PRESIDENT'S OFFICE
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENTS
SAME AND MWANGA SECONDARY SCHOOLS EXAMINATION SYNDICATE (SAMWASSES)

FORM SIX PRE - MOCK EXAMINATIONS - 2022
BASIC APPLIED MATHEMATICS
MARKING SCHEME

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1. a) i) 1.0400………… $(03\ marks)$

ii) $-\ 4.274\times10^{1}$ ………… $(03\ marks)$

b) 7.630×10^{-2} ………… $(04\ marks)$

2. a) $f(x) = \{12x + 5 \quad if\ x > 1 \quad x - 4 \quad if\ x\leq1$

$-\frac{1}{8}\ is\ found\ in\ x\leq1$

$f(x) = x - 4$

$f\left(-\frac{1}{8}\right) = -\frac{1}{8} - 4$

$= -\frac{33}{8}$

$= -4\frac{1}{8}$ ………… $\left(01\frac{1}{2}\ marks\right)$

$f(2)$, $2\ is\ found\ in\ x > 1$

$f(x) = 12x + 5$

$f(2) = 12(2) + 5$

$f(2) = 24 + 5 = 29$............ $\left(01\frac{1}{2}\ marks\right)$

$f(-3)$, -3 *is found in* $x \leq 1$

$f(x) = x - 4$

$f(-3) = -3 - 4$

$f(2) = -7$............ $\left(01\frac{1}{2}\ marks\right)$

b)

$f(x) = \frac{1}{2-x}$

$V.A;\quad 2 - x = 0$

$x = 2$............ $\left(00\frac{1}{2}\ marks\right)$

$H.A;\ \ y = \frac{1}{2-x} = \frac{\frac{1}{x}}{\frac{2}{x} - \frac{x}{x}} = \frac{0}{0-1} = \frac{0}{1} = 0$

$y = 0$............ $\left(00\frac{1}{2}\ marks\right)$

$x - intercept\ ,\ y = 0$

$y = \frac{1}{2-x}\ ;\ 0 = \frac{1}{2-x}\ ;\ 0 = 1$

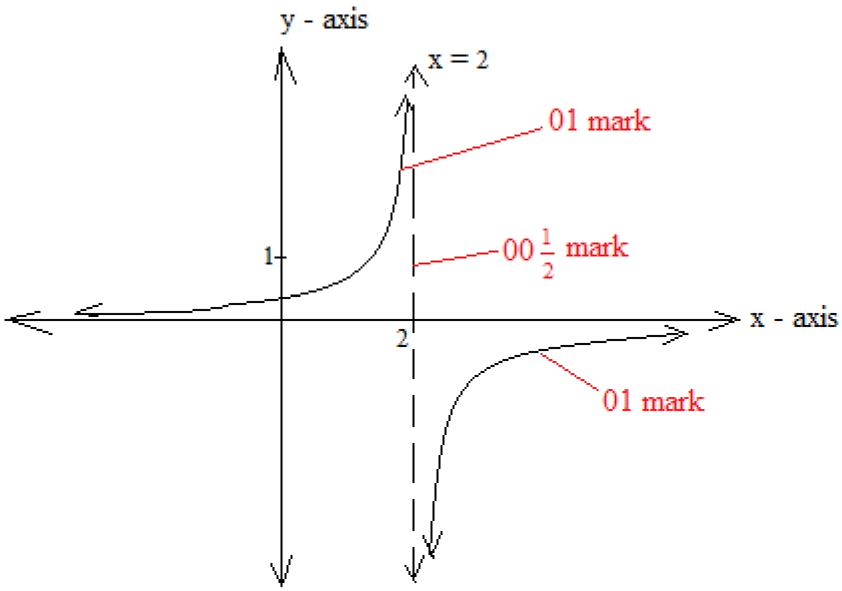
$no\ x - intercept$............ $\left(00\frac{1}{2}\ marks\right)$

$y - intercept;\ \ x = 0$

$y = \frac{1}{2-x}\ ;\ y = \frac{1}{2-0}\ ;\ y = \frac{1}{2}$

$y - intercept; \ y = \frac{1}{2}$............ $\left(00\frac{1}{2}\ marks\right)$

Graph:



$Domain = \{x: x \in R; x \neq 2\}$............ $\left(00\frac{1}{2}\ marks\right)$

$Range = \{y: y \in R; y \neq 0\}$............ $\left(00\frac{1}{2}\ marks\right)$

3. a) $5^{th}\ term = 6\left(2^{nd}\ term\right)$............ $\left(00\frac{1}{2}\ marks\right)$

$A_2 + A_3 = 11$............ $\left(00\frac{1}{2}\ marks\right)$

$Then\ A_5 = 6\left(A_2\right)$

$From\ A_n = A_1 + (n-1)d$

$A_1 + (5-1)d = 6\left(A_1 + (2-1)d\right)$

$5A_1 + 2d = 0$.......... (i)............. $(01\ mark)$

$A_2 + A_3 = 11$

$\left(A_1 + (2-1)d\right) + A_1 + (3-1)d = 11$

$2A_1 + 3d = 11$...............(ii)............. $(01\ mark)$

$\{5A_1 + 2d = 0\ 2A_1 + 3d = 11$

$d = \frac{-5A_1}{2}$............. $\left(00\frac{1}{2}\ marks\right)$

$from\ \ 2A_1 + 3d = 11$

$2A_1 + 3\left(\frac{-5A_1}{2}\right) = 11$

$4A_1 + \left(-15A_1\right) = 22$

$A_1 =- 2$

$d = \frac{-5A_1}{2} = \frac{-5(-2)}{2} = 5$

$d = 5$

i) $A_1 =- 2$............. $(01\ mark)$

ii) $d = 5$............ $(01\ mark)$

iii) $S_{10} = \ ?$

$from\ S_n = \frac{(A_1 + A_n)n}{2}$............ $\left(00\frac{1}{2}\ marks\right)$

$S_n = \frac{(2A_1 + (n-1)d)n}{2}$

$S_{10} = \frac{10}{2}(2(-\ 2) + (10\ -\ 1)5)$

$S_{10} = 205$............ $(01\ mark)$

b) Given ratio of gold and silver $= 1\frac{1}{2} : 2\frac{1}{4}$

$Total\ of\ ratio\ = 1\frac{1}{2} + 2\frac{1}{4} = 3\frac{3}{4} = \frac{15}{4}$............ $(01\ mark)$

$Total\ mass\ = \frac{24g \times \frac{15}{4}}{\frac{3}{2}}$

i) $Total\ mass\ = 60g$............ $(01\ mark)$

ii) $Mass\ of\ silver\ = (60\ -\ 24)g$
$= 36g$............ $(01\ mark)$

4. a) i) $yx^2 - y^2x + 5y - 20x = 14$

$x^2\frac{dy}{dx} + 2xy - 2yx\frac{dy}{dx} - y^2 + 5\frac{dy}{dx} - 20 = 0$

$\frac{dy}{dx}\left(x^2 - 2xy + 5\right) = 20 + y^2 - 2yx$

$\frac{dy}{dx} = \frac{20 + y^2 - 2yx}{x^2 - 2xy + 5}$

$\frac{dy}{dx} = \frac{y^2-2yx+20}{x^2-2xy+5}$ .... $(02\ marks)$

ii)

$y = \frac{x-3}{x^2+2}$

$Let\ u = x - 3 \quad \frac{du}{dx} = 1$

$v = x^2 + 2 \quad \frac{dv}{dx} = 2x$

$\frac{dy}{dx} = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

$\frac{dy}{dx} = \frac{(x^2+2)(1)-(x-3)(2x)}{(x^2+2)^2}$

$\frac{dy}{dx} = \frac{x^2+2-2x^2+6x}{(x^2+2)^2}$

$\frac{dy}{dx} = \frac{-x^2+6x+2}{(x^2+2)^2}$ .... $(02\ marks)$

b) Given

$f(x) = 2x^2$

$f(x + b) = 2(x + h)^2 = 2(x^2 + 2xh + h^2)$

$f(x + b) = 2x^2 + 4xh + 2h^2$

$Then$:

$\frac{dy}{dx} = \frac{f(x+h)-f(x)}{h}$ .... $\left(00\frac{1}{2}\ marks\right)$

$= \left(\frac{2x^2+4xh+2h^2-2x^2}{h}\right)$

$= \left(\frac{4xh+2h^2}{h}\right)$ ………… $(01\ mark)$

$= \left(h\left(\frac{4x+2h}{h}\right)\right)$ ………… $\left(00\frac{1}{2}\ marks\right)$

$= (4x + 2h)$ ………… $\left(00\frac{1}{2}\ marks\right)$

$as\ h \quad \rightarrow 0\ ,\ 2h = 0$

$= 4x + 0$

$= 4x$………… $(01\ mark)$

$\therefore \frac{d}{dx}\left(2x^2\right) = 4x$………… $\left(00\frac{1}{2}\ marks\right)$

c) $y = x^2 - 2x + 1$

$\frac{dy}{dx} = 2x - 2$………… $(01\ mark)$

$at\ x = 2$

$\frac{dy}{dx} = 2(2) - 2 = 2$

$\therefore Gradient = 2$………… $(01\ mark)$

5. a) i)

$$\int_{2}^{4} 3x^2\left(x^3 + 5\right)dx$$

$Let\ u = x^3 + 5\ ,\ \frac{du}{dx} = 3x^2 \quad dx = \frac{du}{3x^2}$

$Then$

$$\int_{2}^{4} 3x^2\left(x^3 + 5\right)dx = \int_{2}^{4} 3x^2 u \bullet \frac{du}{3x^2}$$

$$= \int_{2}^{4} udu$$

$$= \frac{u^2}{2}\Big|_{2}^{4}$$

$$= \frac{1}{2}\left(4^2 - 2^2\right)$$

$$= \frac{1}{2}(12)$$

$$= 6$$

$\int_{2}^{4} 3x^2\left(x^3 + 5\right)dx = 6$………… $(03\ marks)$

ii)

$\int \sin sin\ (2x + 4)\ dx$

$Let\ u = 2x + 4\ ,\ \frac{du}{dx} = 2\ ,\ dx = \frac{du}{2}$

$\therefore \int \sin sin\ (2x + 4)\ dx = \int \sin sin\ u\ \frac{du}{2}$

$= -\frac{1}{2}\cos cos\ u\ + A$

$\therefore \int \sin sin\ (2x + 4)\ dx = -\frac{1}{2}\cos cos\ (2x + 4)\ + A$............ $(03\ marks)$

b)

$A = \int_{x=a}^{x=b} ydx = \int_{0}^{1}\left(4x^2 - 1\right)dx$

$= \int_{0}^{1} 4x^2dx - \int_{0}^{1} dx$

$= \frac{4}{3}\ x^3\ \Big|_0^1 - \ x\ \Big|_0^1$

$= \frac{4}{3}(1 - 0) - (1 - 0)$

$= \frac{4}{3} - 1$

$= \frac{1}{3}$

$\therefore The\ area\ is\ \frac{1}{3}\ square\ units$............ $(04\ marks)$

6.

Class interval	x	f	d	u	fu	$c{\cdot}f$
$1 - 10$	5.5	0	-20	-2	0	0
$11 - 20$	15.5	6	-10	-1	-6	6

$21 - 30$	25.5	3	0	0	0	9
$31 - 40$	35.5	6	1	1	6	15
$41 - 50$	45.5	3	2	2	6	18
$A = 25.5$			$\left(00\frac{1}{2}\ mark\right)$	$\left(00\frac{1}{2}\ mark\right)$	$\left(00\frac{1}{2}\ mark\right)$	$\left(00\frac{1}{2}\ mark\right)$

a) Mean

$$Mean = A + \frac{cfu}{\Sigma f}$$

$but\ A = 25.5\ ,\ c = 10\ ,\ fu = 6$ and $\sum f = 18$

$$Mean = 25.5 + \frac{10\times6}{18}$$

$\therefore Mean = 28.83333$............ $(02\ marks)$

b) Semi – Interquartile Range

$$Q_3 = L + \left(\frac{\frac{3}{4}N - \Sigma f_b}{f_w}\right)c$$

$\frac{3}{4}N = \frac{3}{4}(18) = 13.5\ ,\ \sum f_b = 9,\quad f_w = 6,\ c = 10\ ; L = 31 - 0.5 = 30.5$

$$Q_3 = 30.5 + \left(\frac{13.5-9}{6}\right)\times10$$

$Q_3 = 38$ .... $(02\ marks)$

$$Q_1 = L + \left(\frac{\frac{1}{4}N - \Sigma f_b}{f_w}\right)c$$

$\frac{1}{4}N = \frac{1}{4}(18) = 4.5$, $\sum f_b = 0$, $f_w = 6$, $c = 10$; $L = 10.5$

$Q_1 = 10.5 + \left(\frac{4.5-0}{6}\right)\times 10$

$Q_3 = 18$ ………… $(02\ marks)$

$\therefore Semi - I.Q.R = \frac{38-18}{2} = 10$………… $(02\ marks)$

7. a) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$ $and\ P(C) = \frac{1}{3}$

i) $P(A\cap B) -$ $for\ independent\ events$

$P(A\cap B) = P(A)\cdot P(B)$

$= \left(\frac{1}{2}\right)\bullet\left(\frac{1}{4}\right)$

$P(A\cap B) = \frac{1}{8}$………… $(02\ marks)$

ii) $P(A\cup C) -$ $for\ mutually\ Exclusive\ events$

$P(A\cup C) = P(A) + P(B) - P(A\cap C)$

$but P(A\cap C) = 0$

$P(A\cup C) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

$P(A\cup C) = \frac{5}{6}$………… $(02\ marks)$

b) $n(cards) = 52\,,\ \ n(sides) = 6$

$Required\ P\left(C^{'}n > 4\right) = \ ?$

$Let\ C^{'} - be\ not\ a\ club,\ and$

$G - Greater\ than\ 4$ $(01\ mark)$

$n(G) = \{5, 6\}$

$P\left(C^{'}\cap G\right) = P\left(C^{'}\right)\times P(G)$

$= (1 - P(C)) \times (P(G))$………… $(01\ mark)$

$= \left(1 - \frac{13}{52}\right) \times \frac{2}{6}$

$= \frac{1}{4}$

$\therefore P\left(C^{'}\cap G\right) = 0.25\ or\ \frac{1}{4}$………… $(01\ mark)$

c) Given

$^{n}P_{4} = 42\left(^{n}P_{4}\right)$

$\frac{n!}{(n-4)!} = 42\left(\frac{n!}{(n-2)!}\right)$

$\frac{n!}{(n-4)!} = \frac{42n!}{(n-2)!}$ ………… $(01\ mark)$

$\frac{1}{(n-4)!} = \frac{42}{(n-2)!}$

$\frac{1}{(n-4)!} = \frac{42}{(n-2)(n-3)(n-4)!}$

$\frac{1}{1} = \frac{42}{(n-2)(n-3)}$

$(n - 2)(n - 3) = 42$............ $\left(00\frac{1}{2}\ marks\right)$

$n^2 - 3n - 2n + 6 = 12$

$n^2 - 5n - 36 = 0$

$n_1 = 9\ and\ n_2 =- 4$

$Since\ n\geq 0$

$Therefore$

$n = 9$............ $\left(00\frac{1}{2}\ marks\right)$

$Then\ {}^{n}P_4 = {}^{9}P_4 = \frac{9!}{(9-4)!} = \frac{9\times8\times7\times6\times5!}{5!}$

$\therefore {}^{9}P_4 = 15,120$............ $(01\ mark)$

8. a) $\{x = 1 + \cos cos\ 2\theta$ .... (i) $y = 4 \sin sin\ \theta$ .... (ii)

$from\ eqn\ (i)$

$x = 1 + \cos cos\ 2\theta\ ;\ but\ \cos cos\ 2\theta = 1 - 2\theta$

$x = 1 + (1 - 2\theta)$

$x = 2 - 2\theta$ .... $(01\ mark)$

$but\ from\ eqn\ (ii)\ \sin sin\ \theta = \frac{y}{4}$

$x = 2 - 2\left(\frac{y}{4}\right)^2$

$x = 2 - \frac{2y^2}{16}$ ………… $(01\ mark)$

$x = 2 - \frac{y^2}{8}$

$8x = 16 - y^2$

$y^2 + 8x = 16$ ………… $(01\ mark)$

b)

$\cot cot\,(A + B) = \frac{\cot cot\,A \cot cot\,B - 1}{\cot cot\,A + \cot cot\,B}$

Consider L. H. S

$\cot cot\,(A + B) = \frac{\cos cos\,(A+B)}{\sin sin\,(A+B)}$

$= \frac{\cos cos\,A \cos cos\,B - \sin sin\,A \sin sin\,B}{\cos sin sin\,A\,cos\,B + \sin cos cos\,A\,sin\,B}$ ………… $(01\ mark)$

Divide by $\sin sin\,A\ \sin sin\,B$ *through out*

$= \frac{\frac{\cos cos\,A \cos cos\,B}{\sin sin\,A \sin sin\,B} - \frac{\sin sin\,A \sin sin\,B}{\sin sin\,A \sin sin\,B}}{\frac{\cos sin sin\,A\,cos\,B}{\sin sin\,A \sin sin\,B} + \frac{\sin cos cos\,A\,sin\,B}{\sin sin\,A \sin sin\,B}}$ ………… $(01\ mark)$

$= \frac{\cot cot\,A \cot cot\,B - 1}{\cot cot\,B + \cot cot\,A}$

$= \frac{\cot cot\,A \cot cot\,B - 1}{\cot cot\,A + \cot cot\,B}$ *R. H. S* ………… $(01\ mark)$

SInce L. H. S = R. H. S

Hence Proved

c) $\sin sin\, 2\theta \ =-\ \sin sin\, \theta$

$\sin sin\, 2\theta \ +\ \sin sin\, \theta \ =\ 0$

$2 \sin sin\, \theta\ \cos cos\, \theta \ +\ \sin sin\, \theta \ =\ 0$

$\sin sin\, \theta\ (2 \cos cos\, \theta \ +\ 1) =\ 0$............ $(01\ mark)$

$either\ \ \sin sin\, \theta \ =\ 0\ \ or\ 2 \cos cos\, \theta \ +\ 1\ =\ 0$

$\sin sin\, \theta \ =\ 0\ \ and\ \ \cos cos\, \theta \ =-\ \frac{1}{2}$............ $(01\ mark)$

$for\ interval\ \ -\ \pi \leq \theta \leq \pi$

$\sin sin\, \theta \ =\ 0\ \ implies\ that\ \theta \ =\ 0°\ and\ \theta \ =\ 180°$

$and,\ \ \cos cos\, \theta \ =-\ \frac{1}{2}\ \ implies\ that\ \theta \ =\ 120°\ and\ \theta \ =-\ 120°$

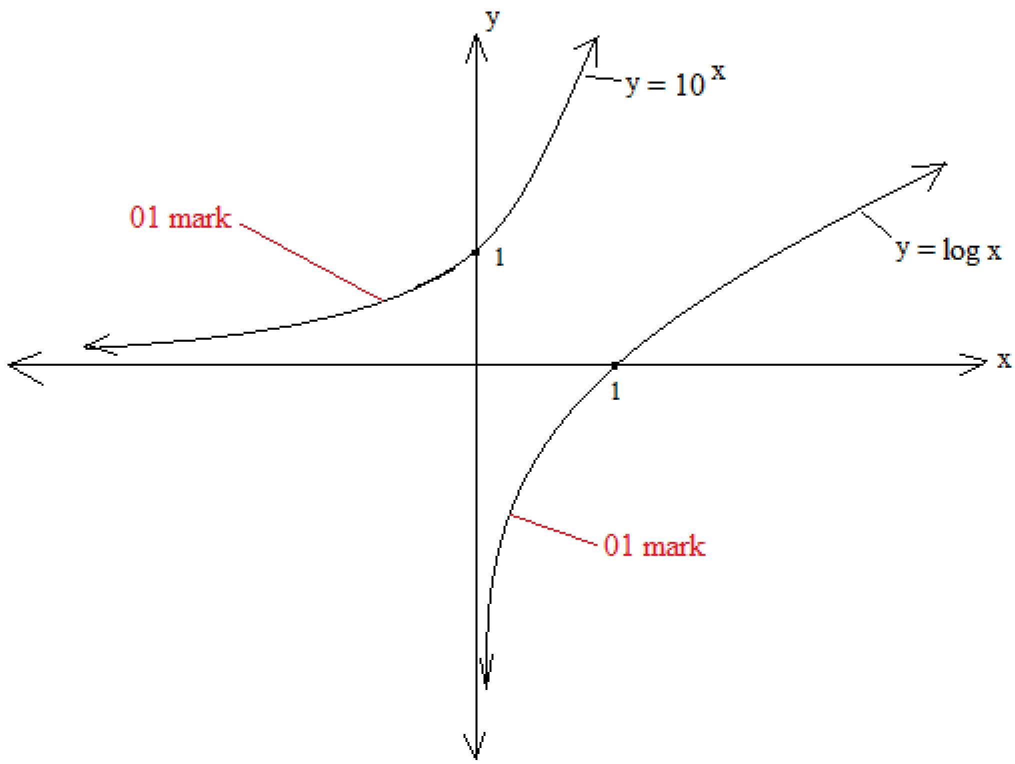
$\therefore \theta \ =-\ 120°,\ 0°\ ,120°\ ,180°\ \ \ or$............ $(01\ mark)$

$\theta \ =-\ \frac{2}{3}\pi\ \ ,\ 0°\ \ ,\frac{2}{3}\pi\ \ ,\ \pi$ .... $(01\ mark)$

9. a) i) $y\ =\ 10^{x}\ ,\ y\ =\ \log log\, x$

x	$-\ \infty$	$-\ 4$	$-\ 2$	0	2	4	∞
10^{x}	0	0.0001	0.01	1	100	⋯	⋯∞
$\log log$	–	–	–	–	0.3	0.6	

$(01\ mark)$



ii) $Domain = \{x: x \in R\}$

$Range = \{y: y \in R\}$............ $\left(00\frac{1}{2}\ marks\right)$

$y = 10^{x} \rightarrow cross\ y - axis\ at\ y = 1$............ $\left(00\frac{1}{2}\ marks\right)$

$y = \log log\ x \rightarrow cross\ x - axis\ at\ x = 1$............ $\left(00\frac{1}{2}\ marks\right)$

$y = 10^{x} \rightarrow cannot\ cross\ x - axis$

$y = \log log\ x \rightarrow cannot\ cross\ y - axis$ .... $\left(00\frac{1}{2}\ marks\right)$

b) i) $y = 6^{x}$

$\ln ln\ y = \ln ln\ 6^{x}$

$\ln ln\ y = x \ln ln\ 6$

$\frac{1}{y}\frac{dy}{dx} = \ln ln\ 6$

$\frac{dy}{dx} = y \ln ln\ 6$

$\frac{dy}{dx} = 6^{x} \ln ln\ 6$ .... $(01\ mark)$

ii) $y = \ln ln\, x$

$\frac{dy}{dx} = \frac{1}{x}$ .... $(01\, mark)$

c) $y = x^{x^2}$

$\ln ln\, y = \ln ln \left(x^{x^2}\right)$

$\ln ln\, y = x^2 \ln ln\, x$

$\frac{1}{y}\frac{dy}{dx} = x^2\left(\frac{1}{x}\right) + 2x \ln ln\, x$ .... $(01\, mark)$

$\frac{dy}{dx} = (x + 2x \ln ln\, x)y$

$\frac{dy}{dx} = (x + 2x \ln ln\, x)\left(x^{x^2}\right)$

$\frac{dy}{dx} = (1 + 2 \ln ln\, x)(x)\left(x^{x^2}\right)$............ $(01\, mark)$

$\frac{dy}{dx} = (1 + 2 \ln ln\, x)\left(x^{x^2+1}\right)$

$\therefore \frac{dy}{dx} = \left(1 + 2 \ln ln\, x^2\right)\left[x^{x^2+1}\right]$ .... $(01\, mark)$

Hence shown

10. a) $A = [2 \ - 6\ 2\ 4\ 2 \ - 8\ 12 \ - 14\ 16\ ]$

$|A| = 2|2 \ - 8 \ - 14\ 16\ | + 6|4 \ - 8\ 12\ 16\ | + 2|4\ 2\ 12 \ - 14\ |$

$|A| = 640$ .... $(01\, mark)$

$Minor\ of\ A = \{|2\ -8\ -14\ 16\ |\ |4\ -8\ 12\ 16\ |\ |4\ 2\ 12\ -14\ |\ |-6\ 2\ -14\ 16\ |\ |2\ 2\ 12\ 16\ |\ |2\ -6\ 12\ -$

$A = (-80\ 160\ -80\ -64\ 8\ 44\ 44\ -24\ 28\ )$............ $(01\ mark)$

$Cofactor\ of\ A = (-80\ -160\ -80\ 64\ 8\ -44\ 44\ 24\ 28\ )$

$Adjoint\ of\ A = (Cofactor\ of\ A)^T$............ $(01\ mark)$

$Adj\ A = (-80\ 68\ 44\ -160\ 8\ 24\ -80\ -44\ 28\ )$............ $(01\ mark)$

$Inverse\ of\ A = \frac{1}{|A|} \cdot Adj\ A$

$= \frac{1}{640}(-80\ 68\ 44\ -160\ 8\ 24\ -80\ -44\ 28\ )$

$A^{-1} = \left(-\frac{1}{8}\ \frac{17}{160}\ \frac{11}{160}\ -\frac{1}{4}\ \frac{1}{80}\ \frac{3}{80}\ -\frac{1}{8}\ -\frac{11}{160}\ \frac{7}{160}\right)$............ $(01\ mark)$

b) Let x – be number of scientific calculators
y – be number of graphic calculators

$from\ information\ provided$

$x \geq 100$

$y \geq 80$

$x \leq 200$

$y \leq 170$

$x + y \geq 200$

$x\ ,\ y \geq 0$ $(01\ mark)$

$Maximizing\ profit\ f(x, y) = -20,000x + 50,000y$............ $(01\ mark)$

Mathematical formulation

$Maximize\ f(x, y) = -20,000x + 50,000y$

$Subject\ to\ x \geq 100$

$y \geq 80$

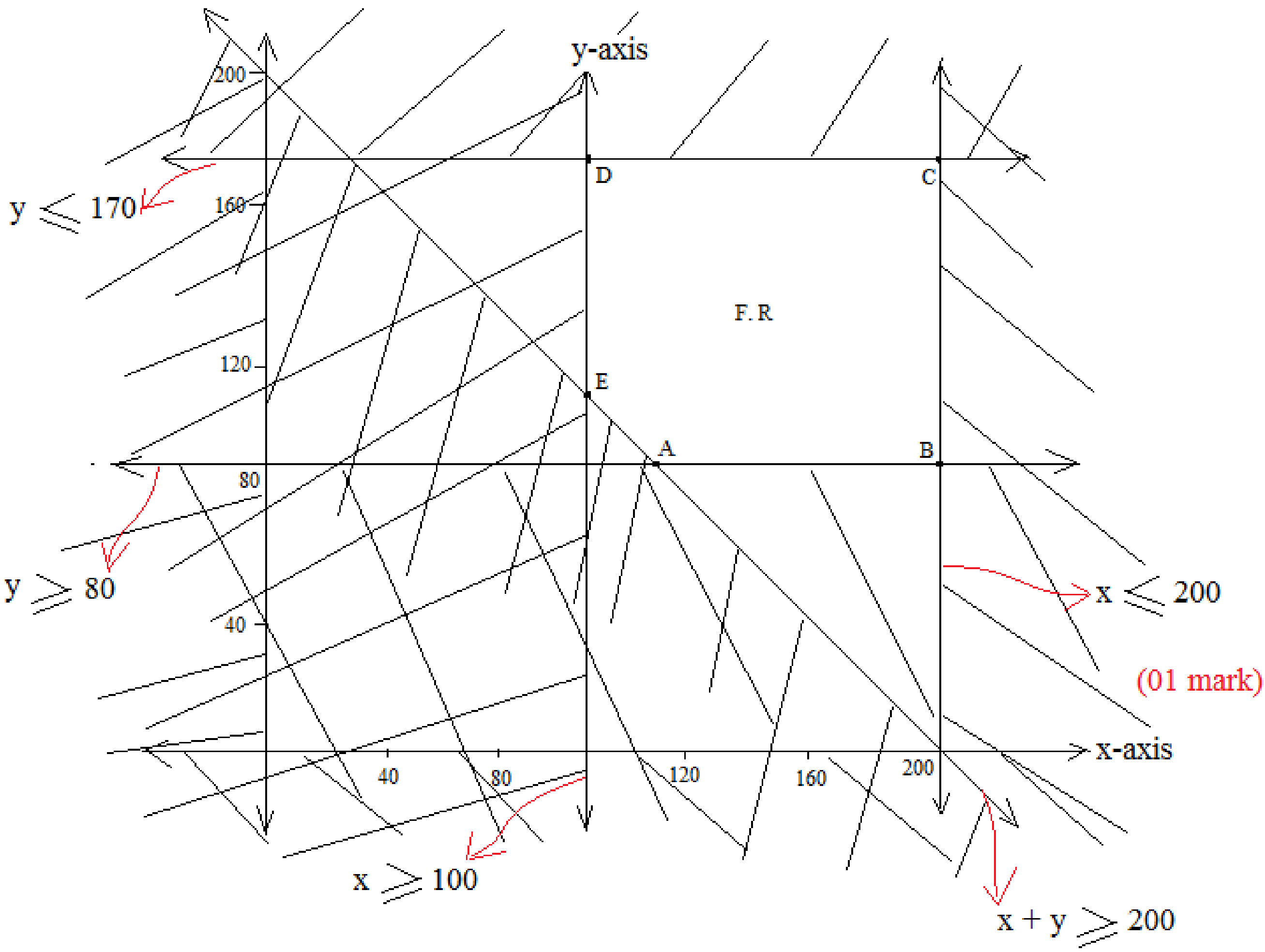
$x \leq 200$

$y \leq 170$

$x + y \geq 200$

$x, y \geq 0$

Graphical representation of inequalities



Corner Points	Max: $f(x, y) = -20,000x + 50,000y$
$A = (120, 80)$	1,600,000/=
$B = (200, 80)$	0
$C = (200, 170)$	4,500,000/=
$D = (100, 170)$	6,500,000/=
$A = (100, 100)$	3,000,000/=

(01 *mark*)

They should make 100 scientific calculators and 170 graphical calculators to maximize the net profit of 6,500,000/=............ (01 *mark*)