

FINALTERM EXAMINATION

Fall 2009

MTH301- Calculus II

Time: 120 min

Marks: 80

Question No: 1 (Marks: 1) - Please choose one

π is an example of -----

► Irrational numbers

► Rational numbers

► Integers

► Natural numbers

Question No: 2 (Marks: 1) - Please choose one

Straight line is a special kind of -----

- ▶ Surface
- ▶ Curve
- ▶ Plane
- ▶ Parabola

Question No: 3 (Marks: 1) - Please choose one

An ordered triple corresponds to ----- in three dimensional space.

- ▶ A unique point
- ▶ A point in each octant

- ▶ Three points
- ▶ Infinite number of points

Question No: 4 (Marks: 1) - Please choose one

The angles which a line makes with positive x ,y and z-axis are known as -----

- ▶ Direction cosines
- ▶ Direction ratios
- ▶ Direction angles

Question No: 5 (Marks: 1) - Please choose one

Is the function $f(x, y)$ continuous at origin? If not, why?

$$f(x, y) = 4xy + \sin 3x^2y$$

► $f(x, y)$ is continuous at origin

► $f(0, 0)$ is not defined

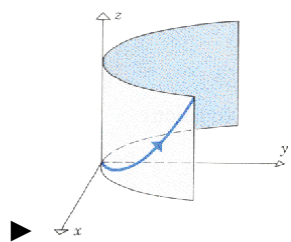
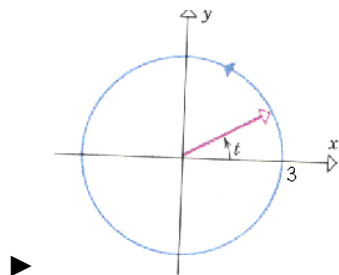
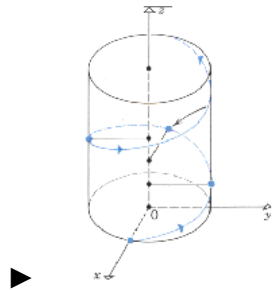
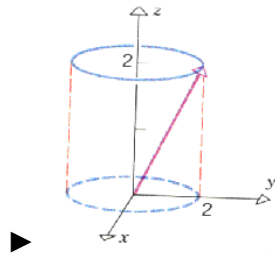
► $f(0, 0)$ is defined but $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist

► $f(0, 0)$ is defined and $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exists but these two numbers are not equal.

Question No: 6 (Marks: 1) - Please choose one

Match the following vector-valued function with its graph.

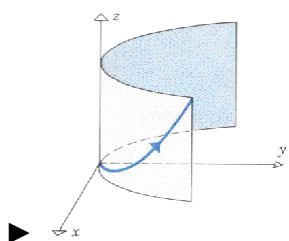
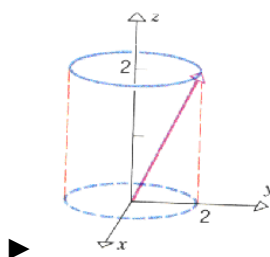
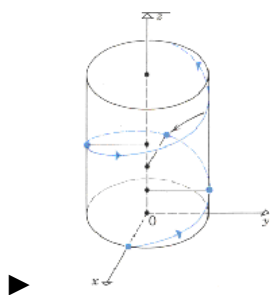
$$r(t) = 3 \cos t \hat{i} + 3 \sin t \hat{j} \quad \text{and} \quad 0 \leq t \leq 2\pi$$

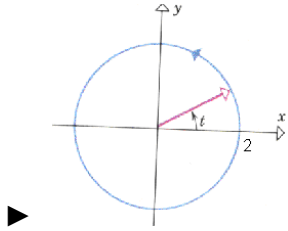


Question No: 7 (Marks: 1) - Please choose one

Match the following vector-valued function with its graph.

$$r(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k} \quad \text{and} \quad t \geq 0$$





Question No: 8 (Marks: 1) - Please choose one

What are the parametric equations that correspond to the following vector equation?

$$\vec{r}(t) = \sin^2 t \hat{i} + (1 - \cos 2t) \hat{j}$$

► $x = \sin^2 t$, $y = 1 - \cos 2t$, $z = 0$

► $y = \sin^2 t$, $x = 1 - \cos 2t$, $z = 0$

► $x = \sin^2 t$, $y = 1 - \cos 2t$, $z = 1$

► $x = \sin^2 t$, $y = \cos 2t$, $z = 1$

Question No: 9 (Marks: 1) - Please choose one

Is the following vector-valued function $\vec{r}(t)$ continuous at $t = 0$? If not, why?

$$\vec{r}(t) = (4\cos t, \sqrt{t}, 4\sin t)$$

- ▶ $\vec{r}(0)$ is not defined
- ▶ $\vec{r}(0)$ is defined but $\lim_{t \rightarrow 0} \vec{r}(t)$ does not exist
- ▶ $\vec{r}(0)$ is defined and $\lim_{t \rightarrow 0} \vec{r}(t)$ exists but these two numbers are not equal.
- ▶ $\vec{r}(t)$ is continuous at $t = 0$

Question No: 10 (Marks: 1) - Please choose one

What is the derivative of following vector-valued function?

$$\vec{r}(t) = (\cos 5t, \tan t, 6 \sin t)$$

► $\vec{r}'(t) = \left(\frac{\sin 5t}{5}, \sec t, 6 \cos t \right)$

► $\vec{r}'(t) = \left(\frac{-\sin 5t}{5}, \sec t, 6 \cos t \right)$

► $\vec{r}'(t) = (-5 \sin 5t, \sec^2 t, 6 \cos t)$

► $\vec{r}'(t) = (\sin 5t, \sec^2 t, -6 \cos t)$

Question No: 11 (Marks: 1) - Please choose one

The following differential is exact

$$dz = (3x^2y + 2) dx + (x^3 + y) dy$$

► True

► False

Question No: 12 (Marks: 1) - Please choose one

The following differential is exact

$$dz = (3x^2 + 4xy) dx + (2x^2 + 2y) dy$$

► True

► False

Question No: 13 (Marks: 1) - Please choose one

Which one of the following is correct Wallis Sine formula when n is odd and $n \geq 3$?

► $\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{\pi}{2} \frac{(n-1)}{n} \frac{(n-3)}{(n-2)} \frac{(n-5)}{(n-4)} \cdots \frac{5}{6} \frac{3}{4} \frac{1}{2}$

► $\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{\pi}{2} \frac{(n)}{(n-1)} \frac{(n-2)}{(n-3)} \frac{(n-4)}{(n-5)} \cdots \frac{6}{5} \frac{4}{3} \frac{2}{1}$

► $\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{(n-1)}{n} \frac{(n-3)}{(n-2)} \frac{(n-5)}{(n-4)} \cdots \frac{6}{7} \frac{4}{5} \frac{2}{3}$

► $\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{(n)}{(n-1)} \frac{(n-2)}{(n-3)} \frac{(n-4)}{(n-5)} \cdots \frac{6}{5} \frac{4}{3} \frac{2}{1}$

Question No: 14 (Marks: 1) - Please choose one

Which of the following is correct?

► $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx = \frac{3}{16}$

► $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx = \frac{3\pi}{16}$

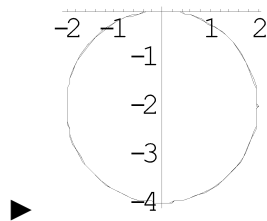
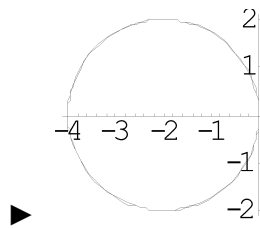
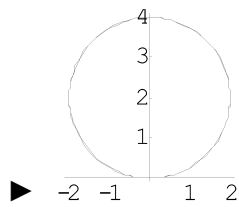
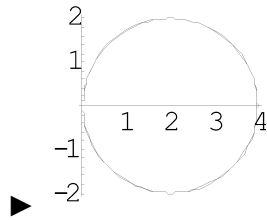
► $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx = \frac{3}{8}$

► $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx = \frac{2\pi}{3}$

Question No: 15 (Marks: 1) - Please choose one

Match the following equation in polar co-ordinates with its graph.

$r = 4 \sin \theta$



Question No: 16 (Marks: 1) - Please choose one

If the equation of a curve, in polar co-ordinates, remains unchanged after replacing (r, θ) by $(r, \pi - \theta)$ then the curve is said to be symmetric about which of the following?

► Initial line

► y-axis

► Pole

Question No: 17 (Marks: 1) - Please choose one

What is the period of a periodic function defined by $f(x) = \sin \frac{x}{2}$?

► $\frac{\pi}{2}$

► π

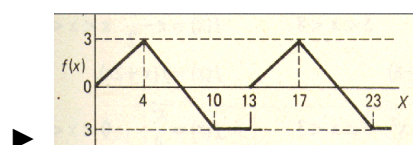
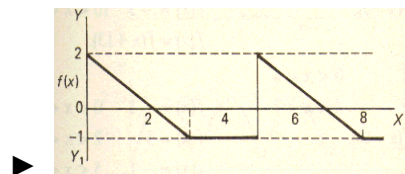
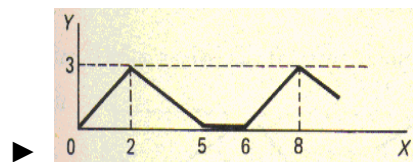
► $\frac{3\pi}{2}$

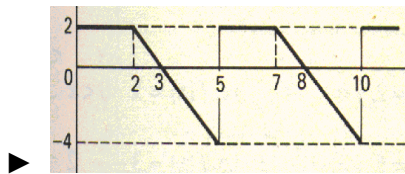
► 4π

Question No: 18 (Marks: 1) - Please choose one

Match the following periodic function with its graph.

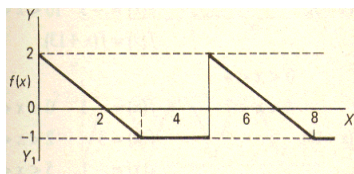
$$f(x) = \begin{cases} \frac{3}{4}x & 0 < x < 4 \\ 7 - x & 4 < x < 10 \\ -3 & 10 < x < 13 \end{cases}$$





Question No: 19 (Marks: 1) - Please choose one

What is the period of periodic function whose graph is as below?



▶ 2

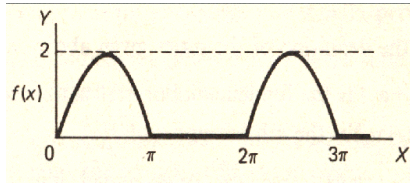
▶ 3

▶ 4

▶ 5

Question No: 20 (Marks: 1) - Please choose one

What is the period of periodic function whose graph is as below?



► 0

► 2

► π

► 2π

Question No: 21 (Marks: 1) - Please choose one

Polar co-ordinates of a point are $\left(-2, \frac{-3\pi}{2}\right)$. Which of the following is another possible polar co-ordinates representation of this point?

► $\left(2, \frac{-\pi}{4}\right)$

► $\left(2, \frac{-\pi}{2}\right)$

► $\left(2, \frac{-\pi}{3}\right)$

► $\left(2, \frac{3\pi}{4}\right)$

Question No: 22 (Marks: 1) - Please choose one

The function $f(x) = x^3 e^x$ is -----

► Even function

- ▶ Odd function
- ▶ Neither even nor odd

Question No: 23 (Marks: 1) - Please choose one

The graph of an even function is symmetrical about -----

- ▶ x-axis
- ▶ y-axis
- ▶ origin

Question No: 24 (Marks: 1) - Please choose one

At which point the vertex of parabola, represented by the equation $y = x^2 - 4x + 3$, occurs?

► (0, 3)

► (2, -1)

► (-2, 15)

► (1, 0)

Question No: 25 (Marks: 1) - Please choose one

The equation $y = x^2 - 4x + 2$ represents a parabola. Find a point at which the vertex of given parabola occurs?

► (2, -2)

► (-4, 34)

► (0, 0)

► $(-2, 14)$

Question No: 26 (Marks: 1) - Please choose one

Is the function $f(x, y)$ continuous at origin? If not, why?

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

► $f(x, y)$ is continuous at origin

► $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist

► $f(0, 0)$ is defined and $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exists but these two numbers are not equal.

Question No: 27 (Marks: 1) - Please choose one

Sign of line integral is reversed when -----

- ▶ path of integration is divided into parts.
- ▶ path of integration is parallel to y-axis.
- ▶ direction of path of integration is reversed.
- ▶ path of integration is parallel to x-axis.

Question No: 28 (Marks: 1) - Please choose one

What is Laplace transform of a function $F(t)$?

(s is a constant)

▶ $\int_0^s e^{-st} F(t) dt$

▶ $\int_0^{\infty} e^{st} F(t) dt$

► $\int_{-\infty}^{\infty} e^{-st} F(t) dt$

► $\int_0^{\infty} e^{-st} F(t) dt$

Question No: 29 (Marks: 1) - Please choose one

What is the value of $L\{e^{5t}\}$ if L denotes laplace transform?

► $L\{e^{5t}\} = \frac{1}{s-5}$

► $L\{e^{5t}\} = \frac{s}{s^2 + 25}$

► $L\{e^{5t}\} = \frac{5}{s^2 + 25}$

► $L\{e^{5t}\} = \frac{5!}{s^6}$

Question No: 30 (Marks: 1) - Please choose one

What is the Laplace Inverse Transform of $\frac{1}{s+1}$

► $L^{-1}\left\{\frac{1}{s+1}\right\} = t+1$

► $L^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t} + e^t$

► $L^{-1}\left\{\frac{1}{s+1}\right\} = e^t$

► $L^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}$

Question No: 31 (Marks: 1) - Please choose one

What is Laplace Inverse Transform of $\frac{5}{s^2 + 25}$

► $L^{-1}\left\{\frac{5}{s^2 + 25}\right\} = \sin 5t$

► $L^{-1}\left\{\frac{5}{s^2 + 25}\right\} = \cos 5t$

► $L^{-1}\left\{\frac{5}{s^2 + 25}\right\} = \sin 25t$

► $L^{-1}\left\{\frac{5}{s^2 + 25}\right\} = \cos 25t$

Question No: 32 (Marks: 1) - Please choose one

What is $L\{-6\}$ if L denotes Laplace Transform?

► $L\{-6\} = \frac{1}{s+6}$

► $L\{-6\} = \frac{-6}{s}$

► $L\{-6\} = \frac{s}{s^2 + 36}$

► $L\{-6\} = \frac{-6}{s^2 + 36}$

Question No: 33 (Marks: 1) - Please choose one

Evaluate the line integral $\int_C (3x + 2y) \, dx + (2x - y) \, dy$ where C is the line segment from $(0, 0)$ to $(2, 0)$.

► 6

► -6

► 0

► Do not exist

Question No: 34 (Marks: 1) - Please choose one

Evaluate the line integral $\int_C (2x + y) dx + (x^2 - y) dy$ where C is the line segment from $(0, 0)$ to $(0, 2)$.

► -4

► -2

► 0

► 2

Question No: 35 (Marks: 1) - Please choose one

Plane is an example of -----

► Curve

► Surface

► Sphere

► Cone

Question No: 36 (Marks: 1) - Please choose one

If $R = \{(x, y) / 0 \leq x \leq 2 \text{ and } -1 \leq y \leq 1\}$, then

$$\iint_R (x + 2y^2) dA =$$

► $\int_{-1}^1 \int_0^2 (x + 2y^2) dy dx$

► $\int_0^2 \int_1^{-1} (x + 2y^2) dx dy$

► $\int_{-1}^1 \int_0^2 (x + 2y^2) dx dy$

► $\int_1^2 \int_{-1}^0 (x + 2y^2) dx dy$

Question No: 37 (Marks: 1) - Please choose one

To evaluate the line integral, the integrand is expressed in terms of x, y, z with

► $dr = dx \hat{i} + dy \hat{j}$

► $dr = dx \hat{i} + dy \hat{j} + dz \hat{k}$

► $dr = dx + dy + dz$

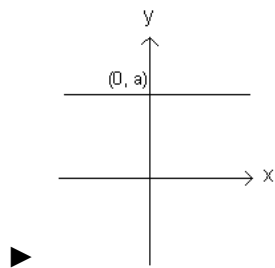
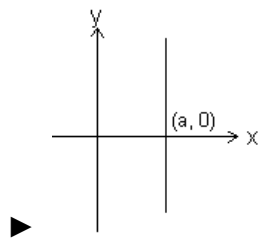
► $dr = dx + dy$

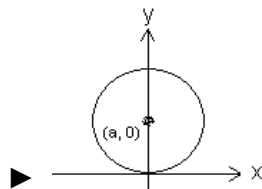
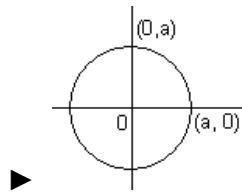
Question No: 38 (Marks: 1) - Please choose one

Match the following equation in polar co-ordinates with its graph.

$r = a$

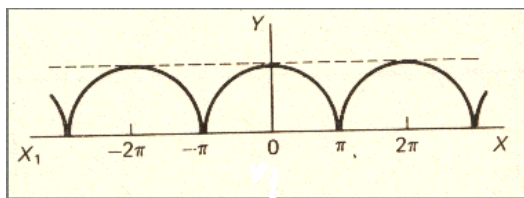
where a is an arbitrary constant.





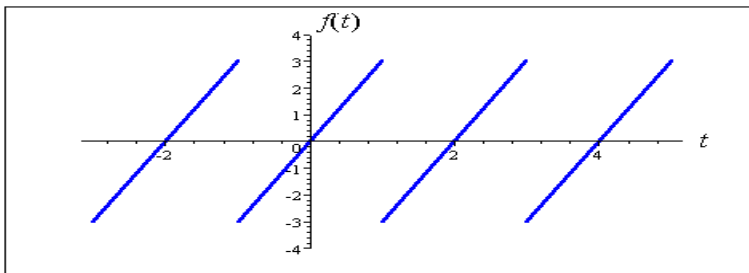
Question No: 39 (Marks: 1) - Please choose one

Which of the following is true for a periodic function whose graph is as below?



- Even function
- Odd function
- Neither even nor odd function

Question No: 40 (Marks: 1) - Please choose one



The graph of "saw tooth wave" given above is -----

► An odd function

► An even function

► Neither even nor odd

Question No: 41 (Marks: 2)

Equation of a curve in polar co-ordinates is $r^2 = 4 \cos 2\theta$

Discuss the symmetry of graph of this curve about y-axis.

Question No: 42 (Marks: 2)

Determine whether the following differential is exact or not.

$$dz = (1 + 8xy) dx + 5x^2 dy$$

Question No: 43 (Marks: 2)

Prove whether the following function is even, odd or neither.

$$f(x) = x^3 + 4x$$

Question No: 44 (Marks: 3)

Find $\text{curl } \vec{F}$, if $\vec{F} = (3x + y)\hat{i} + xy^2z\hat{j} + (xz^2)\hat{k}$

Question No: 45 (Marks: 3)

Determine the fourier co-efficient a_0 of the following function.

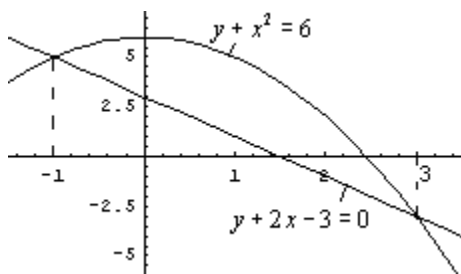
$$f(x) = x^2 \quad 0 < x < 2\pi$$

Question No: 46 (Marks: 3)

Find all possible polar co-ordinates representations of the point P whose rectangular co-ordinates are $(1, 1)$.

Question No: 47 (Marks: 5)

Using definite integral, find area of the region that is enclosed between the curves $y + x^2 = 6$ and $y + 2x - 3 = 0$



Question No: 48 (Marks: 5)

Evaluate the following double integral

$$\int_0^{\pi} \int_0^{1-\sin\theta} r^2 \cos\theta \, dr \, d\theta$$

Question No: 49 (Marks: 5)

Determine the fourier co-efficient b_n of the following function.

$$f(x) = x^2 \quad 0 < x < 2\pi$$

Question No: 50 (Marks: 10)

Let S is the surface bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and

$z = 1$. If $\vec{F} = 4x\hat{i} - 3y\hat{j} + 7z\hat{k}$, use Divergence Theorem to find $\int_S \vec{F} \cdot \hat{n} \, dS$ where $\hat{n}$ denote the unit outer normal to S .