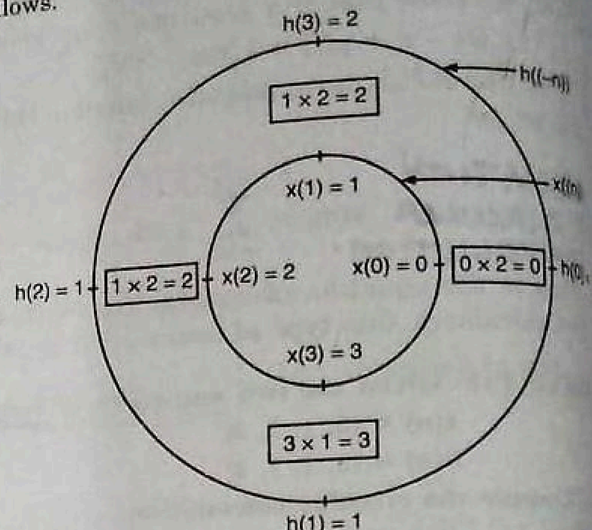


Fig. 2.19. (d) $\sum_{n=0}^{3} x(n)h((-n))_4$.

(iii) Calculation of y(1)

Substituting m = 1 in equation (ii), we have

$$y(1) = \sum_{n=0}^{3} x(n)h((1-n))_4 \quad ...(iv)$$

Here, $h((1-n))_4$ is same as $h((-n+1))_4$. This indicates delay of h((− n)) by 1 sample. This is obtained by shifting h((− n)) in anti-clockwise direction by 1 sample, as shown in figure 2.19(e).

We have already drawn the sequence x(n) as shown in figure 2.19(a). To do the calculations, according to equation (iv), two sequences x(n) and $h((1-n))_4$ are plotted on two concentric circles as shown in figure 2.19(f). Also, y(1) is obtained by adding the product of individual terms.

Therefore, we write

$$y(1) = (0 \times 1) + (3 \times 1) + (2 \times 2) + (1 \times 2) = 0 + 3 + 4 + 2$$

or $$y(1) = 9$$

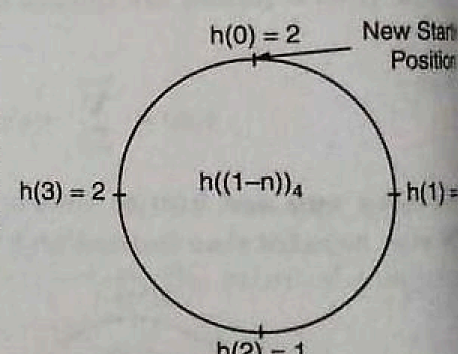


Fig. 2.19. (e). $h((-n+1))_4$.

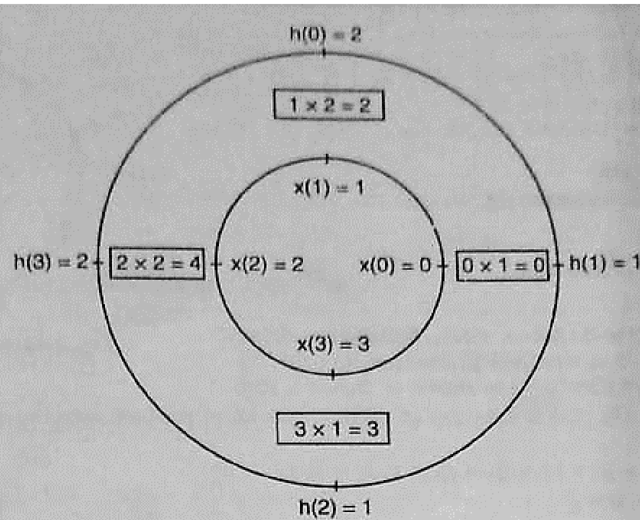


Fig. 2.19. (f) $y(1) = \sum_{n=0}^{3} x(n)h((1-n))_4$.

(iv) Calculation of y(2)

Substituting m = 2 in equation (ii), we obtain,

$$\therefore \quad y(2) = \sum_{n=0}^{3} x(n)h((2-n))_4 \qquad \text{...(v)}$$

Here, $h((2-n))_4$ is same as $h((-n+2))_4$. It indicates delay of $h((-n))_4$ by 2 samples. It is obtained by shifting $h((-n))_4$ by two samples in anti-clockwise direction as shown in figure 2.19(g).

According to equation (iv), the value of y(2) is obtained adding individual product terms as shown in figure 2.19(h).

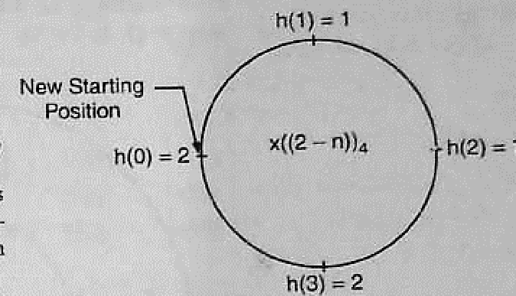


Fig. 2.19. (g) $h((-n+2))_4$.

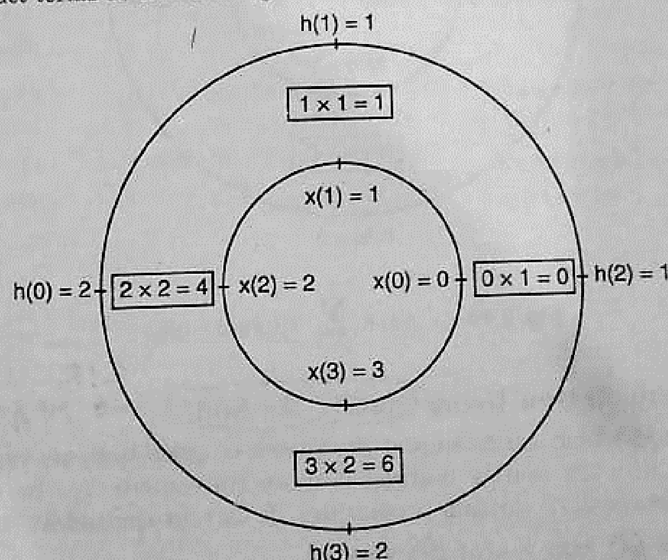


Fig. 2.19. (h) $y(2) = \sum_{n=0}^{3} x(n)\, h((2-n))_4$.

Solution. Each sequence consists of four nonzero points. For the purposes of illustrating the operations involved in circular convolution, it is desirable to graph each sequence as points on a circle. Thus the sequences $x_1(n)$ and $x_2(n)$ are graphed as illustrated in Fig. 7.2.2(a). We note that the sequences are graphed in a counterclockwise direction on a circle. This establishes the reference direction in rotating one of the sequences relative to the other.

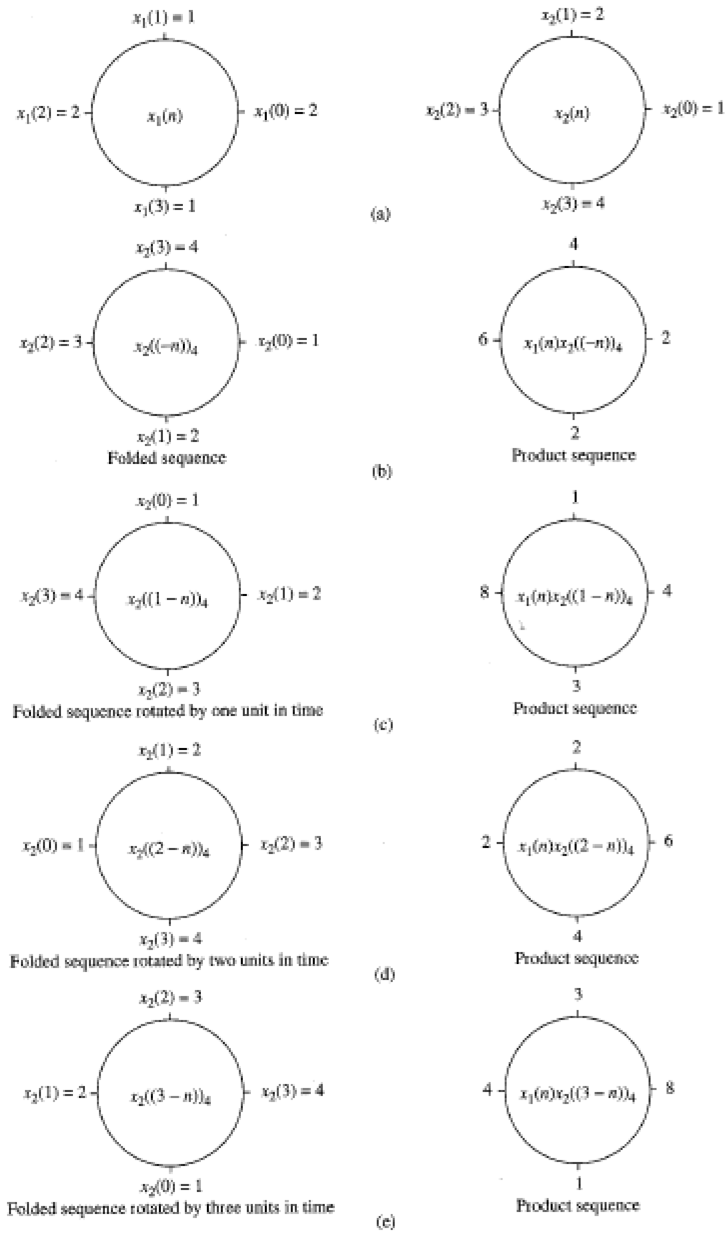
Now, $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified by (7.2.39). Beginning with $m = 0$ we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)x_2((-n))N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle as illustrated in Fig. 7.2.2(b). In other words, the folded sequence is simply $x_2(n)$ graphed in a clockwise direction.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$, point by point. This sequence is also illustrated in Fig. 7.2.2(b). Finally, we sum the values in the product sequence to obtain

$$x_3(0) = 14$$



$x_1(1) = 1$
$x_1(2) = 2$
$x_1(n)$
$x_1(0) = 2$
$x_1(3) = 1$
$x_2(1) = 2$
$x_2(2) = 3$
$x_2(n)$
$x_2(0) = 1$
$x_2(3) = 4$
(a)
$x_2(3) = 4$
$x_2(2) = 3$
$x_2((-n))_4$
$x_2(0) = 1$
$x_2(1) = 2$
Folded sequence
4
6
$x_1(n)x_2((-n))_4$
2
2
Product sequence
(b)
$x_2(0) = 1$
$x_2(3) = 4$
$x_2((1-n))_4$
$x_2(1) = 2$
$x_2(2) = 3$
Folded sequence rotated by one unit in time
1
8
$x_1(n)x_2((1-n))_4$
4
3
Product sequence
(c)
$x_2(1) = 2$
$x_2(0) = 1$
$x_2((2-n))_4$
$x_2(2) = 3$
$x_2(3) = 4$
Folded sequence rotated by two units in time
2
2
$x_1(n)x_2((2-n))_4$
6
4
Product sequence
(d)
$x_2(2) = 3$
$x_2(1) = 2$
$x_2((3-n))_4$
$x_2(3) = 4$
$x_2(0) = 1$
Folded sequence rotated by three units in time
3
4
$x_1(n)x_2((3-n))_4$
8
1
Product sequence
(e)

For $m = 1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)x_2((1-n))_4$$

It is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated counterclockwise by one unit in time as illustrated in Fig. 7.2.2(c). This rotated sequence multiplies $x_1(n)$ to yield the product sequence, also illustrated in Fig. 7.2.2(c). Finally, we sum the values in the product sequence to obtain $x_3(1)$. Thus

$$x_3(1) = 16$$

For $m = 2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence in Fig. 7.2.2(b) rotated two units of time in the counterclockwise direction. The resultant sequence is illustrated in Fig. 7.2.2(d) along with the product sequence $x_1(n)x_2((2-n))_4$. By summing the four terms in the product sequence, we obtain

$$x_3(2) = 14$$

For $m = 3$ we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)x_2((3-n))_4$$

The folded sequence $x_2((-n))_4$ is now rotated by three units in time to yield $x_2((3-n))_4$ and the resultant sequence is multiplied by $x_1(n)$ to yield the product sequence as illustrated in Fig. 7.2.2(e). The sum of the values in the product sequence is

$$x_3(3) = 16$$

We observe that if the computation above is continued beyond $m = 3$, we simply repeat the sequence of four values obtained above. Therefore, the circular convolution of the two sequences $x_1(n)$ and $x_2(n)$ yields the sequence

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$

Q2)

Solution: $x(n) = \left(\frac{1}{5}, \underset{\uparrow}{\frac{1}{5}}, \frac{1}{5}\right)$

We know that, $X(k) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

at $\omega = \frac{2\pi k}{N}, k = 0, 1, ..., N-1$

$$X(k) = \frac{1}{5}[e^{j\omega} + 1 + e^{-j\omega}]_{\text{at } \omega = \frac{2\pi k}{N}} = \frac{1}{5}[1 + 2\cos\omega]$$

where $\omega = \frac{2\pi k}{N}$

and $N = 3$

or $X(k) = \frac{1}{5}\left[1 + 2\cos\left(\frac{2\pi k}{3}\right)\right]$

Thus, $X(k) = \frac{1}{5}\left[1 + 2\cos\left(\frac{2\pi k}{3}\right)\right]$

where $k = 0, 1, ... N-1$ **Ans.**